$$c = \frac{R}{G^*} \quad \text{and} \quad b = \frac{R}{2G^*} \quad \text{and} \quad \frac{h}{b} = h^*G^{*2}$$

$$\therefore \frac{h}{b} - \frac{h_0^1}{b} = 1 - \iint \Phi(r^*, r_1^*, \theta_1, \theta, p_1^*)$$

(b) Reynold's Equation

Now

$$\frac{1}{r}\frac{\partial}{\partial r}\left(rh^3\frac{\partial p_0}{\partial r}\right) + \frac{1}{r^2}\left(h^3\frac{\partial p_0}{\partial \theta}\right) = 6U\eta_0 \cos\theta \frac{\partial h}{\partial r}$$

Nondimensionalising,

$$\therefore \frac{\partial}{\partial r^*}\left(r^*h^{*3}\frac{\partial p^*}{\partial r^*}\right) + \frac{\partial}{\partial \theta}\left(h^{*3}\frac{\partial p^*}{\partial \theta}\right)$$

$$= 24\left(\frac{U}{R}\frac{\eta_0}{E_r}\right)(E_r\alpha)^4 = 24U^*G^{*4}$$

(c) Load

$$W = \iint pr\,dr\,d\theta = -\frac{c^2}{\alpha}\iint \ln(1-p^*)dr^*d\theta$$

$$\therefore \frac{W\alpha}{c^2} = -\iint \ln(1-p^*)r^*dr^*d\theta = W^*G^{*3}$$

$\therefore$ The π's are

$$\pi_1 = \frac{h}{R}G^{*2}, \qquad \pi_2 = U^*G^{*4}, \qquad \pi_3 = W^*G^{*3}$$

$$\therefore h^* = \frac{1}{G^{*2}}\Phi[U^*G^{*4}, W^*G^{*3}] \tag{11}$$

Rolling Friction Parameters. Using a similar procedure, the rolling friction coefficient is

$$\mu = \frac{1}{G^*}\Phi'[U^*G^{*4}, W^*G^{*3}] \tag{12}$$

References

1 Dowson, D., and Higginson, G. T., "A Numerical Solution to the Elastohydrodynamic Problem," *Journal of Mechanical Engineering Science*, Vol. 2, No. 3, p. 188.

2 Crook, A. W., *Proceedings of the Institute of Mechanical Engineers*, London, 1957, pp. 171, 187.

3 Dyson, A., Naylor, H., and Wilson, A. R., "The Measurement of Oil-Film Thickness in Elastohydrodynamic Contacts," *Proceedings of the Institute of Mechanical Engineers*, 1965–1966 180 (Pt 38), 119.

4 Sibley, B., and Orcutt, F. F., "Elastohydrodynamic Lubrication of Rolling Contact Surfaces," *Trans. American Society of Lubrication Engineers*, Vol. 4, No. 2, p. 234.

5 Cameron, A., and Gohar, R., "Theoretical and Experimental Studies of the Oil Film in Lubricated Point Contact," *Proceedings of the Royal Society*, Series A, 1966, pp. 291, p. 520.

6 Archard, J. F., and Cowking, E. W., "Elastohydrodynamic Lubrication at Point Contacts," *Proceedings of the Institute of Mechanical Engineers*, Vol. 180, Part 3B, 1965–1966, p. 47.

7 Cheng, H. S., "A Numerical Solution of the Elastohydrodynamic Film Thickness in an Elliptical Contact," *Trans. of the American Society of Lubrication Engineers*, 69-Labs-17.

8 Archard, J. F., and Kirk, M. T., "Lubrication at Point Contacts," *Proceedings of the Royal Society*, Series A, 1961, pp. 261, 532.

9 Kirk, M. T., "Hydrodynamic Lubrication of Perspex," *Nature*, London, 1962, Vol. 194, p. 965.

10 Gohar, R., and Cameron, A., "The Mapping of Elastohydrodynamic Contacts," ASLE/ASME Lubrication Conference (Minneapolis) 1966, Preprint No. 66 LC 21.

11 Foord, C. A., Hammann, W. C., and Cameron, A., "Evaluation of Lubricants Using Optical Elastohydrodynamics," *ASLE Trans.*, Vol. 11, 1968, pp. 31–43.

12 Dowson, D., and Jones, D. A., "Lubricant Entrapment Between Approaching Elastic Solids," *Nature*, London, 1967, Vol. 214 No. 5091, p. 947.

13 Westlake, F., and Cameron, A., "Fluid Film Interferometry in Lubrication Studies," *Nature*, Vol. 214, No. 5085, May 1967.

14 Wedeven, V., and Cameron, A., "A Study of Elastohydrodynamic Lubrication in Rolling Bearings Using Optical Interference," Inst. Mech. Engrs. Sympos. Experimental Methods in Tribology, Part 3G, 1967–1968.

15 Gohar, R., "A Ball and Plate Machine for Measuring Elastohydrodynamic Oil Films," Inst. Mech. Engrs. Sympos. Experimental Methods in Tribology, 1967–1968, Vol. 182, Part 3G.

16 Chu, P. S. Y., and Cameron, A., "Pressure Viscosity Characteristics of Lubricating Oils," *Journal of the Institute Petroleum*, Vol. 48, No. 461, May 1962.

17 Crook, A. W., "The Lubrication of Rollers—IV. Measurements of Friction and Effective Viscosity," *Philosophical Transaction*, Series A, 1963, pp. 255, 281.

18 Jefferis, J. A., and Johnson, K. L., "Sliding Friction Between Lubricated Rollers," *Proceedings of the Institute of Mechanical Engineers*, Part 1, Vol. 182, 1967–1968.

19 Plint, M. A., "Traction in Elastohydrodynamic Contacts," *Proceedings of the Institute of Mechanical Engineers*, pp. 1967–1968, Vol. 182, Part 1.

20 Dowson, D., and Whomes, T. L., "Effect of Surface Finish Upon the Traction Characteristics of Lubricated Cylindrical Contacts," *Proceedings of the Institute of Mechanical Engineers*, 1967–1968, Vol. 182, Part 1.

21 Evans, D. and Cameron, A. To be published.

22 Johnson, K. L., Discussion of reference [6], *Proceedings of the Institute of Mechanical Engineers*, Vol. 180, Part 3B, 1965–1966, p. 79.

23 Cameron, A., *Principles of Lubrication*, Longmans, 1966, p. 190.

Acknowledgments

I would like to offer my thanks to the Polytechnic Regent Street School of Photography for assistance in taking the photos, and to third year student M. Moussavi for assistance in some of the experiments. The research was supported by a grant from the Science Research Council.

DISCUSSION

D. P. Townsend[2]

The program conducted by the author provides some very interesting quantitative and qualitative results for a wide range of materials of interest in EHD lubrication. It is somewhat unfortunate that the point of most interest in the evaluation of the EHD parameters (i.e., the lowest value of the modulus of elasticity) is extrapolated to such an extent as to throw considerable doubt on its accuracy.

The author has taken constant values of the nondimensional velocity parameter U^* and load parameter W^* and from these have determined the film thickness h^* as a function of G^* over a wide range of the modulus of elasticity E. The EHD film thickness is a very strong function of velocity, inlet viscosity and radius (i.e., $(\eta_0, U)^{0.7}$ and $(R)^{0.43}$) and only lightly dependent on load and elastic modulus (i.e., $(W)^{-0.13}$ and $(E^1)^{0.3}$). Therefore, it is the discusser's opinion that it would have been much more appropriate to run the tests at a constant velocity while varying the inlet viscosity or radius to maintain a constant U^* with changing elastic modulus. If this procedure is utilized it should give a more realistic evaluation of film thickness parameter h^* as a function of the material parameter G^*.

The photograph of film shapes in Fig. 3 is very interesting, especially at the sides of the contact where it appears that side leakage causes the minimum film thickness to occur. It would be much more valuable if the author could show the film shapes and thickness in three dimensional perspectives or even two dimensional traverses to give the reader a better feel for the magnitude of the film shape variations.

[2] NASA—Lewis Research Center, Cleveland, Ohio

W. R. D. Wilson[3] and J. W. Kannel[3]

We would like to congratulate Dr. Gohar on an excellent piece of research. The data he presents will be of great value in the design of real bearing systems. The data presented on the least film thickness at the edge of contact is of particular interest to us.

For a number of years we at Battelle have been obtaining data on film thicknesses in highly loaded line contacts by the X-ray technique. This data shows a much greater load dependence than that calculated by Grubin or Dowson and Higginson. In general, we have found that the measured film thickness is approximately inversely proportional to the maximum Hertz pressure.[4] It is very gratifying to discover that the least film thickness data given in Fig. 4 seems to exhibit a similar variation with load. This implies that the X-ray technique provides a good measure of the minimum film thickness in a highly loaded contact. Since the minimum film thickness governs a lubricants ability to prevent surface damage, X-ray measurements can serve as a useful screening technique for lubricants. We look forward to seeing further data from Imperial College, especially some film thickness maps of line contacts.

R. J. Boness[5]

The author is to be congratulated on presenting some interesting results of film thickness and friction forces in lubricated point contacts.

Although the author established that a good correlation between experimental and theoretical rolling friction values existed, no attempt was made to examine theoretically the sliding friction component.

This contribution is concerned with the tractive forces present in an elliptic contact when small degrees of sliding are present.

The analysis, although based upon the assumptions made by the author, emphasizes the danger in using an exponential pressure viscosity relationship when calculating elastohydrodynamic sliding friction forces. The assumptions are made that the pressure distribution between heavily loaded lubricated contacts is approximately Hertzian and that the film thickness is constant over the contact area. Furthermore the analysis is limited to small amounts of sliding as no allowance is made for viscosity changes due to temperature variations in the lubricant film.

The friction force is found by integrating the viscous shear forces occuring in the contact region.

Newton's law of viscous flow gives the shear stress τ as

$$\tau = \eta \partial u/\partial y \tag{13}$$

It can be shown by considering the equilibrium of an element of oil that

$$\frac{\partial p}{\partial x} = \frac{\partial \tau}{\partial y} = \eta \frac{\partial^2 u}{\partial y^2} \tag{14}$$

Integrating equation (14) with the boundary conditions $u = U_2$ at $y = h$ and $u = U_1$ at $y = 0$ leads to

$$\frac{\partial u}{\partial y} = \frac{1}{\eta}\frac{\partial p}{\partial x}(y - h/2) + \frac{U_2 - U_1}{h} \tag{15}$$

The viscous shear stress may therefore be written as

$$\tau_{0,h} = \eta(\partial u/\partial y) = \mp\frac{h}{2}\frac{\partial p}{\partial x} + \eta/h(U_2 - U_1) \tag{16}$$

where the suffixes 0 and h refer to the surfaces where $y = 0$ and $y = h$.

The total friction force for the pressure zone is

$$\begin{aligned} F_{0,h} &= \int_{y_1}^{y_2}\int_{x_1}^{x_2} \tau dxdy \\ &= \int_{y_1}^{y_2}\int_{x_1}^{x_2} \mp h/2 \frac{\partial p}{\partial x} dxdy \\ &\qquad + \int_{y_1}^{y_2}\int_{x_1}^{x_2} \eta/h(U_2 - U_1)dxdy \end{aligned} \tag{17}$$

The first integral can be identified with the rolling friction and the second gives the contribution due to sliding.

The author has considered theoretically the rolling friction term and at speeds of sliding above a few cm/sec has shown that the sliding component is predominate.

Turning therefore to the sliding friction force F_s, assuming the film thickness h to be constant over the contract region F_s may be written as

$$F_s = \frac{U_2 - U_1}{h}\int_{y_1}^{y_2}\int_{x_1}^{x_2} \eta dxdy \tag{18}$$

Rewriting equation (18) in the usual dimensionless terms, using

$$V = \frac{(U_2 - U_1)\eta_0}{E^1 R}, \qquad \bar{\eta} = \eta/\eta_0, \qquad \bar{x} = x/a, \qquad \bar{y} = y/b,$$

gives

$$\frac{F_s}{E^1 R^2} = V\left(\frac{R}{h}\right)\left(\frac{a}{R}\right)\left(\frac{b}{R}\right)\int_{-1}^{+1}\int_{-1}^{+1}\bar{\eta} d\bar{x} d\bar{y} \tag{19}$$

where the shape and size of the contact region may be obtained after Hertz from

$$a = a^{*3}\sqrt{\frac{3WR}{E^1}} \tag{20}$$

$$b = b^{*3}\sqrt{\frac{3WR}{E^1}} \tag{21}$$

$$p_{\max} = \frac{3W}{2\pi ab} \tag{22}$$

$$p = P_{\max}\left(1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}\right)^{1/2} \tag{23}$$

In order to evaluate this integral an equation describing the variation of viscosity with pressure is required. In keeping with the author's assumptions the standard exponential form was chosen,

$$\eta = \eta_0 \exp(\alpha p) \tag{24}$$

After substituting equations (20), (21), (23), and (24) into

$$I = (a/R)(b/R)\iint \bar{\eta} d\bar{x} d\bar{y}$$

the integral was evaluated using a digital computer.

The results obtained indicate that the integral I may be written in terms of the maximum Hertzian pressure as

$$\log I = m(p_{\max} - D) + \log C$$

where m and C are dependent on the pressure viscosity coefficient and the a/b ratio, respectively. For the case of sphere loaded against a flat plate these values may be tabulated as Table 1.

[3] Battelle's Columbus Laboratories, Columbus, Ohio.

[4] Gohar, R., "Oil Film Thickness and Rolling Friction in Elastohydrodynamic Point Contact," JOURNAL OF LUBRICATION TECHNOLOGY, TRANS. ASME, Series F, Vol. 93.

[5] Royal Military College of Science, Shrivenham, Wilts, England.

Table 1

Pressure viscosity coefficient $dyne^{-1}$ cm^2	m lbf^{-1} in^2	C	D lbf/in^2
2.23×10^{-9}	6.7×10^{-5}	.0135	3.75×10^4
1.56×10^{-9}	4.8×10^{-5}	.0032	3.75×10^4
1.12×10^{-9}	3.45×10^{-5}	.0012	3.75×10^4

As an example of the application of these theoretical results consider the result shown in the author's Fig. 9.

load = 3579 grmf
rolling speed = 195 cm/s
viscosity = 4.2×10^{-6} Reyns
$h/R = 10.3 \times 10^5$
$R = 0.25$ in.
pressure viscosity coefficient 2.48×10^{-9} $dyne^{-1}$ cm^2

Extrapolating from Table 1 and solving equation (13) leads to an integral I value of approximately 9.9.

The sliding friction force can be evaluated from

$$F_s = V\eta_0 R \cdot \left(\frac{R}{h}\right) I$$

For $V = 2.54$ cm/sec

$$F_s = \underline{460 \text{ grmf}}$$

This result indicates that the theoretical calculation of sliding friction force based upon Grubin type assumptions grossly over-estimates the experimental value (12 grmf).

This discrepancy between the theoretical and experimental values is unlikely to be due to the assumption of a constant film thickness over the contact area, as the author's photographs of film shape illustrates large horseshoe areas of constant film thickness. The most likely sources of error would therefore be the choice of an exponential pressure viscosity relationship together with the assumption that the pure rolling pressure viscosity coefficient remains unchanged when sliding is present. It is interesting to note that using a pressure viscosity relationship of the form $\eta = \eta_0 \exp.\left(\frac{3747\bar{p}}{1 + 50.4\bar{p}}\right)$ reduces the sliding friction force by a factor of 200.

Author's Closure

The author thanks the discussers for the points they have raised. Mr. Boness has shown that if the viscosity is taken to be the only pressure dependent (assuming a Hertzian distribution) then the calculated sliding friction is well above experiment. By assuming that the theoretical rolling friction is most significant in the inlet part of the contact, where pressures have not yet become excessive, the author avoided the troublesome question of how viscosity behaves in the parallel film high pressure region.

When slight sliding is present in point or line contact various effects must combine, to a greater or lesser extent, to reduce the pressure dependent viscosity. These are:

1 The effective viscosity depends on rolling velocity [17, 18]. Fig. 15 shows $\frac{F_s}{W}$ plotted against $\frac{2(U_1 - U_2)}{U_1 + U_2}$. A single straight line results. The points are from Figs. 8 and 9. $F_s = F + F_r$ and is the sliding friction. One of Crook's [17] results for disks is also shown. Assuming $h^* = 1.73\,(U^*G^*)^{5/7}(W^*)^{-1/21}$ (24) and $\eta^* = \frac{\bar{\eta}}{\eta_0}$, where $\bar{\eta}$ is the "effective" viscosity based on isothermal Newtonian conditions [18].

Then

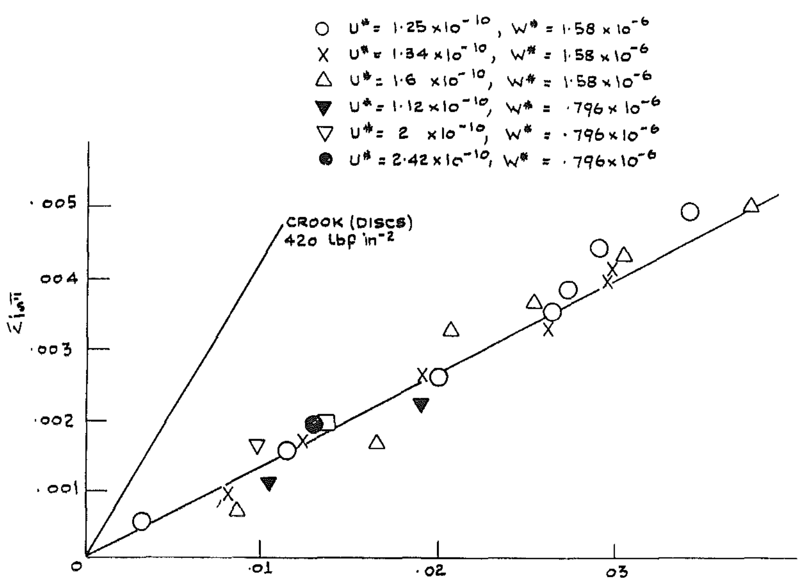
$$\eta^* = 0.0564 W^{*2/7} U^{*-2/7} G^{*5/7} \tag{13}$$

The effective viscosity therefore decreases with rolling velocity.

2 Thermal effects which include compressibility (reference [23], p. 223).

3 That the pressure levels within the contact are often lower than they would be had Hertzian conditions been assumed [25].[6] There must also be a sharp fall of pressure within the edges of the contact. The side lobes in Fig. 3 are inside the Hertz radius (reference [3], p. 254). As these lobes commence, the pressure falls and the surface springs outwards giving the least film thickness (Figs. 4 and 5). Further evidence of this fall of pressure is adduced when it is noted that the film thickness at the side lobes is more sensitive to load than in the central region [24]. There, an increase in load results in a downward move-

[6] Numbers [24–25] in brackets designate Additional References at end of Closure.



$$\frac{2(U_1 - U_2)}{U_1 + U_2}$$

Fig. 15 Coefficient of sliding friction against slip-roll ratio ment of the ball center together with an upward distortion of the contact region, the net effect being hardly any change in thickness. The side lobes do not feel the upward distortion due to pressure change.

A combination of the aforementioned effects together with the fact that anyway an exponential viscosity pressure relationship is not accurate, even statically, at high loads, must result in a considerable reduction in the theoretically predicted sliding traction.

The author agrees with the observations of Wilson and Kannel. Their carefully conducted experiments using X rays have an advantage over our method of interferometry in that they can use steel surfaces. We are working at present on projects involving the general line contact mapping problem, as well as on end blending effects.

Mr. D. P. Townsend's observations are welcome. It would have been difficult to run experiments at constant velocity while controlling inlet viscosity as it itself depends on velocity. Radius is more easily controlled but depends on availability of a selection of super precision balls. We are currently running experiments using an annular conforming groove instead of a flat plate. Slight changes in ball radius will then become much more inportant. It is unfortunate that perspex has an E value one twentieth that of glass and therefore the W^* must, of necessity, be large. Incidentally, the experimental results can be plotted as

$$\frac{h^*}{W^*G^*} = \Phi \left\{ \left[\frac{U^*}{W^{*3/2}G^{*1/2}} \right], [W^*G^{*3}] \right\}$$

For the harder material combinations, $(W^*G^*)^3$ has little effect on the film thickness. For steel on steel (quoted in the above paper) we get

$$\frac{h^*}{W^*G^*} = 1.28 \left(\frac{U^*}{W^{*3/2}G^{*1/2}} \right)^{7} (W^*G^{*3})^{0.05}$$

Note that two of the groups do not contain E and are therefore valid for an undistorted contact with pressure dependent viscosity. The graph of the results based on the new groups are shown in Fig. 16 together with the theory of Wedeven, Evans, and Cameron [24]. Traverses of point contact EHL shapes can be seen in references [5 and 11].

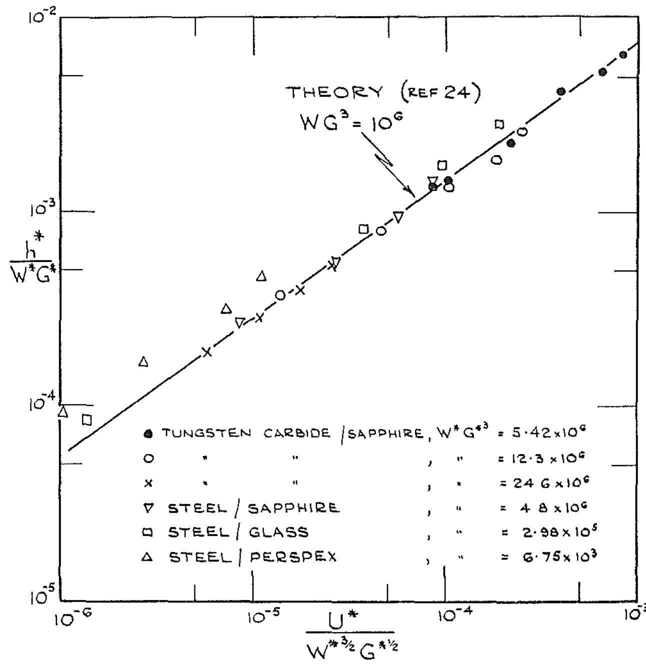


Fig. 16

Additional References

24 Wedeven, L. D., Evans, D., and Cameron, A., "Optical Analysis of Ball Bearing Starvation," ASME/ASLE Lubrication Conference, Oct. 1970, Preprint 70-Lub. 19.

25 Klemz, B. L., "Stresses Measured by Photoelasticity in Lubricated Line Contact," PhD thesis, University of London, 1970.