large shearing stress at the coating/substrate interface, leading toward delamination of the coating.

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APPENDIX

$$A_1 = [N_2(i-1, j) + N_3(i, j-1)]/(4\xi_{\bar\xi}\eta_{\bar\eta}), \tag{17}$$

$$A_2 = N_1\left(i-\frac{1}{2}, j\right)/\xi_{\bar\xi}{}^2 + N_1(i, j).\xi_{\bar\xi\bar\xi}/(2\xi_{\bar\xi}{}^3), \tag{18}$$

$$A_3 = -[N_2(i-1, j) + N_3(i, j+1)]/(4\xi_{\bar\xi}\eta_{\bar\eta}), \tag{19}$$

$$A_4 = N_3\left(i, j-\frac{1}{2}\right)/\eta_{\bar\eta}{}^2 + N_3(i, j).\eta_{\bar\eta\bar\eta}/(2\eta_{\bar\eta}{}^3), \tag{20}$$

$$\begin{aligned} A_5 = &-\left\{\left[N_1\left(i+\frac{1}{2}, j\right) + N_1\left(i-\frac{1}{2}, j\right)\right]/\xi_{\bar\xi}{}^2 + \right. \\ &\left. + \left[N_3\left(i, j+\frac{1}{2}\right) + N_3\left(i, j-\frac{1}{2}\right)\right]/\eta_{\bar\eta}{}^2\right\}, \end{aligned} \tag{21}$$

$$A_6 = N_3\left(i, j+\frac{1}{2}\right)/\eta_{\bar\eta}{}^2 - N_3(i, j)\eta_{\bar\eta\bar\eta}/(2\eta_{\bar\eta}{}^3), \tag{22}$$

$$A_7 = -[N_2(i+1, j) + N_3(i, j-1)]/(4\xi_{\bar\xi}\eta_{\bar\eta}), \tag{23}$$

$$A_8 = N_1\left(i+\frac{1}{2}, j\right)/\xi_{\bar\xi}{}^2 - N_1(i, j).\xi_{\bar\xi\bar\xi}/(2\xi_{\bar\xi}{}^3), \tag{24}$$

$$A_9 = [N_2(i+1, j) + N_3(i, j+1)]/(4\xi_{\bar\xi}\eta_{\bar\eta}), \tag{25}$$

$$B_1 = [N_2(i, j-1) + N_3(i-1, j)]/(4\xi_{\bar\xi}\eta_{\bar\eta}), \tag{26}$$

$$B_2 = N_3\left(i-\frac{1}{2}, j\right)/\xi_{\bar\xi}{}^2 + N_3(i, j).\xi_{\bar\xi\bar\xi}/(2\xi_{\bar\xi}{}^3), \tag{27}$$

$$B_3 = -[N_2(i, j+1) + N_3(i-1, j)]/(4\xi_{\bar\xi}\eta_{\bar\eta}), \tag{28}$$

$$B_4 = N_1\left(i, j-\frac{1}{2}\right)/\eta_{\bar\eta}{}^2 + N_1(i, j).\eta_{\bar\eta\bar\eta}/(2\eta_{\bar\eta}{}^3), \tag{29}$$

$$\begin{aligned} B_5 = &-\left\{\left[N_3\left(i-\frac{1}{2}, j\right) + N_3\left(i+\frac{1}{2}, j\right)\right]/\xi_{\bar\xi}{}^2 + \right. \\ &\left. + \left[N_1\left(i, j+\frac{1}{2}\right) + N_1\left(i, j-\frac{1}{2}\right)\right]/\eta_{\bar\eta}{}^2\right\}, \end{aligned} \tag{30}$$

$$B_6 = N_1\left(i, j+\frac{1}{2}\right)/\eta_{\bar\eta}{}^2 - N_1(i, j)\eta_{\bar\eta\bar\eta}/(2\eta_{\bar\eta}{}^3), \tag{31}$$

$$B_7 = -[N_2(i, j-1) + N_3(i+1, j)]/(4\xi_{\bar\xi}\eta_{\bar\eta}), \tag{32}$$

$$B_8 = N_3\left(i+\frac{1}{2}, j\right)/\xi_{\bar\xi}{}^2 - N_3(i, j).\xi_{\bar\xi\bar\xi}/(2\xi_{\bar\xi}{}^3), \tag{33}$$

$$B_9 = [N_2(i, j+1) + N_3(i+1, j)]/(4\xi_{\bar\xi}\eta_{\bar\eta}), \tag{34}$$

where $\bar\xi$ and $\bar\eta$ are the coordinates of the transformed plane.

DISCUSSION

F. F. Ling[1]

Once again, Professor Ju and his proteges have extended their interesting and useful series of solutions of problems in surface mechanics. In recent years they have tackled the problem set of thermal mechanical effects due to moving load; moreover, these problem sets include layer media and cavities.

One of the authors' conclusions, (iii), is "for the same Peclet number (R_1), the critical ligament thickness (L_{cr}) is smaller than the critical depth (η_{cr}) of a coating material. This coincides with this discusser's intuition. Their conclusion (ii) seems to be a redundant one. Conclusion (ii) states "the location of the cavity influences thermal stress, which reaches a maximum, . . . , at the critical ligament thickness of $L_c = 0.094$ for both cases of a single material with a cavity and a layer medium with a cavity." First of all, the authors are talking about same material for the single as well as the layer material. Earlier on in the paper, the author wrote " . . . maximum tensile thermal stress in the case of a single material with a rectangular cavity is much higher . . . the occurrence of the maximum tensile stress is at the trailing corner of the cavity, which defines a critical thickness L_{cr} . . . ." As such the single material with a cavity and the layer medium with a cavity is one and the same.

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Authors' Closure

Professor Ling's observation is very much to the point. We are grateful to have the opportunity to clarify some points which we should have done so in the main text. The critical ligament thickness is indeed intuitively apparent to be smaller than the critical depth of the coating material. The paper did quantify the critical ligament thickness through a numerical extremizing process as shown in Fig. 4. The relationship can be expressed algebraically as:

$$R(L_{cr})^{1.83} = 17.53,$$

where R is the Peclet Number and L_{cr} is the critical ligament thickness. The authors agree that the conclusion in (ii) could have been included in (iii). As for the critical depth and the critical ligament thickness, both are characteristics of the coating material as a single material. However their effect on the thermo-mechancial field in a coated medium is very significant, if the coating thickness is in the neighborhood of the critical ligament thickness as demonstrated in Fig. 7. In the figure the same coating material, with changes in stiffness, is shown to result in much higher thermal principal stress than the case of single materials.