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APPENDIX

The equilibrium of a differential control volume gives us

$$\frac{dp}{dx} = \frac{\partial \tau}{\partial y} \tag{A-1}$$

Integrating to obtain the dimensionless shear stress as

$$\frac{\tau}{\tau_0} = \frac{y}{\tau_0}\frac{dp}{dx} + \frac{\tau_1}{\tau_0} \tag{A-2}$$

where τ_1 is the shear stress on the lower surface ($y = 0$) to be determined. The Eyring's nonlinear fluid model is expressed as:

$$\frac{\partial u}{\partial y} = \frac{\tau_0}{\eta} \sinh\left(\frac{\tau}{\tau_0}\right) \tag{A-3}$$

Using the relation

$$\sinh(x) = \frac{e^x - e^{-x}}{2} \tag{A-4}$$

and substituting (A-2) into (A-3) gives

$$\frac{\partial u}{\partial y} = \frac{\tau_0}{2\eta}\left[\text{Exp}\left(\frac{y}{\tau_0}\frac{dp}{dx}\right)\text{Exp}\left(\frac{\tau_1}{\tau_0}\right) - \text{Exp}\left(-\frac{y}{\tau_0}\frac{dp}{dx}\right)\text{Exp}\left(-\frac{\tau_1}{\tau_0}\right) \right] \tag{A-5}$$

Integrating again to obtain

$$u = \int_0^h \frac{\tau_0}{2\eta}\left[\text{Exp}\left(\frac{y}{\tau_0}\frac{dp}{dx}\right)\text{Exp}\left(\frac{\tau_1}{\tau_0}\right) - \text{Exp}\left(-\frac{y}{\tau_0}\frac{dp}{dx}\right)\text{Exp}\left(-\frac{\tau_1}{\tau_0}\right) \right]dy + C_1 \tag{A-6}$$

The integration constant C_1 can be determined by using the boundary conditions, i.e.,

$$y = 0, \ u = u_1$$
$$y = h, \ u = u_2$$

Define sliding velocity u_d as

$$u_d = u_2 - u_1 \tag{A-7}$$

therefore, the following expression can be obtained:

$$u_d = \frac{\tau_0}{2}\left\{ \text{Exp}\left(\frac{\tau_1}{\tau_0}\right)\int_0^h \frac{1}{\eta}\text{Exp}\left(\frac{y}{\tau_0}\frac{dp}{dx}\right)dy - \text{Exp}\left(-\frac{\tau_1}{\tau_0}\right)\int_0^h \frac{1}{\eta}\text{Exp}\left(-\frac{y}{\tau_0}\frac{dp}{dx}\right)dx \right\} \tag{A-8}$$

Let

$$A_1 = \int_0^h \frac{1}{\eta}\text{Exp}\left(\frac{y}{\tau_0}\frac{dp}{dx}\right)dy \tag{A-9}$$

$$A_2 = \int_0^h \frac{1}{\eta}\text{Exp}\left(-\frac{y}{\tau_0}\frac{dp}{dx}\right)dy \tag{A-10}$$

equation (A-8) can be rewrite as

$$\frac{2u_d}{\tau_0} = A_1\text{Exp}\left(\frac{\tau_1}{\tau_0}\right) - A_2\text{Exp}\left(-\frac{\tau_1}{\tau_0}\right) \tag{A-11}$$

Solving equation (A-11), the variable τ_1/τ_0 can be obtained as:

$$\frac{\tau_1}{\tau_0} = \ln\left[\frac{u_d + \sqrt{u_d^2 + \tau_0^2 A_1 A_2}}{\tau_0 A_1} \right] \tag{A12}$$

DISCUSSION

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The inclusion of the energy equation in any non-Newtonian analysis of elastohydrodynamic lubrication is an important step. The rheological and thermal properties of the liquid are dependent upon local temperature. The discussers are, however, puzzled by the rheological model selected by the authors. The logarithmic shear stress nature of the Eyring model with properties averaged over the entire contact corresponds well with the average shear stress observed in EHD traction. However, when applied locally as a rheological equation of state, it fails to represent the most pronounced non-Newtonian effect observed in primary measurements—that of a rate independent limiting shear stress (Bair and Winer, 1979, 1982, 1990 and Ramesh and Clifton, 1987). It is interesting to note that the authors reference our 1979 paper showing limiting shear stress behavior for EHD lubricants at high pressure and shear stress.

This limiting stress has been observed in concentric cylinder rheometers (Bair and Winer, 1979, 1982, 1990) with a pressurization time on order of minutes, and impact pressure shear plate experiments (Ramesh and Clifton, 1987) for which the pressurization time was a few hundred nanoseconds. In the 1990 paper it was shown that the apparent logarithmic variation of average shear stress with apparent shear rate (which has been a justification of the Eyring model) for EHD is a consequence of the growth of the plastic controlled region with increasing shear rate. No primary evidence exists which supports Eyring's Sinh Law behavior in lubricants at high pressure. Why then has so much time been spent incorporating it into an EHD analysis?

Additional References

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