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DISCUSSION

J. C. Nicholas[3] and P. E. Allaire[3]

This work represents a welcome addition to the literature on stepped bearings with a more practical outlook than some of the previous studies. Optimization of finite sliders has been carried out but only limited applications exist.

It should be noted that this analysis neglects the pressure drop at the step due to inertia effects [10, 11],[4] which has been shown to be significant even when the Reynolds' number for the slider is small. The pressure profile is strongly affected but often the load capacity is fairly close due to the integration of pressures above and below the laminar results. Thus, the results given here should be taken as an approximation to the optimum sector thrust bearing. At that it still represents the best treatment currently available.

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I. Etsion[5]

The authors mention an optimum pad angle β of 150° for radius ratio of 0.5. They also find that for radius ratio of ⅓ an optimum has not been reached even at pad angle β of 180°. Do the authors have any physical explanation for this behavior?

S. M. Rohde[6] and G. T. McAllister[7]

The authors are to be congratulated on their attack of a classical problem in hydrodynamic lubrication theory—the optimum stepped sector pad profile. Their use of pattern search methods furthermore introduces another technique to the lubrication field. These methods may be of use in other lubrication problems.

A minor point is the interpretation of the stiffness characteristics of the optimum geometry. As is well known the load capacity of a bearing varies essentially as the reciprocal of the square of the minimum film thickness. We use the word essentially to caution that a change in the minimum film thickness changes the load variable since the (machined) profile remains constant. Hence optimum or near optimum profile configurations which are designed to operate at a specific minimum film thickness sometimes do not have good overload characteristics as discussed in [12].[8]

Could the authors also comment on the effect of mesh size on their results?

Recently we also have examined some bearing profile optimization problems in detail [13, 14]. In that work we have devised some new algorithms for treating this class of problems. The latter fall into the category of distributed parameter optimization problems.

The algorithms presented in [13, 14] make no a-priori assumptions regarding the shape of the profile. Both finite difference and finite element discretization methods (as well as other methods) can be used for the constructions. We will now sketch the algorithm. Complete details, proofs, and results can be found in the references. For brevity we will present the equations in the continuous rather than the discrete forms and in Cartesian coordinates. The reader should note that the continuous problems shown are to be simply replaced by appropriate discretizations.

The Problem: Find $H^* \geq 1$ which maximizes

$$L(P_H) = \int\int_\Omega P_H(X, Y)dXdY \qquad (1)$$

over all $H \geq 1$ and when the pressure, P_H, satisfies

$$\nabla \cdot H^3 \nabla P_H = \frac{\partial H}{\partial X}, \qquad (X, Y)\epsilon\Omega \qquad (2)$$

$$P_H \rfloor_{\partial\Omega} = 0,$$

and where Ω is the pad area.

The Algorithm:

(1) Choose an initial film profile H_0 such that $P_{H_0} \geq 0$ ($H_0 \equiv 1$ is usually our choice), set $R = 0$ and $\tilde{P}_0 = P_{H_0}$.

(2) Solve the "squeeze film" problem for P_{1H_k}

$$\nabla \cdot H_k{}^3 \nabla P_{1H_k} = -1, \qquad (X, Y)\epsilon\Omega$$

$$P_{1H_k}\rfloor_{\partial\Omega} = 0.$$

(3) Let $H_{k+1}(X, Y)$ be that value of Z, $1 \leq Z \leq M$ which maximizes $F(Z)$ at each value of (X, Y) where

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[6] Research Laboratories, General Motors Corp., Warren, Mich.
[7] Lehigh University, Bethlehem, Pa.
[8] Numbers 12–14 in brackets designate Additional References at end of discussion.

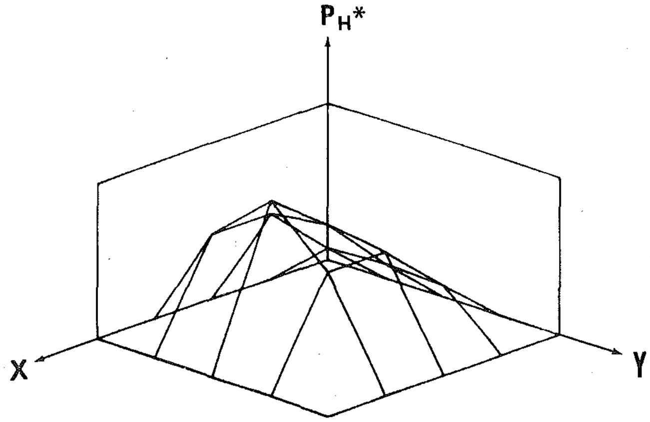

Fig. 7(*a*) Optimum pressure distribution, $d = 1/4$

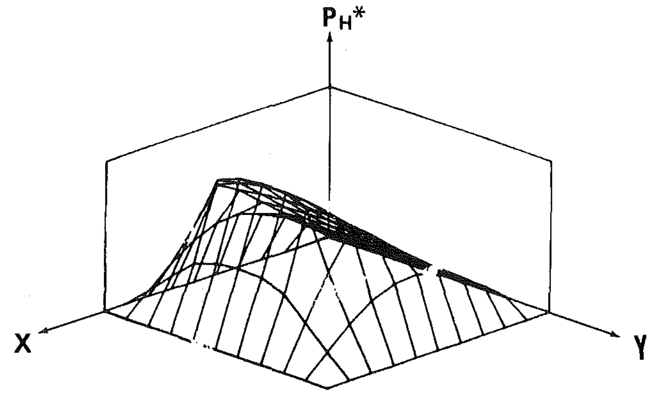

Fig. 8(*a*) Optimum pressure distribution, $d = 1/10$

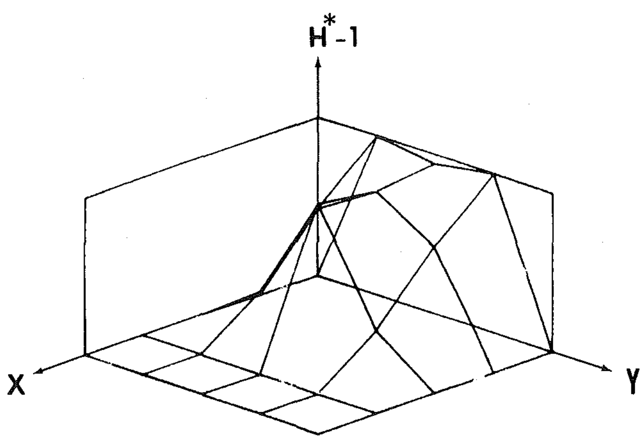

Fig. 7(*b*) Optimum film shape, $d = 1/4$

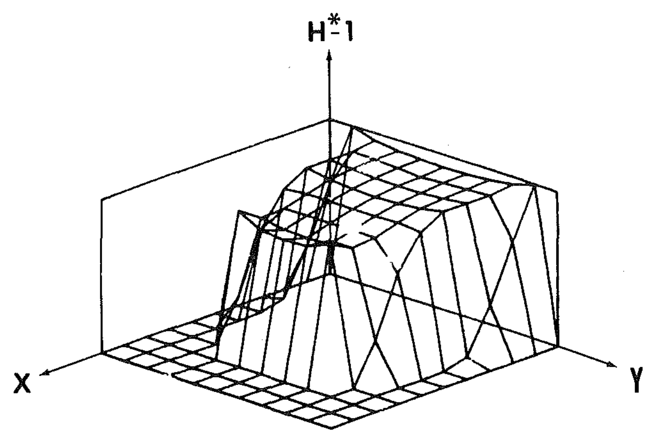

Fig. 8(*b*) Optimum film shape, $d = 1/10$

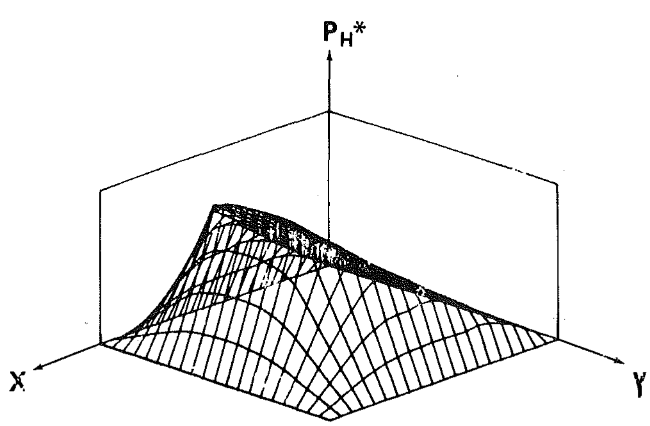

Fig. 9(*a*) Optimum pressure distribution, $d = 1/20$

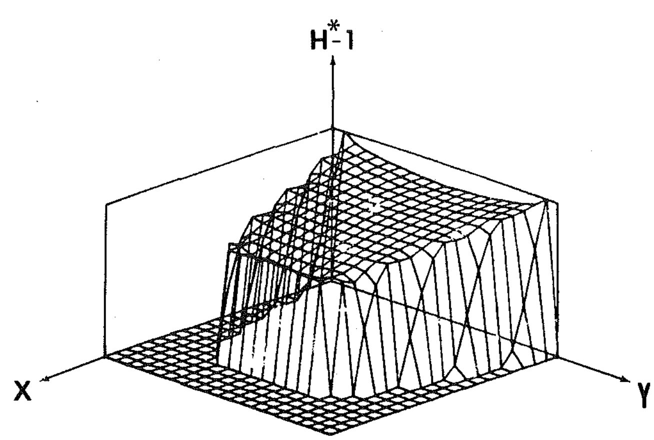

Fig. 9(*b*) Optimum film shape, $d = 1/20$

$$F(Z) = -Z^3\nabla\tilde{P}_k \cdot \nabla P_{1H_k} + Z\frac{\partial P_{1H_k}}{\partial X}.$$

(4) Let $\xi(X, Y)$ be the solution of

$$\nabla \cdot H_k{}^3\nabla\xi = -\frac{1}{\gamma}\left\{\nabla \cdot H_{k+1}{}^3\nabla\tilde{P}_k - \frac{\partial H_{k+1}}{\partial X}\right\}, \quad (X, Y)\epsilon\Omega$$

$$\xi\rfloor_{\partial\Omega} = 0.$$

Set $\tilde{P}_{k+1} = \tilde{P}_k + \xi$.

(5) Check for convergence, i.e.,

$$\text{is } \|H_{k+1} - H_k\| < \epsilon_1$$

$$\text{and } \|\tilde{P}_{k+1} - \tilde{P}_k\| < \epsilon_2.$$

(6) If the convergence criterion is satisfied then $H^* = H_{k+1}$ and $P_{H^*} = \tilde{P}_{k+1}$. If not, set $k = k + 1$ and go to step (2).

In this algorithm M and γ are constants as discussed in [13, 14]. M is usually selected to be about 3 whereas γ is reduced as the iteration proceeds.[9]

This algorithm was found to be extremely efficient, requiring only two solutions of the differential equation per iteration. Likewise few iterations were required to obtain four place accuracy. H was allowed as many as eight hundred degrees of freedom in some of these computations!

Figs. 7–9 show some finite difference results for a square slider obtained using this algorithm for decreasing mesh size d. Note how a pocket *naturally* forms. No assumptions regarding shape have been made.

The extensions of these methods to other areas are currently being pursued by the discussors. In particular the optimization of finite journal bearings is straightforward using this type of algorithm.

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[9] Starting with $\gamma = 100$, γ can be reduced to 2 within a few iterations.

14 Rohde, S. M., and McAllister, G. T., "On the Optimization of Fluid Film Bearings," *Proceedings of the Royal Society of London*, Series A, Nov. 1976.

Authors' Closure

We are grateful to the discussors for their interest and comments about this work and appreciate the various questions they have raised. Our reply to Professors Nicholas and Allaire is that we recognize the inertia effects at a step 1) may influence significantly the details of the pressure profile near the step, and 2) may affect the estimate of power loss. Our investigation, however, did not include such effects on the assumption that they were localized and would not influence materially the load capacity of the bearing. We understand that work on inertia effects is being carried out by other investigators and we await the results of their work. Perhaps the nature of these results will suggest those aspects of stepped bearing solutions, so far obtained, which should be re-examined.

In reply to Dr. Etsion, we note that the optimum bearing dimensions correspond approximately to a given "length to width ratio" of 3.5–4.0. We cannot attach any particular physical significance to this result; we note only that it happened.

Drs. Rohde and McAllister have described a useful algorithm for the determination of optimum bearing profiles. We look forward to the publication of the work upon which much of their discussion is based. The stiffness characteristics of the optimum sector pad were determined numerically and it was verified that the load capacity does vary inversely as the square of the minimum film thickness. Our conclusion is that the optimum profiles possess the ability to sustain overloads better than other configurations. The mesh size was selected on a trial-and-error basis. Further refinements did not produce significantly different numerical results which already showed substantial agreement with selected other published results. In fact, further reductions in mesh dimensions introduced an unexpected problems with the pattern search subroutine. As the "rectangles" became smaller, numerical errors produced an artificially increasing load capacity and the search method caused some of the "rectangles" to collapse in one dimension in order to find a "higher load capacity." The mesh size we selected represents a good compromise and we are confident about the validity of our results.