

## An Accurate Solution of the Gas Lubricated, Flat Sector Thrust Bearing<sup>1</sup>

V. Castelli.<sup>2</sup> the authors present useful information about the equilibrium characteristics of Flat Sector Bearings operating with compressible lubricants.

For the purpose of clarification this writer would like to ask the following questions:

1) What grid spacings and convergence criteria were used in the numerical solution?

2) What accuracy checks were employed aside from the comparison with a rectangular slider solution?

3) What results are presented in reference [6] that are not presented here?

4) The range of compressibility numbers covered in the paper is very small since it barely enters the compressibility region (the largest presented deviation from the incompressibile solution is forty percent in the paper). Is this done because higher values are deemed practically useless or because numerical difficulties barred the road?

5) Is the generating computer program maintained and available for general use?

The authors' patience in providing the answers is deeply appreciated.

# **Authors' Closure**

The authors wish to thank Professor Castelli for his comments and interest in the paper. The answers to his questions follow.

(1) The grid spacing was 14 mesh points radially and 20 circumferentially. Between successive iterations the change in the variable  $Q = (PH)^2$  at each grid point is obtained. The largest change is then tested against a convergence criterion which was  $10^{-5}$ .

(2) Different grid spacings were examined to check their effect on the solution. It was found that a mesh as coarse as 8 by 12 gave results for the pressure distribution over the pad area which differed trivially from pressure values calculated with the finer mesh size. The overall performances were also compared with the results of reference [4] in those cases where the pads were the same. (3) Bearing performance characteristics in the same form as in this paper are presented in reference [6] for the whole range of radius ratios 0.3, 0.5, and 0.7 at pad angles of 30, 45, and 60 degrees.

(4) The range of compressibility numbers was selected from practical considerations [10]. No attempt was made to solve for compressibility numbers higher than 100. However, no numerical difficulties were encountered in the range covered. Convergence was usually achieved after 40 to 50 iterations and took about 1 to 2 seconds for each case.

(5) The computer program is available from COSMIC, Barrow Hall, University of Georgia, Athens, GA 30601. The listing and instructions for use are in [8], which has been published, subsequent to the preprinting of this paper, as NASA TM X-73595, 1977.

#### **Additional Reference**

10 Etsion I., A Cantilever Mounted Resilient Pad Gas Thrust Bearing," JOURNAL LUBRICATION TECHNOLOGY, TRANS. ASME, Series F, Vol. 99, No. 1, Jan. 1977, pp. 95–100.

#### Dynamic Life Estimation for Track Surfaces under Periodic Loadings and Nonuniform Backup Supports<sup>1</sup>

**J. Y. Liu.**<sup>2</sup> The authors are to be commended for using a relatively simple procedure to solve the complicated problem of estimating the fatigue life of a roller track system with complex geometry.

For a track support with complex cross sectional design, the load deflection relationship can only be analyzed using a numerical approach, such as the finite element method employed by the authors. In the sample problem of the paper, the track support is approximated by an infinite cantilever plate, which, however, does not possess a complex cross sectional shape. The problem of an infinite cantilever plate carrying a concentrated load was solved by T. J. Jaramillo (see p. 336 of reference [7] of the paper). For a group of concentrated loads acting on the plate, the solution can simply be obtained by applying the principle of superposition. It would be of interest if the authors could compare their finite element solution with that obtained by Jaramillo's method.

It will be recalled that the Lundberg and Palmgren fatigue life theory is based on the assumptions that in a rolling contact the contacting bodies are of the same material and the risks of fatigue failure for them are equally great. If the roller and the track considered by

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<sup>&</sup>lt;sup>1</sup> By I. Etsion and D. P. Fleming, published in the January, 1977, issue of the JOURNAL OF LUBRICATION TECHNOLOGY, TRANS. ASME, Series F, Vol. 99, pp. 82–88.

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<sup>&</sup>lt;sup>1</sup> By M. J. Hartnett and A. N. Palazotto, published in the October, 1976, issue of the JOURNAL OF LUBRICATION TECHNOLOGY, TRANS. ASME, Vol. 98, pp. 602–606.

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the authors are of the same material, it would be meaningful to estimate the life of the whole assembly.

# **Authors'** Closure

The authors wish to thank Dr. Liu for his discussion. The discussor suggests that the agreement between deflections of the cantilever plate predicted by the finite element method be compared to the results of the explicit solution. We agree that whenever it is possible to make such comparisons that they be made. However, the example considered was chosen in order to demonstrate the technique of analysis rather than solve a particular problem and for this reason comparisons with Jaramillo's results were not made. In practice, where more complex cross sections are normally encountered, explicit form results are often precluded. In these instances characteristics of the load distribution can be found with relative ease by employing the methods indicated herein.

With regard to the question of assembly life estimate, life prediction for the assembly is an important design consideration and can be determined by computing individual component lives and combining these as demonstrated in equation (17).

### First Effects of Stokes Roughness on Hydrodynamic Lubrication<sup>1</sup>

**K. Tonder.**<sup>2</sup> The authors are to be congratulated for having attacked a very important problem in lubrication. Their work is an impressive piece of mathematics and puts a question-mark to some of the basic assumptions implicit in Reynolds' equation.

However, such a question-mark must also, according to this discussor, be put on their results. The latter are closely tied up with expressions (37). These are in fact the decisive factor in the deviation from previous roughness theories, allowing an infinite increase in the load capacity.

The authors are aware of this since they write, "Expressions (37) might leave the impression that the roughness effects could become unbounded if  $\beta$  or  $\lambda$  were chosen arbitrarily small. But these parameters are closely tied with the roughness spacing. If the roughness spacing is smaller than the film thickness, the RHS of equations (2) and (3) are dominant and then the iterative method fails. In fact, our results should be increasingly better when the RHS of equations (2) and (3) are increasingly small."

Certainly these parameters are tied up with the roughness spacing, and so the approach would fail for a very dense roughness pattern. But basically the applicability of the results cannot be assessed from the physics of the problem because it is the mathematical *form* of (37) that is important. Which parameter values are permissible?

This discussor finds it very unlikely that the inclusion of the Stokes terms in (2) and (3) would produce an infinite load capacity, even theoretically, when the roughness spacing tends to zero. Therefore, since formally expressions (37) may become unbounded, an additional correction must also have the possibility of tending to infinity with decreasing roughness spacing, in order to cancel the unboundedness. But then this term should also be included in the solution.

This has to do with the possibility that the iterative process may be nonconvergent. This has not been considered by the authors. What is the reason for this?

Alternatively, assume that the process is (rapidly) convergent with a proper initial iterant.

In order that the load expression of (31) may be bounded, its two last bracketed terms (or their "correct" equivalents) must be of the order of  $\beta^2$ ,  $\lambda^2$  and  $\beta^4$ ,  $\lambda^4$ , respectively. This means that one is facing a case of subtraction of terms consisting of components of very different orders of magnitude.

As is well known this may cause serious errors if the terms involved are not extremely accurate. Though the pressure profile expressed by (7) may not deviate much from the correct one satisfying (1)-(3), it may be quite inadequate as an initial iterant because of the high derivatives appearing in the expressions—which may deviate widely from the true value—and, possibly, also because of its combination with the velocity expressions of (7).

The discussor has not been able to find any formal error in the analysis, and though he has doubts about the correctness of the direct results, he feels that in many ways the approach is interesting and should be pursued further.

The discussor would appreciate the authors' comments on the above suggestions.

## **Authors' Closure**

The authors commend Professor Tonder for the reiteration of the validity and limitations of their results. The main contribution of the paper was represented by equations (31) and (32), while in equation (36) a general form of the autocorrelation function (ACF) of roughness height was postulated. As stated in the paper and also pointed out by Professor Tonder, the two parameters  $\beta$  and  $\lambda$  used to characterize the ACF relate directly to roughness spacing. They are of the same order of magnitude of  $\epsilon$  and can not be infinitesimal.

The use of the Stokes equations inherently assumes that continuum mechanics applies and inertia terms are negligible. It is the iteration scheme used in the paper to solve the Stokes equations which imposes a limitation on the smallness of  $\beta$  and  $\lambda$ . Thus we are not considering the case of roughness frequency approaching infinity. The parameters  $\beta$  and  $\lambda$  dictate the rate of convergence of the iteration solution. For large  $\beta$  and  $\lambda$ , the Stokes terms of the solution diminish and the Reynolds solution is resulted. The iteration solution is not valid for very small  $\beta$  and  $\lambda$  (compared to  $\epsilon$ ) as discussed in the paper. Professor Tonder is right that the problem of convergence of the iterative solution should be pursued further.

<sup>&</sup>lt;sup>1</sup> By Dah-chen Sun and Kuo-Kuang Chen, published in the January, 1977, issue of the JOURNAL OF LUBRICATION TECHNOLOGY, TRANS. ASME, Series F, Vol. 98, pp. 2–9.

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