perature in the absence of conductivity occurs with the amount [2]

$$T_{pi} - T_{pb} = \frac{\sigma_0 \xi (\Delta v_t)}{\sqrt{3}\rho C_p v_n J} \quad (99)$$

where

$$\Delta v_t = \frac{2v_i}{\alpha} \int_0^\alpha \cos \alpha' d\alpha', \qquad v_n = \frac{2v_i}{\alpha} \int_0^\alpha \sin \alpha' d\alpha' \quad (100)$$

Here $\Delta v_t$ is the discontinuity in the tangential velocity, and $v_n$ is the velocity normal to the discontinuity. Substituting (100) into (99), one obtains

$$T_{pi} = T_{pb} + \frac{\sigma_0 \xi \sin \alpha}{\sqrt{3}\rho C_p J (1 - \cos \alpha)} \quad (101)$$

# APPENDIX 2

Referring to Fig. 1, the heat equation is

$$\frac{k}{\rho C_p} \frac{\partial^2 T_{s2}}{\partial^2 x_1'} = \frac{D T_{s2}}{Dt} \quad (102)$$

Here $D(\ )/Dt$ is the material derivative. The transformation between two coordinates is

$$x_1 = x_1' + v_i t, \qquad r = r \quad (103)$$

since

$$\frac{D T_{s2}}{Dt} = \frac{\partial T_{s2}}{\partial t} + \frac{\partial T_{s2}}{\partial x_1} \frac{\partial x_1}{\partial t} \quad (104)$$

For the quasi-stationary state, $\partial T_{s2}/\partial t = 0$ in (104) gives, eliminating subscript $s2$ of $T$ for simplicity,

$$\frac{DT}{Dt} = \frac{\partial x_1}{\partial t} \frac{\partial T}{\partial x_1} \quad (105)$$

Differentiate $x_1$ in (103) with respect to $t$ and substitute $\partial x_1/\partial t$ into (105)

$$\frac{DT}{Dt} = v_i \frac{\partial T}{\partial x_1} \quad (106)$$

Substituting equation (106) into (102), with $\frac{\partial T}{\partial x_1} \equiv \frac{\partial T}{\partial x_1'}$ and $T = T(x_1)$, one obtains

$$\frac{d^2 T}{dx_1^2} - \frac{\rho C_p v_i}{k} \frac{dT}{dx_1} = 0 \quad (107)$$

The boundary conditions are

$$x_1 = 0, \qquad T = T_{pb}; \qquad x_1 = l_2, \qquad T = T_{pi} \quad (108)$$

The solution in equation (107) subjected to (108) is solved

$$T = T_{pb} + \frac{e^{\frac{M_i x_1}{l_2}} - 1}{e^{M_i} - 1} (T_{pi} - T_{pb}) \quad (109)$$

# DISCUSSION

## V. DePierre[6]

Several assumptions (such as isothermal die and container surface, homogeneous material deformation and uniform velocity across the die cross section, and pressure on container equal to pressure on the billet material) on which the author bases his mathematical solutions do not represent actual conditions in hot extrusion of high strength materials. Therefore the author's theory cannot be accepted for practical application without serious reservations. These reservations are strengthened by the author's selective selection of test results from one extrusion of Rene 41 nickel-base alloy performed at the Air Force Materials Laboratory, WPAFB, Ohio, and the erroneous test values used in his calculations to verify his mathematical solutions.

In reference [7] (Table 5, p. 118) of the subject paper, the following test results of two Rene 41 extrusions are reported:

| Extrusion no. | Temp °F | Die angle | Billet lubrication | Reduction ratio | Running die pressure (ksi) | Running ext. speed in./sec |
|---|---|---|---|---|---|---|
| 1775 | 2000 | 60 | 0010 glass | 3.9:1 | 103 | 2.25 |
| 1776 | 2000 | 60 | 0010 glass | 3.9:1 | 80 | 2.50 |

Measured running die pressures were significantly different for the two extrusions. The difference was attributed to friction calculated from measured forces to be as follows:

| Extrusion no. | Average running container friction (ksi) | Average running die friction (ksi) |
|---|---|---|
| 1775 | 21.0 | 13.5 |
| 1776 | 8.6 | 3.2 |

The higher friction values for extrusion 1775 was due to cooling of the glass lubricant during transfer of the heated billet from the furnace to the extrusion press. This transfer time was 30 sec compared to normal transfer time of 6–10 sec.

The author's selection of test results from extrusion 1776 and not from extrusion 1775 to verify his theory was correct although without detailed knowledge of the difference in transfer time, the selection appears biased. However, one test is not considered a sound basis for experimental verification.

The following data used by the author to calculate extrusion pressure at the die with his theory are erroneous:

$$l_2 = 6 \text{ in. } (l_2 \text{ should be } 4.5 \text{ to } 5.0 \text{ in.})$$
$$r_1 = 1.425 \text{ in. } (r_1 \text{ should be } 1.516 \text{ in.})$$
$$h_e = 10^{-4} \text{ in. } (h_e \text{ is approximately } 0.005 \text{ in.})$$

when hydrodynamic conditions have been achieved at the Air Force Materials Laboratory.

The film thickness, 0.000153 inches, calculated by the author for film thickness in Zone II (container zone) is significantly different from the value of 0.010 inches measured at the Air Force Materials Laboratory.

From the foregoing statements, it is evident that the mathematical solutions presented in the subject paper are not acceptable as a basis for quantitative studies of hot extrusion of high-strength materials under present practical processing conditions.

## W. R. D. Wilson[7]

The author has used the same yield condition

$$\sigma_x + p = \sigma_0 \quad (2)$$

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[6] Air Force Materials Laboratory, Wright-Patterson Air Force Base, Ohio.
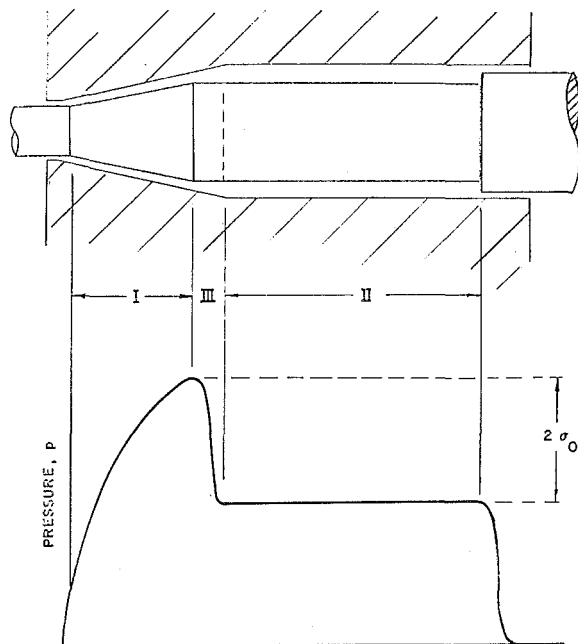[7] Battelle Memorial Institute, Columbus, Ohio.

**Fig. 23**

in both zones I and II, but the deformation states in the two zones are quite different. In Zone I, the workpiece is being elongated in the axial direction and compressed in the radial direction and the relationship is correct. However, in Zone II, the workpiece is tending to be upset under the action of the punch load. Thus, it is tending to expand radially and contract axially and the yield criterion should read

$$\sigma_x + p = -\sigma_0. \tag{110}$$

The condition given in equation (2) requires that the axial compressive stress exerted by the punch is less than the radial pressure of the lubricant. If this were true, lubricant would be forced into the space between the billet and punch and the extrusion process would become hydrostatic.

Now the equation of axial equilibrium requires that the axial stress $\sigma_x$ must be continuous between Zones I and II. Thus, the use of equation (2) in Zone I and equation (110) in Zone II requires a step change in the lubricant pressure $p$ by an amount $2\sigma_0$ on passing from Zone I to Zone II. This can only be accommodated by assuming that a short Zone III in which the material is rigid exists between Zones I and II as shown in Figure 23. This will allow a sudden decrease in film thickness between Zones II and I with the consequent increase in lubricant pressure. This type of hydrodynamic inlet region has been discussed by Walowit and Wilson[8] with reference to drawing and rolling.

Since the amount of lubricant passing out through the die throat is now no longer equal to the volume swept out by the stem some compensating leakage past the stem must be allowed. If this does not occur, then the pressure in the lubricant film will rise to a greater value than the stem pressure and the extrusion will become hydrostatic.

The approach outlined in the foregoing will allow the calculation of the exit film thickness $h_e$ in terms of the other process parameters including the applied coating thickness. If the coating thickness is large compared with the exit thickness (as is usually the case) then the exit thickness is almost independent of the applied coating thickness and the billet/container friction is small.

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[8] Walowit, J. A., and Wilson, W. R. D., "The Mechanics of Hydrodynamic Lubrication in Steady Deformation Processes," ASTME Preprint MF 69-102 presented at the ASTME Annual Engineering Conference, Chicago, 1969.

## Author's Closure

The author wishes to thank Drs. DePierre and Wilson for their comments on his paper.

The author would like to refute the claim by Dr. DePierre that the solutions presented in his paper are not acceptable as a basis for quantitative studies of hot extrusion of high-strength materials under present practical processing conditions.

Since the hot extrusion process is of short duration, the die and container surface can be assumed to be isothermal. In continuum mechanics, alloys are usually assumed to be homogeneous materials. That the uniform velocity is across the die cross-section is applicable for the practical case (see Reference [3 and 4] in the present paper). For a *thin* lubricant film such as 0.01 in., the variation of pressure across the film can be neglected compared with one along the container and die. So the pressure on the container can be approximated to be equal to one on the billet.

Because of different running extrusion speeds for extrusion No. 1775 and 1776, both of them are not tested under the identical conditions. As such, the average experimental quantity of running die pressure cannot be used to check the theoretical one. The running extrusion speed of either 2.25 in./sec or 2.5 in./sec must be used to calculate theoretical running die pressure which will be checked against either extrusion No. 1775 or 1776. As far as experimental running extrusion speed is concerned, the extrusion No. 1775 and 1776 are not identical. From Dr. DePierre's comment, the author also noticed that the transfer time was 30 seconds for extrusion 1775 compared to 6–10 seconds for extrusion 1776. The transfer time is defined by Dr. DePierre as one which is needed to transfer the heated billet from the furnace to the extrusion press. The longer the transfer time, the more the heat is transferred. As such, the preheat billet temperature at ram of extrusion 1775 must be lower than one of extrusion 1776. So the same measured billet temperature for both extrusion 1775 and 1776 is in question. Due to lower preheat billet temperature, the lubricant film temperature of extrusion 1775 is lower than one of 1776. Further, the lower film temperature of extrusion 1775 is responsible for higher friction, as stated in Dr. DePierre's comment.

The data used by the author to calculate extrusion pressure at the die with his theory are not erroneous. The value of $r_1 = 1.425$ in. stated in Dr. DePierre's comment must be $r_i = 1.4575$ in. as noted in the present paper. The value of $r_i = 1.4575$ in. and $l_2 = 6$ in. are chosen based upon Table 2 (p. 102, Reference [8]). Since only the dimensionless $H$ appears in (50), there is no effect of $h_e$ on $P$. Hence, the theoretical extrusion pressure at die remain unchanged regardless of the value of $h_e$. Besides, in Reference [8], only an approximate range of the thickness of the lubricant film is given as $0.01 \sim 0.015$ in. for all tests. The author was not sure if $h_e$ was within that range or not. As given in Dr. DePierre's comment, $h_e = 0.005$ in. is out of the range of $h$ in reference [8]. The reason that the author chooses the small quantity of $h_e = 10^{-4}$ in. is because the theoretical pressure at die is not affected by $h_e$ and the extrusion process may result in hydrodynamic lubricant film. Noted in (46), (55), (57), (84), and (92), $h_e$ also has no effect on $H$, $\theta_{m1}$, $\bar{L}_1$, $\theta_{m2}$, and $\bar{L}_2$. For instance, for $h_e = 0.005$ in. which is given in Dr. DePierre's comment, the corresponding film thickness $h$ in Zone II can be calculated from $H = 1.75$ as shown in Fig. 6 in the present paper. As such, $h = 1.75 \times 0.005 = 0.00875$ in. This theoretical value is in good agreement with the measured value of 0.01 in. as given in Dr. DePierre's comment.

However, if the different value of $h_e$ is used, the value of $\tau_{s1}, f_d, \tau_{s2}, f_c$, and $P$ will be affected accordingly. In order to compare the average theoretical running container and die frictions with average experimental ones for extrusion 1775 and 1776 as stated in Dr. DePierre's comment, $\tau_{s1}$ and $\tau_{s2}$ can be calculated from (48) and (75) for extrusion 1775 ($h_e = 0.005$ in., $v_i = 2.25$ in./sec, $T_{pb}$ is lower than one for extrusion 1776) and for extrusion 1776 ($h_e = 0.005$ in., $v_i = 2.5$ in./sec, $T_{pb} = 2000$ deg F). Then the

average quantity is obtained by $(\tau_{s1})_{\text{ave}} = \left( \int_0^{l_1} \tau_{s1} dx \right) \bigg/ l$, and $(\tau_{s2})_{\text{avg}} \equiv \tau_{s2}$.

For the present paper, in the absence of rigid material between Zone I and II, $\sigma_x + p = \sigma_0$ is valid for both Zone I and II, in view of the continuity of $p$ and $\sigma_x$ at the interface of two zones. However, if rigid material, suggested by Dr. Wilson in Zone III, is introduced to exist between Zones I and II, the expression of $\sigma_x + p = -\sigma_0$ is than valid for Zone II. This is due to rigid body on the left side of Zone II.