5 Dowson, D., and Higginson, G. R., *Elastohydrodynamic Lubrication,* Pergamon Press, 1966.

6 Cameron, A., Gohar, R., "Theoretical and Experimental Studies of the Oil Film in Lubricated Point Contact," *Proceedings of Royal Society,* London, Series A, No. 291, 1966, pp. 520–536.

7 Dowson, D., Elastohydrodynamics, *Proceedings of the Institution of Mechanical Engineers,* Vol. 182, Part 3A, 1968, pp. 151–167.

8 Archard, J. F., and Cowking, E. W., "Elastohydrodynamic Lubrication of Point Contacts," *Proceedings of the Institution of Mechanical Engineers,* Vol. 180, Part 3B, 1965–1966, pp. 47–56.

9 Ranger, A. P., Ettles, C. M. M., and Cameron, A., "The Solution of the Point Contact Elasto-Hydrodynamic Problem," *Proceedings of Royal Society,* London, Vol. 346, 1975, pp. 227–244.

## DISCUSSION

### R. K. Kunz[2] and W. O. Winer[2]

The authors are to be commended for developing usable film thickness formulas from their complete numerical solution of the isothermal point contact problem as described in their previous papers. Such formulas are not only useful for design applications, but also provide a convenient means for assessing the effects of the various parameters on the EHD film thickness.

Tables 5 and 6 indicate that the authors' formulas for minimum and central film thickness (equations (28) and (33), respectively) yield results accurate to within 10 percent of the values computed from their more complete numerical analysis. However, one should not expect to obtain such accuracy when comparing calculated film thicknesses to those found in actual EHD contacts. For the purposes of making such a comparison, Tables 7 and 8 were constructed based on film thickness measurements made in the tribology laboratory at Georgia Tech. The experimental apparatus consisted of a steel ball which was rolling and sliding on a sapphire plate (i.e., $k = 1$). Measurements were made using the optical interference technique described in [10].[3] Table 7 shows the results of both calculations and measurements for three lubricants in pure sliding, each under two different loads and three speeds. Using the authors' notation, $\tilde{H}_c$ and $\tilde{H}_{\min}$ denote the dimensionless film thickness calculated from equations (28) and (33), respectively. $H_c$ and $H_{\min}$ are the measured dimensionless center and minimum film thickness. Figs. 10 and 11 show comparisons between the calculated and measured film thicknesses for the two loads.

For the low load case ($W = .1238 \times 10^{-6}$), Fig. 10, the results compare quite well, although the calculations are not in general within 10 percent of the measurements. Clearly, however, the experimental error in taking the measurements must also be taken into account in assessing the validity of the calculations. The ratios between center and minimum film thickness are similar for the calculations and measurements, and the dependence of film thickness on speed appears to be well represented by the authors' equations.

For the high load case ($W = .9287 \times 10^{-6}$) Fig. 11, the agreement is not as good, with the equations predicting consistently larger film thicknesses than measured. There are several possible explanations for this result at the higher load. Because the measurements were made in a condition of pure sliding, thermal effects may enter at the high load. The value of viscosity used in the calculations of Table 7 was that at the temperature of the bath. If thermal effects become important, the authors' isothermal assumption is violated, and the value of the viscosity to be used in their formulas is somewhat arbitrary. In addition, at the more severe conditions imposed by the higher load, the lubricant may no longer behave as Newtonian liquid, which would violate the authors' assumptions. The point is that care must be taken in applying the authors' film thickness formulas to actual EHD contacts. The conditions in the contact must be examined to see how well they match the assumptions underlying the formulas derived by the authors.

Table 8 shows the results of calculations and measurements for a single lubricant in pure rolling under several loads and speeds. A comparison between calculations and measurements is shown in Fig. 12. For this case, calculated film thickness tends to be lower than measured, although the agreement is quite good. The inclusion of some sliding does not affect the calculations as long as $u$ remains

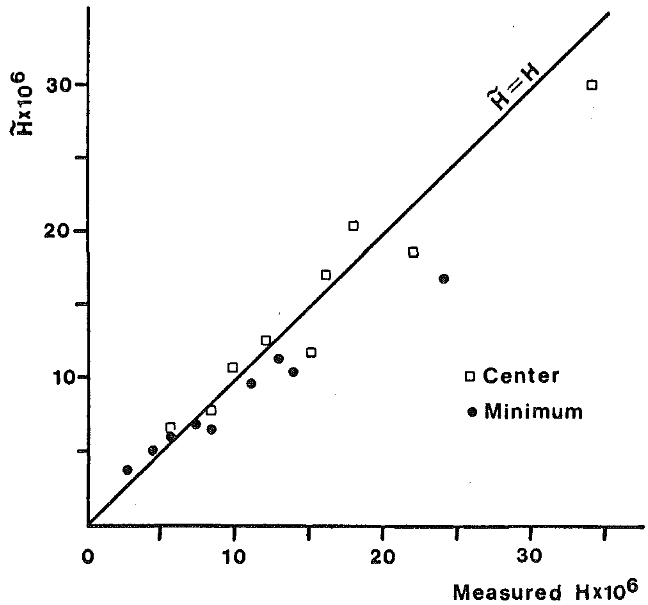


Fig. 10 $\tilde{H}$ versus $H$ for pure sliding, $W = .1238 \times 10^{-6}$

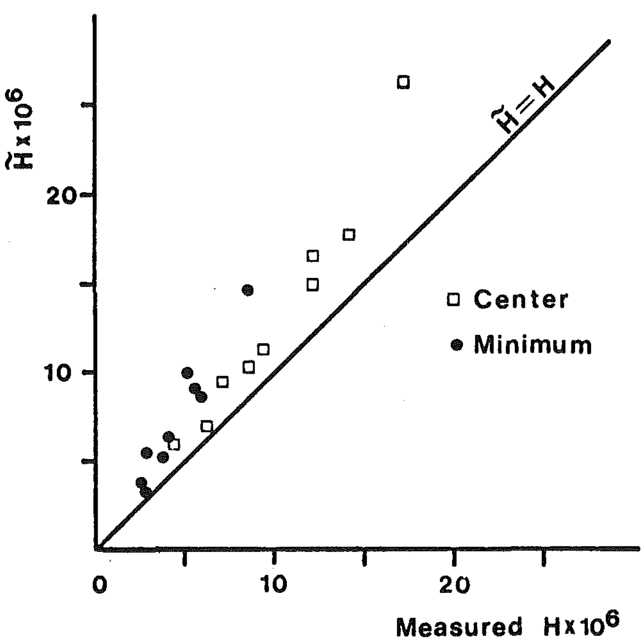


Fig. 11 $\tilde{H}$ versus $H$ for pure sliding, $W = .9287 \times 10^{-6}$

[2] Georgia Institute of Technology, Atlanta, Ga.

[3] Number 10 in brackets designates an Additional Reference at end of discussion.

Table 7 Comparison between calculated ($\tilde{H}_c$, $\tilde{H}_m$) and measured ($H_c$, $H_m$) film thickness

| Lubricant | U | W = .1238 x $10^{-6}$ | | | | W = .9287 x $10^{-6}$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | $\tilde{H}_c$ | $H_c$ | $\tilde{H}_m$ | $H_m$ | $\tilde{H}_c$ | $H_c$ | $\tilde{H}_m$ | $H_m$ |
| Polyalkyl Aromatic | .1963 x $10^{-11}$ | 6.64 x $10^{-6}$ | 5.7 x $10^{-6}$ | 3.87 x $10^{-6}$ | 2.8 x $10^{-6}$ | 5.96 x $10^{-6}$ | 4.3 x $10^{-6}$ | 3.33 x $10^{-6}$ | 2.8 x $10^{-6}$ |
| $\alpha$ = 1.58 x $10^{-8}$ $(N/m^2)^{-1}$ | .3926 | 10.9 | 9.9 | 6.20 | 5.7 | 9.50 | 7.1 | 5.35 | 4.3 |
| $\eta$ = .0255 $Ns/m^2$ | .7866 | 17.3 | 16.0 | 9.93 | 11.0 | 15.1 | 12.0 | 8.58 | 5.7 |
| G = 4507 | | | | | | | | | |
| Synthetic Hydrocarbon | .2637 | 11.9 | 15.0 | 6.58 | 8.4 | 10.4 | 8.4 | 5.68 | 2.8 |
| $\alpha$ = 3.11 x $10^{-8}$ $(N/m^2)^{-1}$ | .5274 | 18.9 | 22.0 | 10.5 | 14.0 | 16.6 | 12.0 | 9.11 | 5.6 |
| $\eta$ = .0343 $Ns/m^2$ | 1.057 | 30.2 | 34.0 | 17.0 | 24.0 | 26.4 | 17.0 | 14.6 | 8.4 |
| G = 8874 | | | | | | | | | |
| Modified Polyphenyl Ether | .2268 | 8.04 | 8.2 | 4.53 | 5.0 | 7.01 | 6.3 | 3.92 | 2.6 |
| $\alpha$ = 1.79 x $10^{-8}$ $(N/m^2)^{-1}$ | .4536 | 12.8 | 12.0 | 7.27 | 7.5 | 11.2 | 9.4 | 6.27 | 3.8 |
| $\eta$ = .0295 $Ns/m^2$ | .9089 | 20.6 | 18.0 | 11.6 | 13.0 | 17.8 | 14.0 | 10.1 | 5.0 |
| G = 5107 | | | | | | | | | |

Table 8 Comparison between calculated ($\tilde{H}_c$, $\tilde{H}_{min}$) and measured ($H_c$, $H_{min}$) film thickness-pure rolling

| U | W | $\tilde{H}_c$ | $H_c$ | $\tilde{H}_{min}$ | $H_{min}$ |
|---|---|---|---|---|---|
| .2433 x $10^{-11}$ | .4643 x $10^{-6}$ | 8.57 x $10^{-6}$ | 11.3 x $10^{-6}$ | 4.76 x $10^{-6}$ | 4.8 x $10^{-6}$ |
| .3643 | .4643 | 11.2 | 13.9 | 6.27 | 5.6 |
| .3643 | .2613 | 11.7 | 15.1 | 6.55 | 11.3 |
| .3643 | .9287 | 10.7 | 13.9 | 5.96 | 4.8 |
| .2470 | .9287 | 8.27 | 11.3 | 4.57 | 4.8 |
| .3507 | .9287 | 10.5 | 13.9 | 5.81 | 4.8 |
| .4866 | .9287 | 13.0 | 15.1 | 7.25 | 8.0 |
| .6076 | .9287 | 15.1 | 17.6 | 8.44 | 11.2 |

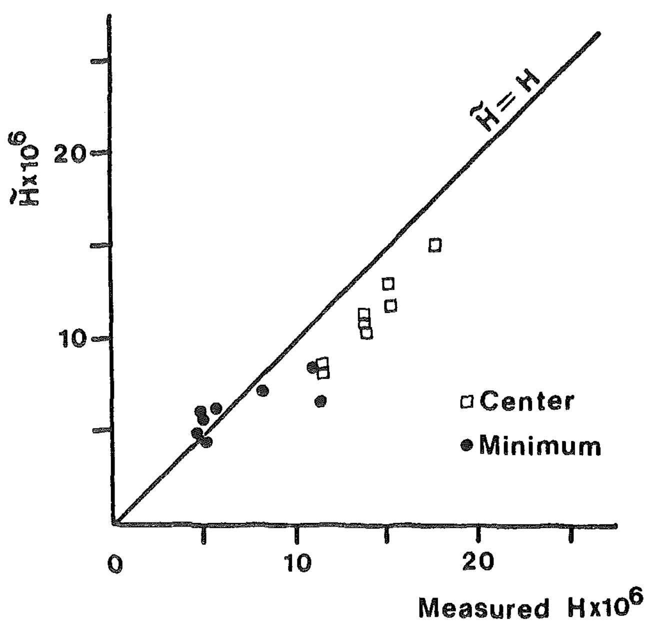


Fig. 12 $\tilde{H}$ versus $H$ for pure rolling

constant, while the measured film thickness decreases slightly. However, the results for combined rolling and sliding appear similar to those in Fig. 12.

In reference to Fig. 3(*b*) for $k = 1.25$, the authors make the statement that the bearing surfaces are not parallel in the central region of the contact due to the inclusion of lubricant compressibility in the theory. We believe this statement is misleading to a certain extent. The film profile results from the elastic deformation of the solid surfaces, and therefore is dependent on the pressure distribution acting on the surfaces. The pressure, in turn, is determined by an interaction of the compressibility, the film shape, lubricant properties, speed, load, and other factors. The film thickness as a whole may be reduced by the inclusion of compressibility. However, we would not expect the addition of compressibility to cause the relatively large local changes in surface deformation required to transform a parallel film shape to that shown in Fig. 3(*b*).

In conclusion, the authors are to be congratulated for their thorough treatment of a most difficult problem. The film thicknesses calculated from their formulas agree quite well with the point contact film thickness measurements available to us, and should prove to be a valuable asset to workers in the field.

## Additional Reference


10 Sanborn, D. M., and Winer, W. O., "Fluid Rheological Effects in Sliding Elastohydrodynamic Point Contacts with Transient Loading: 1-Film Thickness" JOURNAL OF LUBRICATION TECHNOLOGY, TRANS ASME, Series F, Vol. 93, No. 2, 1973.


## Author's Closure

The authors would like to thank Professors Kung and Winer for their full and interesting comparison between their experimental findings and the theoretical predictions presented in our paper. The agreement is most encouraging.