The remaining boundary conditions can be met using the same approach as used for the normal stress conditions [12]. These conditions yield the matrix given in Table 1.

Green's Function for Shear

The tangential deflection on the surface from equation (A1) can be expressed as:

$$u=-\frac{(1-\nu^2)}{\pi E}\int_0^\infty \frac{d^2G}{d\eta^2}\frac{\cos s\zeta}{s}ds \tag{A13}$$

where

$$\frac{d^2G}{d\eta^2}=-2(B_1-D_1)s \tag{A14}$$

The Green's function can be written with reference to an arbitrary displacement V_1 at $\zeta=1$

$$u-u_1=\frac{1-\nu^2}{\pi E}\left\{\int_0^\infty 2(B_1-D_1)s\frac{\cos s\zeta-\cos s}{s}ds-2\beta\ln\zeta\right\} \tag{A15}$$

It can be shown, using the matrix in Table 1 that for larger values of s, $(B_1-D_1)\rightarrow 1/s$. Equation (A15) can be expressed as two integrals $(0<s<s_0)$ and $(s_0<s)$, as given in the text.

If we let

$$c_{ij}=u-u_1$$

then this equation corresponds to equation (5) in the text.

Because the shear deformation is calculated as a relative displacement, a reference point must be selected. The displacements calculated from equation (A15) becomes small for large ζ; therefore a reasonable assumption for the reference point is 5 contact widths $(x=-10b)$. Then the relative tangential deflection (ϵ_i) is:

$$\epsilon_i=\sum_{j=1}^{n}(c_{ij}-c_{0j})\tau_j\Delta x \tag{A16}$$

assuming $i=0$ is the reference point.

DISCUSSION

S. H. Loewenthal[1] and D. A. Rohn[1]

The authors deserve much credit for extending the well known tangentially strained, rolling element contact problem to include the effects of a soft layer. This study is particularly timely since the use of simple, zero backlash, low torque ripple, traction drive positioning mechanisms is currently receiving greater scrutiny [14].

The initial impetus for this line of investigation stems from the classic, 1927 Carter study [6] of a locomotive wheel in driving contact against a rail. The present work is a welcome departure from Carter's and subsequent analyses which are restricted to homogeneous, isotropic bodies in driving contact. The layered cylinder solution not only has relevance to the numerous paper and tape handling machines which generally employ elastomer coated, pinch roller drive mechanisms, but moreover has implications for high precision actuators for spacecraft. Examples include a wide class of pointing and tracking mechanisms such as optical and antenna positioners, robotic hinges and pivots as well as control moment gyro gimbal drives. Here the introduction of a self-lubricating polymeric, organic, or soft metal film coating is particularly advantageous to avoid the difficulties of liquid lubrication in space. However, the detrimental effects of such coatings on open loop positional accuracy needs to be addressed.

In the authors' investigation, the "start-up" problem that is the transition from a stationary, unloaded roller contact to a tangentially loaded rolling one, appears to be handled in two parts, judging from the discussion of Fig. 3. First the static contact is strained to the desired traction level and then the rollers are allowed to rotate under constant traction, that is fresh roller material is allowed to "flow" through the contact under a strain field that is invariant with time. In practice, a two mode start-up would not occur since torque buildup and rolling would occur simultaneously as the shaft and components downstream of the driver roller experienced torsional "windup." In this latter case, "fresh" or unstrained material would sweep through the contact being exposed to a microslip region which was steadily increasing and changing position within the contact with respect to torque, hence time. Intuitively, one would expect a different displacement to occur than when this sweeping action took place while the locked and microslip regions were "frozen" at the pre-selected value of torque. In an attempt to address this time dependent aspect of the start-up problems, a stepwise method was used to combine the static and rolling solution in an approximate fashion in the discussers' analysis of a torsionally strained, point contact roller appearing in [15]. Can the authors' analysis account for the simultaneous torque/rolling buildup case and, if so, what is the magnitude of the error in separating these effects?

In most pointing and tracking mechanisms applications, as with servo positioners, the actuator will be required to "hunt" or "dither" around a demand position or a part may need to be reregistered. For such applications, the roller drive would be torqued while moving in one direction, stop and have negative torque applied while moving in the opposite direction, thereby forming a hysteresis loop. Would the authors' care to comment on how their analysis could be adapted to predict a hysteresis loop? Presumably the width of this loop, hence hysteresis loss, would be quite sensitive to the thickness of the soft layer due to its low stiffness characteristics.

Finally, the discussers would have liked to have seen some comparison of the results of the authors' analysis with some of the classical, non layered contact solutions as identified in the paper's introduction. Can the authors comment as to the extent of agreement they found with previous methods? Are there any experimental data which may help our understanding of the tangentially strained, line contact problem?

In closing, the discussers would like to commend Dr. Kannel and Professor Dow for tackling a relatively formidable problem in contact mechanics. We look forward to subsequent work in this area from them.

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Additional References

14 Loewenthal, S. H., Rohn, D. A., and Steinetz, B. M., "Application of Traction Drives as Servo Mechanisms," *Proceedings of the 19th Aerospace Mechanisms Symposium*, NASA CP-2371, 1985.

15 Rohn, D. A., and Loewenthal, S. H., "An Analysis of Traction Drive Torsional Stiffness," NASA Paper 84-DET-100, 1984.

Author's Closure

We would like to thank the discussers for their comments and questions. The initial "wind up" occurs in order to generate sufficient torque to rotate the driven roller against a fixed traction load (or torque). The analysis does not require that the load be fixed in time, but it does require that the load be known. It would seem that if the model the discussers used involved a small load build up at each step, their predictions would be valid.

The "hunting" phenomenon in the control system always poses backlash related problems. The authors plan to, at least partially, address the problem in a future publication. It is interesting that small levels of hunting (less than a half width of contact) do not produce a large hysteresis loop. Very little microslipage occurs during the initial motion reversal.

The curves of Fig. 3 were developed for correlation with Kalker's [4] curve to verify the solution. The agreement between the two theories is quite good considering the difference of the two approaches. The predictions have also been compared with measurement as will be described in a future publication.