Journal of Vibration, Acoustics, Stress, and Reliability in Design

# Book Reviews

**Nonlinear Oscillations in Physical Systems,**
C. Hayashi,
Princeton University Press, Princeton, N.J.,
1985, 392 pages.

**Reviewed by H. Saunders**

By popular demand, this book is a reprint of the 1964 issue. Nonlinear oscillations revolve about systems whose differential equations cannot be solved exactly. However, great strides have been accomplished in employing approximate solutions. This provides adequate information concerning nonlinear oscillations. This volume is meant to furnish approximate solutions for these nonlinear differential equations. As stated by the author, "That the problems discussed in this book are mostly concerned with forced oscillations; furthermore in most of the text, only systems with one degree of freedom are treated. However, a wide variety of oscillations may occur in these systems under the influence of external forces." This book fulfills the author's intentions and opens up a vast story that makes interesting reading. Although originally pointed towards electrical engineers, it remains a good reference for mechanical engineers plus researchers interested in other disciplines.

The book consists of four parts with a total of 13 chapters, 6 appendices, and an excellent bibliography.

Part I, entitled "Physical methods of nonlinear analysis," contains 3 chapters. The initial chapter presents analytical methods. This includes the powerful perturbation method (autonomous and nonautonomous systems). Examples are provided using Duffing's and Van Der Pol equations. We progress to the iteration method (successive iteration), averaging method (autonomous and nonautonomous), and the principles of harmonic balance with reference to the describing function method. Comparisons are made in the solution of Duffing's equation between perturbation and harmonic balance. Both furnish almost identical answers.

Chapter 2 reports on topological methods which are powerful schemes of investigating various phenomena of nonlinear oscillations. The study of autonomous systems rests upon topological methods. Beginning with integral curves and singular points in the static plane, we continue with the classification of singular points according to the character of the integral curves. The author plunges ahead into canonical forms of the differential equations, singular points of higher order plus an interesting study of limit cycles. The text shows 5 different types of singular points in state space. The next important methods are (*a*) isocline method (graphical), (*b*) Lienard's method dealing with self-excited oscillation, (*c*) higher order approximates (use of eigenvalue and approximations instead of initial values), (*d*) double delta methods, (*e*) slope line method of graphical construction, and (*f*) second order equations of autonomous and nonautonomous systems. Illustrative examples explain these methods using Duffing and Van Der Pol equations. Chapter 3 speaks about stability of nonlinear systems. Liapunov stability opens this chapter. This leads to Routh-Hurwitz criteria, Floquet's theorem, Mathieu's equations, and their respective functions. In the unstable region, Hill's infinite determinant and Whittaker method are the most prominent. The former delves into the stability problem and improved approximation of the characteristic exponent in the odd unstable regions (first and third) and even (second). This chapter contains a large number of illustrative examples.

Part II studies forced oscillations in steady state. Chapter 4 treats stability of periodic oscillations in second order systems. Since the principle of superposition is no longer applicable to nonlinear systems, other means of determining stability is required. Different conditions for stability of second order systems are necessary and encompasses various types of variational equations. It points a finger at the various stability conditions.

Chapter 5 focuses on harmonic oscillations. Here, higher harmonics can be neglected because the fundamental component possesses a period, the same as that of the predominant external force. After deriving the fundamental equation, this continues with periodic states of equilibrium and its stability plus harmonic oscillations with unsymmetric nonlinear characteristics. The chapter concludes with a demonstration of an electrical circuit having unsymmetrical characteristics. Chaper 6 speaks about higher harmonic oscillations. Beginning with higher harmonic oscillations in series-resonance circuits, it migrates to parallel resonance circuits. Chapter 7 deals with important topics of subharmonic oscillations. Relationships between nonlinear characteristics and order of subharmonic oscillations of this study, i.e., quadratic, cubic and fifth order forcing functions are considered. Each of them contribute to a subharmonic order of 1/3. This is extended to dissipative and nondissipative systems. This follows with nonlinear characteristics determined by quintic function. The chapter concludes with nonlinear characteristics represented by a symmetric quadratic function which produces a 1/2 harmonic oscillation. Experimental investigations complement the above subjects.

Part III delves into forced oscillations in the transient state. Chapter 8 opens up with harmonic oscillation and considers periodic solutions and their stability in the transient stage. Harmonic oscillations are analyzed by means of integral curves with and without dissipative systems. In the next chapter, we jump ahead into subcontinua and subharmonic oscillations of order 1/3 and 1/2. Use of previously studied Routh-Hurwitz criterion is applied to stability of periodic solutions. Chapter 10 focuses upon initial conditions leading to different types of periodic oscillations. Initial considerations are given to symmetrical systems and location of fixed points and stability investigations. In turn, one applies this to subharmonic response (1/3 order), harmonic response (no subharmonics) and unsymmetric systems. Chapter 11 dwells upon periodic oscillations. The study includes almost periodic oscillations in a resonant circuit with superimposed direct current, periodic and almost periodic solution. It ends with almost periodic oscillations in a parametric excitation circuit where subharmonic oscillations have order of 1/2. In this part, illustrative examples, numerical solution of the resulting derived equations and experimental investigations comprise this section.

Part IV points out the various aspects concerned with self-oscillations subjected to external forces. Chapter 12 explains entrainment of frequency, i.e., periodic force is applied to a system where free oscillation is of self-excited type. An illustrative example is the Van Der Pol equation containing an additional term for periodic excitation. This follows with harmonic entrainment where the driving frequency is in the neighborhood of unity. We proceed ahead into higher harmonic entrainment and subharmonic entrainment. In all cases, stability investigations and periodic solutions explain in detail the different aspects of the entrainment condition. Chapter 13 considers the subject of almost periodic oscillations. These occur in self-oscillatory systems under periodic excitation. We initiate the topic with Van Der Pol's equation with a forcing term. After completing the analysis, we encounter limit cycles correlated with almost periodic oscillation, transition between external oscillations and almost periodic oscillations, coalescence of singular points at the boundary of harmonic entrainment plus the derivation of a singularity which is a stable focus. We progress to limit cycles correlated with almost periodic oscillations. The authors develops it from higher harmonic oscillation, transition between entrained oscillations and almost periodic oscillations. The chapter concludes with self oscillatory systems with nonlinear restoring force.

Appendix I considers the expansion of Mathieu's function. Appendix II and III delve into the subject of unstable oscillations of Hill's equation and its extended forms. Appendix IV relates the stability criterion by employing the perturbation method. Appendix V focuses upon integral curves, singular points and uses the theorem of Bendixson plus theory of contours. Appendix VI speaks about electronic synchronous switches which proscribe the initial conditions.

In summary, this is a good book. Since almost all nonlinear systems are basically nonlinear, the approximation device (linearization) may range from good to poor. This helps the reader in acquiring a better understanding of nonlinear systems. The reviewer would have preferred seeing a more elaborate section on bifurcation, Nyquist diagrams for nonlinear systems, and employment of Bogoliubov-Metropolsky asymptotic methods. Furthermore, the book should be "updated" to incorporate the digital computer system and associated computer codes. This would definitely enhance the book on this subject.

**Random Vibration of Structures,**
C. Y. Yang,
John Wiley & Sons, New York,
1986, 295 pages.

In the modern day world, most structures and mechanisms are subjected to random forces. The excitation due to dynamic forces of wind pressure, earthquake motion, and ocean waves are usually dealt with in a rather crude fashion. One selects a maximum design load versus time function or the average of several ominous design load time functions. In due consideration both appeals are deficient. At times, they may furnish rational signs of forthcoming load conditions. They do not provide the mechanical or structural engineer with the precise answer to the question of structural safety. The structural response can be highly random in intensity and could contain a wide frequency band. This results in making the problem of structural safety very complex, unwieldy, and hard to solve. The present assumption of mass, stiffness, damping, and failure rules (yield and fatigue) falls within the category of random vibration analysis. It describes the safety of the structure in a probabilistic sense and is most valuable for structural design and planning. As stated by the author, "For practicing structural engineers, the text provides an understanding of the basic concepts and some application of random vibration of structures to enable them to tackle practical problems and to follow the research literature." The author follows his intentions with great vigor.

The book consists of 10 chapters and 3 appendices.

Chapter 1 introduces the idea of random vibration and how it originates. This follows with an interesting discussion of random processes, probability, and statistics. This encompasses frequency definition of probability, probability density, joint probability density, and conditional probability. The statistics entail variance, standard deviation, correlation and its coefficients plus covariance.

Chapter 2 continues with stationary random processes, autocorrelation, and spectral density. The autocorrelation function is defined which then leads to Fourier series and Fourier integral representation. This includes Parseval's relationship. We next meet power spectral density and its kinship with autocorrelation. A slight mention is made of cross correlation but none of cross spectral density. Chapter 3 computes the introduction to random vibration. Beginning with ergodic processes (temporal statistics) and its mathematical relationships, we proceed to autocorrelation and spectral density of one sample function plus an alternative definition of spectral density function directly from the sample function.

Chapter 4 dedicates itself to 3 models of random excitation experienced by a structural system. We superimpose simple fundamental modes with various deterministic parameters and random variables. Finally, we determine the statistical and probabilistic characteristics of the final model. The initial model is a stationary random roadway with mathematical derivations. It starts out with the discrete model and proceeds along to the continuous model. The second is random earthquake motion and is nonstationary and contains both discrete and continuous models. The third is random ocean waves (multivariable stationary model) containing both discrete and continuous models. The spectral density of waves forces conclude the chapter. A most interesting chapter!

Chapter 5 begins the study of the relationship of the transfer or transmission relationship of the random vibration of structures. We initiate this concept by studying the mathematical relationships of the single degree of freedom (SDOF) system. The two solutions are (*a*) frequency domain (FD) and, (*b*) time domain (TD). The frequency response function is determined in FD and employed in a number of illustrative examples plus integration in the complex plane. TD considers an excitation as a superposition of impulses using Duhamel's in-