the shaft and a noncontacting FMR mechanical face seal. The method was then applied to analyze an actual test rig. The results from the CETM method were compared to results of a closed-form solution of an FMR seal, a solution that was limited to rigid body dynamics (i.e., did not include shaft flexibility or axial offset of the rotor center of mass).

The results show that when a seal is being driven by a slender shaft, the seal dynamics is greatly affected by the shaft dynamics even at relatively low operating speeds. This particularly holds in high speed applications. The flexibility of the shaft and the axial offset of the rotor center of mass were found to have an adverse effect on the dynamic behavior of a seal, where the latter enhances the response at resonance.

In the test rig under consideration the driving shaft was especially designed to be very stiff, therefore, its effect on the dynamic response of the seal was negligible in the designed speed range. Here the CETM method and the closed-form solution produced practically identical results. In general seal applications, however, the closed-form solution may not realistically predict the seal dynamic response.

The CETM method established here offers a complete dynamic analysis of a seal tribosystem including the effects of the shaft, fluid film, secondary seal, flexibly mounted rotating element, and axial offset. The method is modular and can accommodate other triboelements such as bearings, gears, and the like, to provide comprehensive analysis of elaborate tribosystems.

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APPENDIX

The numerical values for the various parameters of the actual test rig (Lee and Green, 1992) are

EI	$= 1338.2\ \mathrm{Pa \cdot m^4}$	d	$= 0.005$ m
I_{p1}	$= 3.2847 \times 10^{-6}\ \mathrm{kg \cdot m^2}$	I_{p2}	$= 2.4447 \times 10^{-5}\ \mathrm{kg \cdot m^2}$
I_{p3}	$= 2.5027 \times 10^{-6}\ \mathrm{kg \cdot m^2}$	I_{p4}	$= 2.0652 \times 10^{-5}\ \mathrm{kg \cdot m^2}$
I_{p5}	$= 4.1619 \times 10^{-4}\ \mathrm{kg \cdot m^2}$	I_{t1}	$= 8.3497 \times 10^{-6}\ \mathrm{kg \cdot m^2}$
I_{t2}	$= 1.2513 \times 10^{-5}\ \mathrm{kg \cdot m^2}$	I_{t3}	$= 4.2175 \times 10^{-6}\ \mathrm{kg \cdot m^2}$
I_{t4}	$= 1.0829 \times 10^{-5}\ \mathrm{kg \cdot m^2}$	I_{t5}	$= 2.8032 \times 10^{-4}\ \mathrm{kg \cdot m^2}$
l_1	$= 0.01667$ m	l_2	$= 0.01984$ m
l_3	$= 0.01588$ m	l_4	$= 0.01667$ m
m_1	$= 0.07241$ kg	m_2	$= 0.08621$ kg
m_3	$= 0.05517$ kg	m_4	$= 0.08803$ kg
m_5	$= 0.5198$ kg	γ_{ri}	$= 4 \times 10^{-4}$ rad
K_f	$= 1134.5\ \mathrm{N \cdot m/rad}$	K_s	$= 5.35 + 146.1\ \omega^2/(36.36 + \omega^2)\ \mathrm{N \cdot m/rad}$
D_f	$= 2.1476\ \mathrm{N \cdot m \cdot s/rad}$	D_s	$= 881.4/(36.36 + \omega^2)\ \mathrm{N \cdot m \cdot s/rad}$

The dependency of K_s and D_s upon frequency was obtained from experiments done on a support consisting of a spring and two Buna-N O-rings (Lee, 1992). The fluid film coefficients K_f and D_f were calculated for water pressure of 0.283 MPa and viscosity of 0.89 mPa·s. The rotor input forcing misalignment, $\gamma_{ri} = 0.4$ mrad, is in the bulk of measurements. (The numerical value of the latter is actually insignificant because results are presented in a transmissibility form, i.e., ratio of output to input.) The various lumped masses and moments of inertia were calculated based upon the geometry of the shaft and the rotor, and upon the length of section i (the mass of the spring, however small, is lumped into the mass of the rotor). The index $i = 1$ to 4 corresponds to the shaft, and $i = 5$ represents the rotor. EI is the flexural rigidity of the shaft, and $d = 5$ mm is a generous estimate of the center of mass axial offset.

DISCUSSION

R. Metcalfe[1]

The coupling of shaft and end face seal rotordynamics is an interesting extension of previous work. This has been studied for high speed pumps with annular and labyrinth seals, but for end face seals little is known about their dynamic interactions with machines in which they are installed. In general, this is because few problems of this kind have been identified, though they may be more common than is known.

At the discusser's company, seal ring responses to various misaligned conditions were measured more than a decade ago. When their O-ring supports were tested for stiffness and damping coefficients to use in analysis, it was found they behaved far differently from ideal. Not only were their responses to harmonic displacement dependent on frequency, as mentioned in the authors' appendix, but friction and hysteresis dominated

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their form. It was concluded that any analysis that assumed ideal stiffness and damping was inherently deficient. A further concern was friction from the anti-rotation pins or lugs that normally transmit the driving torque. Did the authors similarly find their O-ring response to be far from ideal? How sensitive were the numerical results to variations of stiffness and damping? Could the authors method be adapted to include the empirically-determined O-ring response, as opposed to the idealistic representation?

Mark S. Darlow[2]

I would like to congratulate the authors on a well thought out paper that advances the state-of-the-art in areas both directly and indirectly related to the subject of mechanical face seals. The contribution to the analysis of noncontacting face seals is obvious in that it illustrates how derived face seal properties, which are shown to be quite linear, can be incorporated into a transfer matrix analysis. The authors show directly how the seal dynamics are influenced by the dynamics of the rotor system and imply the converse when the rotor is not substantially stiffer than the seal.

It is interesting to note that while the introduction of an axial offset to the location of the seal mass has a dramatic effect on the transmissibility and phase of the seal, there is no significant change in the resonant frequency of the system. This is presumably due to the fact that although the mass of the seal, which is by far the largest mass in the model, is moved a significant distance away from the built-in end of the shaft, the angular stiffness of the rotor shaft connection is so much less than that of the shaft itself that we can consider the mass to remain attached to the pivot point with the addition of a small amount of rotatory inertia at that point.

An additional contribution of this paper, which is applicable beyond the area of seal analysis, is the interesting new approach taken to the construction of the transfer matrices. Traditionally, forcing functions (including mass unbalance) are incorporated in the analysis through the use of forcing function vectors that are added to the state vector after multiplication by the corresponding point matrix. This is fine when a step-by-step approach to moving through the model and calculating an overall transfer matrix is used. However, with modern matrix analysis software tools that are now generally available on large, as well as small, computers, the multiplication of a series of point and field matrices is more convenient and less sensitive to the accumulation of round-off errors. It is possible to construct a single equation to represent the overall transfer matrix using the traditional point and field matrix constructions, but the resulting equation will be of the following form

$$S_n = A_n A_{n-1} \cdots A_2 s_1 + \left[\sum_{i=2}^{n-1} (A_n A_{n-1} \cdots A_{i+1}) \begin{Bmatrix} 0 \\ f \end{Bmatrix}_i \right] + \begin{Bmatrix} 0 \\ f \end{Bmatrix}_n$$

which is no more convenient to apply than a step-by-step solution. The complex extended transfer matrix approach, on the other hand, seeks to incorporate the forcing function terms directly into the point matrix by enlarging the point and field matrices by one row and one column. This representation could be used in transfer matrix analysis in general and provides for a much more convenient solution of the problem.

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The model data provided in the appendix is very useful to give the reader a sense of the scale of the model and the magnitude of the response. One additional item that would be of interest to the reader is the initial, axial clearance of the seal.

Author's Closure

The authors thank Drs. Metcalf and Darlow for their interest in the paper and for their thoughtful discussions. The purpose of this paper is to provide a comprehensive analytical tool to analyze complex tribosystems. As with any analysis the results are as good as the assumptions.

It is also the authors' belief that the O-ring secondary seal is the "Achilles heel" of designs that require very small and controllable motions such as in mechanical seals. It is our experience that as long as there is unrestricted small O-ring flexing the modeling of the O-ring as "ideal" stiffness and damping coefficients is quite realistic. This was verified in many repeatable tests such as in Green and Etsion (1986) and Lee (1992). But other design parameters may hinder this representation. For example, it was found by Green and Etsion (1986) that at high pressures the O-ring greatly stiffens effectively locking the flexibly mounted element. Breakaway frictional force is another nonlinear effect occurring at relatively large motions. To overcome some of these problems in the test rig the authors resorted to a two O-ring secondary seal system with a small squeeze, as described in Lee and Green (1992). From a numerical view point it is well known that in seals for incompressible fluids the stiffness and the damping of the fluid are typically a few orders of magnitude higher than those of the O-ring secondary seal. Therefore, the calculated seal response is little affected by the O-ring representation. This, however, may not be true for seals for compressible fluids and low pressure. While it is convenient (and often plausible) to use "ideal" stiffness and damping in analyses, the current method is not limited to such a representation. An empirically-determined O-ring response can replace the frequency-dependent O-ring impedance in Eq. (5).

The presence of friction in any mechanical element will invariably introduce a nonlinear effect. In which case an "exact" closed-form solution would generally not be feasible. This nonlinearity, however, can be "linearized" by translating the dissipating frictional energy into equivalent dissipating viscous-damping energy (see for example the additional reference, Thomson (1988), pp. 70–74). The anti-rotation pins are another source of nonlinearity. Not only because of friction but also because of the uncertainty in the kinematical conditions that they impose. Undoubtedly, three or more active anti-rotation pins will lock the flexibly mounted element in the angular mode and, therefore, no more than two pins should ever be used. Not such attention is typially given to the manufacturing of these pins, and an analysis which accounts for all possible designs is a formidable task. Some of these aspects and the role of the anti-rotation pins are addressed in the additional reference, Green and Etsion (1986).

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