From the heat balance between the inlet heat flow at the rotor face 2 and the outlet heat flow at the rotor outer surface 3 we have

$$k \frac{\partial T}{\partial y}\bigg|_{y=0} 2\pi r dr = q_\xi \tag{A4}$$

Also from the heat balance between q_ξ at the rotor surface 3 and the heat flow convected by the surrounding fluid we have

$$q_\xi = H(T_3 - T_f)2\pi r_0 dz \tag{A5}$$

where (see Fig. A2)

$$dz = dr \cdot \tan \varphi \tag{A6}$$

From Eqs. (7) and (11)

$$\frac{\partial T}{\partial y}\bigg|_{y=0} = \frac{\mu\omega^2 r^2}{kh} \tag{A7}$$

Hence, using Eqs. (A3), (A6), and (A7) in Eqs. (A4) and (A5) we have

$$\frac{2\mu\omega^2 r^3}{h} = \frac{k^*(T_2 - T_3)\sin 2\varphi}{\ln(r_0/r)} \tag{A8}$$

and

$$\frac{k^*(T_2 - T_3)\cos^2 \varphi}{\ln(r_0/r)} = H(T_3 - T_f)r_0 \tag{A9}$$

DISCUSSION

R. A. Burton[1]

This paper is a clearly thought-out and clearly presented contribution to the very difficult problem of the thermotribology of seals. The discusser has been concerned with different aspects of the problem, with emphasis on thermoelastic interactions. Although the assumption of axial symmetry excludes many of these interactions, nevertheless some of the notions of our analyses may shed some light on the findings presented here.

Figure 9 shows data for cooling of the cylindrical surface of a rotating disk. This was published in ASME Transactions [1], and showed that the earlier work on long cylinders applied as well to short cylinders [2, 3]. Subsequent measurements by A. Kistler, of Northwestern University indicate that this effect is not associated with gross movements of the fluid, but is quite local. For example, the film is re-established in a short distance behind a dam. Figure 1 is therefore felt to represent a good estimate of heat transfer from a turning cylinder.

Nusselt number, Nu, can be shown to correspond to D/δ, where D is the diameter, and δ is the thickness of stagnant coolant that would have the same thermal resistance as the convective cooling effects. From Fig. 1 it is seen that δ/D varies from about 0.01 to 0.025 over the range of Reynolds number. The ratio of conductivities, r, for stainless steel and oil is $r \approx 133$, and for stainless steel and water is $r \approx 25$. Consequently, the thermal resistance can be expressed as a thickness of metal, δ_m, for operation in the two fluids, where:

$$\delta_m/D = (\delta/D)r$$

For the above numbers we find:

In oil, $3.32 > \delta_m/D > 1.33$

In water, $0.63 > \delta_m/D > 0.25$

To get a bound on expected radial temperature drop, take the case where all of the heat passes radially outward from r_i to r_0. For $r_0 - r_i = 0.1D$, the temperature drop may be expressed as a fraction of the drop from r_0 to the cooling fluid bulk temperature, t_{coolant}, since:

$$(t_i - t_0)/(t_0 - t_{\text{coolant}}) = (r_0 - r_i)/\delta_m = 0.1D/\delta_m$$

Using the above results to evaluate δ_m/D, one finds:

$(t_i - t_0)/(t_0 - t_{\text{coolant}})$

In oil 0.03 to 0.075

In water 0.16 to 0.4

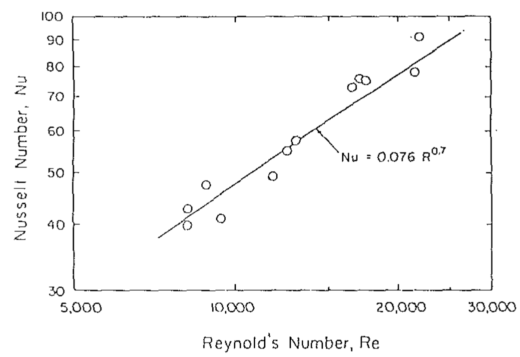

Fig. 9

This suggests that in the oil, the radial temperature drop is not a tremendously important factor, whereas it is much more important in water, which is in accord with the results reported by the authors. The drop of viscosity in the hotter region would bias the heat generation toward the outer edge of the ring, shortening the thermal path through the metal. Coning acts to counter this effect and move the radial distribution of heating inward.

We have recently studied similar interactions in a related problem, the coupling of heating and waviness, which may aid in conceptualizing this coupling more fully [4], in a somewhat different context.

Additional References

1 Heckmann, S. R., and Burton, R. A., "A Theoretical Study of the Effects of Cooling on Thermoelastic Contact Instabilities," ASME Journal of Lubrication Technology, Vol. 99, 1977, pp. 247–253.

2 Wottring, J. M. S., M. S. Project Report, Northwestern University, Evanston, IL, 1975.

3 Etemad, G. A., "Free Convection Heat Transfer From a Rotating Cylinder to Ambient Air With an Interferometric Study of Flow," *Trans. ASME*, 1955, pp. 1283–1289.

4 Burton, R. A., "The Coupling of Waviness and Heating in a Seal," *STLE Trans.*, in press.

Authors' Closure

The authors would like to thank Dr. Burton for his comments and interest in the paper. Figure 9 of the discussion is useful for evaluating the Nusselt number. In the paper we used very similar results that were published in Becker (1963).

Dr. Burton's approach for estimating the radial temperature drop is indeed an interesting one, and we are pleased to see that his results are in accord with ours. It should be noted, however, that although the dimensionless temperature drop, as presented here, is lower in the oil as compared to the water, the actual temperature difference $T_i - T_0$ may be the same. This is because $T_0 - T_{\text{coolant}}$ is higher in the oil.

[1]Burton Technologies Inc., Raleigh, NC 27636-8809.