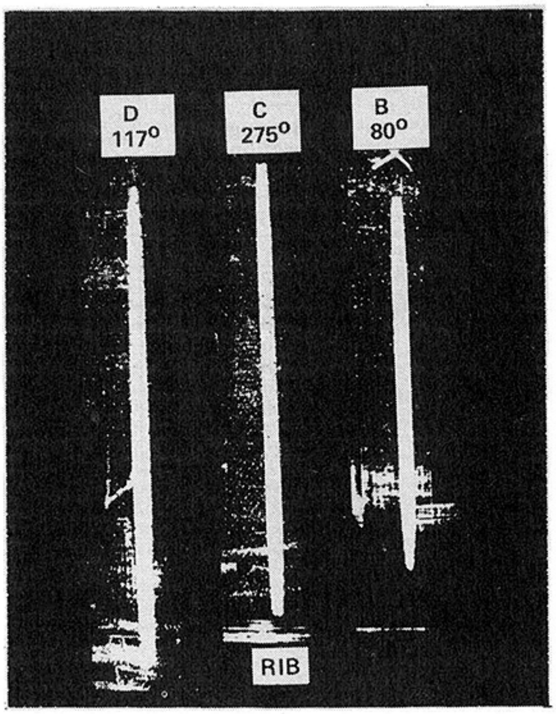

Fig. 6 Heavily loaded contacts in reverse stall loading condition for bearings B, C and D

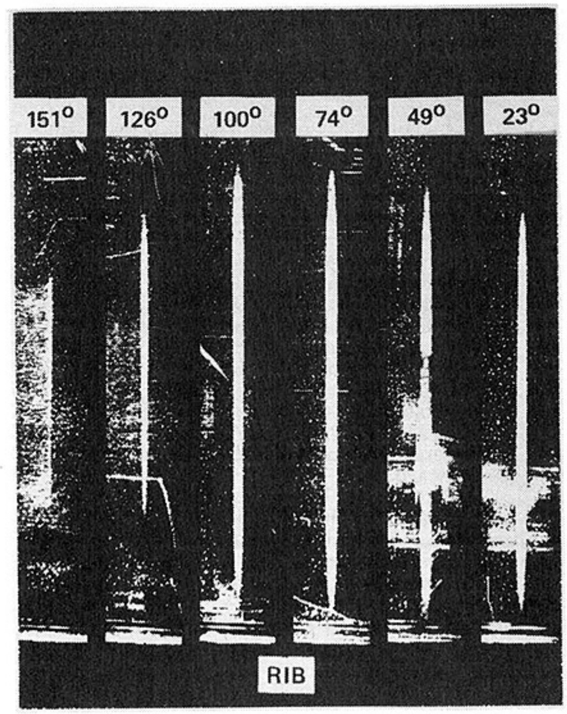

Fig. 7 Contact pattern on bearing C in forward stall loading condition

tions, shows misalignment of a peculiar nature. In forward stall loading condition (Fig. 7), the rollers at 23° and 49° appear to have been pinched at the end close to the rib, the roller at 74° (which is also the most heavily loaded roller) shows no misalignment, and the rollers at 100°, 126° and 151° locations are pinched at the end opposite to the rib. This kind of misalignment in a plane different from the loading plane, if severe, could cause cyclic thrust loading between the rollers and the rib.

Conclusions

1 In all the bearings in the system, the bearing load calculated by utilizing the Contact Pattern Analysis technique agrees well with the load imposed on the bearing by the system, demonstrating the accuracy of the technique.

2 In some cases, the load distribution among rollers in the bearing and in one case the number of rollers loaded, differ significantly from theoretical.

3 In eleven of the sixteen cases the direction of the bearing reaction is significantly different from the applied loading direction.

4 In seven of the sixteen cases, misalignment as calculated from the footprints is .003 to .005 mm/mm which is greater than was anticipated at the design stage.

5 There is evidence of contact truncation and roller edge loading in the stall loading conditions which was not anticipated at the design stage.

6 This systems approach, utilizing the Contact Pattern Analysis technique revealed anomalies that suggest tailoring system parameters, such as housing stiffness and/or roller crown, in order to provide improved performance. These anomalies could not have been discovered by other known, practical, theoretical or experimental techniques.

References

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4 Harris, T. A., *Rolling Bearing Analysis,* Wiley, New York, 1960, pp. 153–155.

5 Harris, T. A., "The Effect of Misalignment on the Fatigue Life of Cylindrical Roller Bearings Having Crowned Rolling Members," JOURNAL OF LUBRICATION TECHNOLOGY, TRANS. ASME, Series F, Vol. 91, No. 2, Apr. 1969, pp 294–300.

DISCUSSION

J. H. Rumbarger[2]

The authors are to be congratulated upon the development of an excellent experimental technique and also for their evaluation which derives much useful design information from the "footprint" study.

In terms of the experimental approach, the method of using the black oxide etch avoids the need for an objectionable etching gas. The use of a flash copper plate exposed to hydrogen sulfide (rotten egg smell) gas has been used successfully in the past. Some success has also been achieved with a light silver plate exposed to an amonia gas. Both gas techniques have objectionable odors and require containment of the gas to the immediate area of the loaded ball or roller.

A gear mesh usually requires some torsional wind up to load the gear teeth especially if one shaft is held and the other shaft is torqued to

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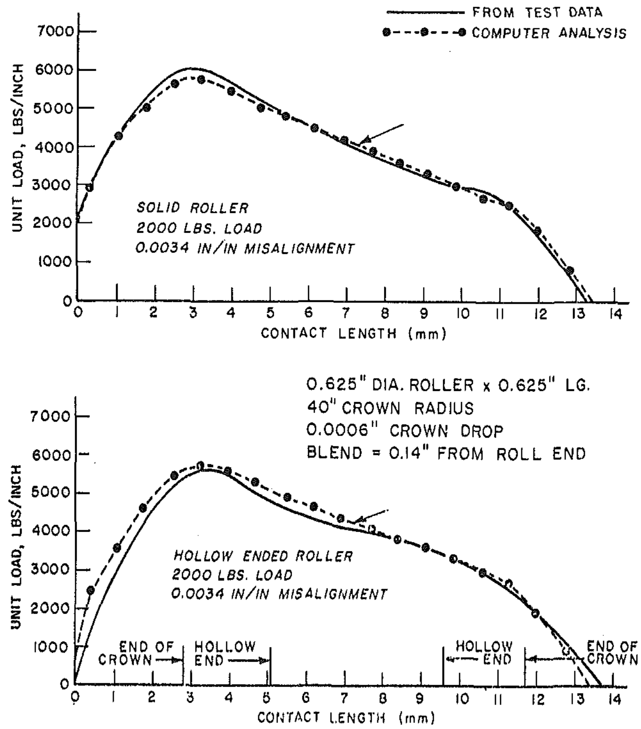

Fig. 8 Roller load analysis comparison with rolling data

apply the load. There is no evidence of smearing of the footprints in the present study. How was the load applied in order to avoid rolling and subsequent smearing of the footprints?

The authors describe some difference between the externally applied load system and the reactions from the footprints. A loaded roller is an excellent load-gage and the authors have demonstrated good correlation with etch correction factors. In the case of differences I would believe the footprint data. Variation in bearing support stiffness can certainly be expected to show differences in the individual bearing internal load distributions.

The paper describes roller edge loading in Figs. 5 and 6 under stall conditions. This is edge loading in the sense that the roller is loaded out to one edge or roller end similar to the load distribution of Fig. 8. Edge stress increases which are detrimental occur when the maximum roller load is at the roller end of contact. All of the footprints of Figs. 5 and 6 show a slight but measurable narrowing at the end of the footprint and the widest portion occurs at the crown blend point. This certainly does not seem to be detrimental especially under stall conditions.

The authors comment that the results achieved could not have been achieved by any other known analytical technique. This is certainly true in a cost effective sense. Sophisticated finite element computer modeling of the shaft, bearings, and housings in conjunction with a complete rolling bearing analysis can determine the effects of support stiffness (reference [6]).[3] Analytical techniques can correlate (reference [7]) analysis and experimental footprints. Fig. 8 shows good agreement between analysis and etched footprint data per (reference [9]). The analysis applied a normal load and angular misalignment to a roller and crown of known dimensions.

The use of both analytical and experimental techniques should be used for cost-effective structures-bearing studies. The differences between an idealized computer solution (no out-of-round, out-of-flat or angular twist) can be compared with the experimental results of the present study. The differences will describe relative structural effects. The computer analysis can then be altered by constants at each roller location to describe noncircular, nonflat rings. This procedure is described in (reference [6]) the only difference being that the structural effects are now determined experimentally. The analysis could then be used to determine optimum roller crowns and internal diametral clearance by means of an inexpensive parametric analysis.

[3] Numbers in brackets designate Additional Reference at end of discussion

Additional References

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7 Rumbarger, J. H. and Jaskowiak, M. J., "GENROL—General Rolling Element Bearing Analysis Program (Computer Program Description)," Report No. 32TR76-1, The Franklin Institute Research Laboratories, Philadelphia, PA, February 1976.

8 Derner, W. J., Goodelle, R. A., Root, L. E. and Rung, R., "The Hollow Ended Roller—A Solution for Improving Fatigue Life in Asymmetrically Loaded Cylindrical Roller Bearings," JOURNAL OF LUBRICATION TECHNOLOGY, TRANS. ASME, Series F, Vol. 94, Apr. 1972.

M. J. Hartnett[4]

The authors have completed an investigation of roller loading in bearing systems by employing an experimental technique that requires extremely painstaking procedures of measurement, and should be commended for their efforts. By far, the majority of published information in this area is of an analytical nature, and it is reassuring to see additional experimental verification of these solutions. Deviations between the results of the mathematical and experimental techniques illustrate the value of such verification, and clearly indicate the necessity of considering parameters external to the bearing, eg., housing structural characteristics, in applications. It would be extremely helpful, however, if more information were made available regarding bearing support, both in terms of structural geometry and material, actual tangential and normal force gear loadings, and finally relevent dimensional aspects of the system, ie., shaft diameters, bearing and gear positioning, etc.; thereby providing the reader a better understanding of the systems flexibility characteristics as well as sufficient information for independent analyses.

Finally, it is difficult to understand in what manner the authors can estimate the actual misalignment across a roller by inspection of the contact area. The dependence of contact width, the $2b_n$ dimension shown in Fig. 3, upon roller shape and applied loading is well known and discussed in the authors' reference [3]. Computational techniques of the authors' reference [5] are predicated upon a two-dimensional solution of roller raceway contact, and thus are insensitive to width changes resulting from the three-dimensional nature of the stress distribution. It would seem this inability to compensate for width changes would preclude accurate misalignment estimation.

J. W. Kannel[5]

The approach presented by the authors for determining circumferential load distributions and axial pressure variations in a bearing is very interesting. The footprint approach appears to be very well suited for use in an actual bearing system, and the results are very informative. I found the level of misalignment in a bearing both interesting and disturbing. In an instrument ball bearing, for example, misalignments considerably (by a factor of 10) lower than given by

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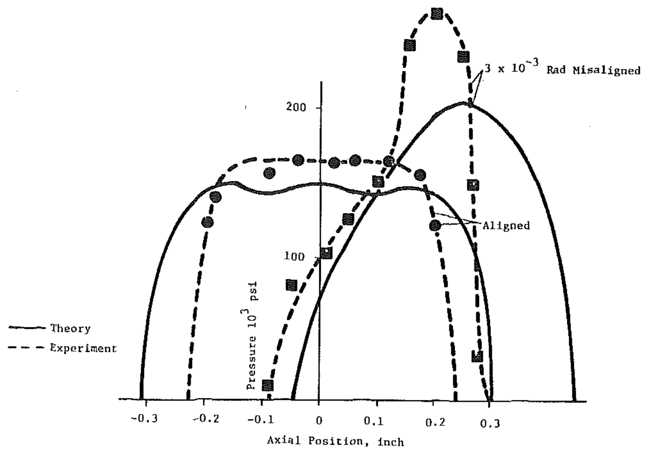

Fig. 9 Comparison of measured with predicted pressure profiles between two 4-in. dia cylinders. Flat region = 0.3 in.; blend edge radius = 24 in.; load = 2150 lb

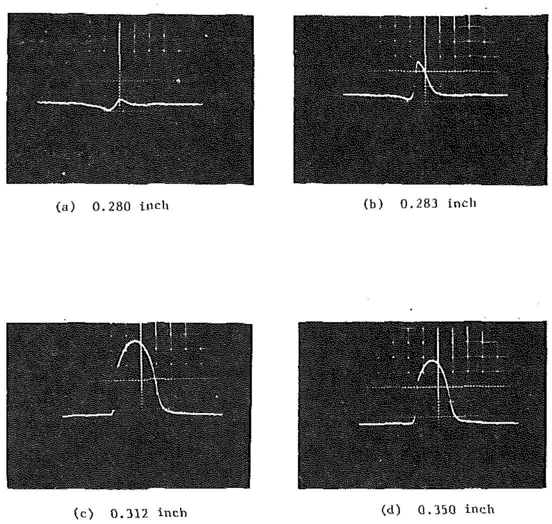

Fig. 10 Typical pressure traces associated with .75-in. wide contoured cylinder 150,000 psi maximum contact pressure
At various axial positions
Dimensions are axial relative micrometer settings
Profile designed for 150,000 psi maximum contact pressure
Rolling direction right to left
Rolling speed 1700 rpm
Oscilloscope sweep speed 20 μs/division

the authors could cause havoc in performance.

We, at Battelle, have measured pressure distributions between rolling-contact elements using a vapor-deposited pressure transducer and have also computed such pressures.[6] Fig. 9 illustrates the strong effect of 3×10^{-3} radial misalignment (which is similar to the authors) both as predicted as well as measured. To further illuminate the effect, several oscilloscope traces of pressure measurements are given in Fig. 10.

The pressure traces were obtained by locating a vapor-deposited pressure-transducer on a pair of rolling disk and of measuring the pressure-dependent resistance change of the manganin as the transducer swept through contact. Axial variations of pressure were obtained by axially shifting the location of the transducer. Fig. 10 shows the pressure traces at various axial locations and illustrates the effect of misalignment on the magnitude and width of the pressure traces. Surely, such pressure peaks can have an emmence effect on cage-loading in a bearing as well as bearing life.

Have the authors evaluated effect of misalignmt in cage or fatigue life? If not, have they seen any unusual wear from cursory evaluations in this regard? What future evaluation in this regard to they foresee?

L. E. Root[7]

A significant contribution toward furthering the understanding of rolling element contact stresses, in real world applications, has evolved in this rigourous, most complete, presentation. One of the more powerful aspects of the Contact Pattern Analysis Technique lies in its ability to qualitatively determine misalignments (the quantitative evaluation is a much more difficult task, though here the authors have, seemingly, done very well); the probability of ascertaining maldistributed loads in "line" contacts by any other currently known technique would, indeed, be small. From the early days of copper-films (1930–1940), through the acid-spray technique (self-photographing the footprint pattern; 1940–1950), the improved and practical experimental techniques (see authors' references [1, 2, 3]) surfacing in the early 1970's have provided a springboard for serious bearing load-distribution analysis.

The authors' observation concerning the agreement between bearing loads calculated by the Contact Pattern Analysis technique and bearing loads calculated from a system static equilibrium is of special interest. Reporting that their cases of least agreement were for the more lightly loaded conditions, this is felt to be rather expected—though perchance not for the reason the authors state; namely, that a reasonable absolute error becomes a significant percentage error. In the discussion to a paper by Goodelle, et al.[8] offered by Messrs. Eschmann and Schreiber (FAG Kugelfischer Georg Schaefer & Co., Germany), the latter pointed to the fact that the more lightly loaded contacts are concomitant with the tendency toward less "rectangular" and more "eliptical" contact pattern shapes. As indicated by the authors [1] in their closure—in TABLE E1-, the percentage difference increased 2-½ times in comparing loads on the most heavily loaded roller with loads on rollers twice removed from this location.

Commenting on Table 4 of the authors' paper, the sixteen individual listed "PERCENT DEVIATION" have a statistical median value of 11 LOW. It is of some passing interest that the reviewer, in conducting similar contact pattern analysis work on identical bearing sizes A, B, D (authors' notations) in 1974, and comprising some, though not all, of the same indicated loading conditions [per author Table 4], for a similar commercial gear train, found that the deviation in load existing between contact pattern and analytical method analyses had a "Percent Deviation" statistical median value of 20 LOW. This latter figure (20 LOW) takes on significance not from its proximity to the authors indicated 11 LOW though the agreement here is not all that bad, but more importantly that it reflects on the possible innate characteristic of the contact pattern analyses to always (?) indicate lesser loads than that determined by classical treatment (viz. system static equilibrium approach.)

The alluded to work of the reviewer also included a determination of probable angular misalignment for two (out of the authors' four) loading conditions on each of the three bearings A, B, D. When comparing these with the authors' indicated misalignments, rather good correlation exists; in particular: (*a*) four of the six average ~60 percent

[6] Kannel, J. W., "Comparison Between Predicted and Measured Axial Pressure Distribution Between Cylinders," JOURNAL OF LUBRICATION TECHNOLOGY, TRANS. ASME, Series F, Vol. 96, No. 3, pp 508–514, July, 1974.

[7] Rollway Bearing Co., Inc., Liverpool, N. Y.

[8] Goodelle, R. A., Derner, W. J., Root, L. E., "Determination of Static Load Distributions from Elastic Contacts in Rolling Element Bearings;" *ASLE TRANS.*, Vol. 14, No. 4, pp. 275–291

of the authors' values; (*b*) two of the six were ~ twice the authors' value.

The early observation of some, concerning the very limited scope of practical application of footprint technique (viz. contact pattern analysis), is proving to be ill-founded. Thanks to the efforts of these three authors the potentialities of this powerful analytical technique are once again amply illustrated.

Authors' Closure

The authors wish to thank the discussors for their comments on this paper. Attainment of a good contact pattern is a task which requires good judgment and carefully controlled procedures during recording. In this case, the contact patterns on the input and intermediate shafts were not obtained at the same time. At any time, only one shaft had the etched inner rings to accept the contact patterns, while the second shaft had unetched rings. The load was applied by holding the test shaft and torquing the other two in opposite directions.

Mr. Rumbarger indicated that a sophisticated finite element computer modeling of the shaft, bearings, and housing, in conjunction with a complete rolling bearing analysis can determine the effects of support stiffness. In the authors' experience, this is possible but involves considerable effort and expense. Moreover, just as in any other analytical model, a number of assumptions will have to be made to arrive at the finite element model. Therefore the information obtained from such a model will not be as effective in revealing actual conditions as is the Contact Pattern Analysis technique. However, analytical techniques such as the finite element model are most important for understanding the results obtained from experimental techniques.

Mr. Hartnett indicated the need to reveal more information concerning the components in the system. The authors do realize the usefulness of this information for readers, as well as for those intending to engage in independent analytical analyses. However, it will not be possible to provide this information, due to its proprietary nature. The discussor raised some questions concerning the method used for calculating the misalignment. The authors' computer program is a modified form of the technique published in the authors' reference [5]. In the computer program the crowned roller is defined by a polynomial equation. In the actual misaligned situation the inner ring tilts and the roller remains unaffected, but in this program the roller is assumed to tilt and the inner ring remains unaffected. With the increase in misalignment, the point of initial roller/race contact moves away from the roller center. This program generates the distance of the roller/race contact point from the end of the contact or roller for various degrees of misalignment. Then, from the contact pattern, the distance of the widest point is determined and is compared to the computed results to determine the degree of misalignment. In the case of a truncated contact, the end of the contact pattern is the true end of the contact, but in the case of a contact which is not truncated the contact pattern will be longer than the true contact, and the true length is determined by compensating for the etch correction factor. The authors agree that the contact width depends upon the load as well as the roller crown, and a three-dimensional approach is required for its determination. But in this case the main focus is on the location of the widest point and not the absolute value of width, and for a given roller crown the location of the widest point is determined by the misalignment.

The data presented by Mr. Kannel on the effect of misalignment on the magnitude and the width of the pressure distribution in the contact zone is very informative. This data is at .003 mm/mm misalignment, which is in the range of authors' data for the misalignment. It shows how the misalignment affects the pressure distribution in the contact zone, which can have considerable effect on the fatigue life. In the authors' bearings there was no cage problem but there were instances of premature fatigue failure, not conclusively caused by misalignment. These bearings had crowned rollers designed to absorb a significant amount of misalignment. The authors have observed in both actual applications and laboratory tests the very destructive effect of misalignment on both ball and cylindrical roller bearing cages, as well as other bearing components. The authors' laboratory has numerous on-going test programs to evaluate bearing components and materials with regard to rolling contact fatigue and cage performance, among other variables. A program is included to evaluate the effect of contact ellipse truncation (edge loading) on fatigue life.

It is very encouraging to learn that Mr. Root also conducted similar contact pattern work utilizing bearings similar to the authors' *A*, *B*, and *D*. The good correlation between the authors' misalignments and those of the discussor again demonstrates the practicality and strength of the Contact Pattern Analysis as an experimental technique.

The least agreement between experimental loads and analytical loads was found in cases of light loads. The discussor contended that it is not due to the fact that a reasonable absolute error becomes a significant percentage error under light loads, as suggested by the authors, but that it is due, instead, to the fact that "lightly loaded contacts are concomittant with the tendency toward less rectangular and more elliptical contact pattern shapes." In support of his contentions the discussor referred to Table E1 in the authors' closure to a paper by Goodelle, et al. (reference [2] in the present paper). This table is reproduced herein for clarity of discussion.

At a load deflection exponent of 1.11 (for the line contact), the table shows the percent of deviation between calculated and experimental roller loads for the second adjacent roller to be 2½ times the deviation for the most heavily loaded roller. This, in the discussor's view, is due to the fact that the second adjacent contact is more elliptical in shape than the most heavily loaded roller. As shown in the table, the deviation between the calculated and the experimental bearing loads increases from −0.8% to −3.7% as the load deflection exponent is increased from 1.11 to 1.17, which is contrary to discussor's contention. Again, the deviations for the most heavily loaded roller at a load deflection exponent of 1.11, 1.17, and 1.21 do not prove the discussor's point. The deviations for the adjacent and the second adjacent rollers show no correlation with the load deflection exponent. In the authors' view the load deviations in this table are erratic in nature and therefore could not be used for drawing any quantitative conclusion.

Table E1 Roll body loads and deviations from experimentally determined values for various exponents, N

		← Roll Body Load-lb (Theoretical) →				← % Diff. Theoretical/Test →			
		For Exponent N Equal				For Exponent N Equal			
Roll Body Designation	Value From Test Observation	1.056	1.11	1.17	1.21	1.056	1.11	1.17	1.21
Q_{max}	2740	2658	2832	2990	3177	−3.0	+3.4	+9.1	+15.9
$Q_{\psi = 27.7°}$	2034	2109	2121	2101	2064	+3.7	+4.3	+3.3	+1.5
$Q_{\psi = 55.4°}$	285	636	310	28	Zero	+123.	+8.8	−90.	Indeterminate
F_r	7000	7116	6941	6743	6832	+1.7	−0.8	−3.7	−2.4