Conclusions

A formula for the thermal and non-Newtonian (Ree-Eyring) film thickness reduction factor, C_r, has been obtained by regression based on numerical results. This formula, equation (65), shows a decrease in C_r with an increase of L_1, ξ, and g_2, and with a decrease of g_1. The change of the reduction factor, C_r, with L_1 is influenced by the value of g_1. The formula also shows that, even when the thermal loading parameter, L_1, approaches zero, there still exists a film thickness reduction due to non-Newtonian effects. The solutions presented are useful in the design of gears, rolling element bearings and cams where elastohydrodynamic line contacts exist.

The changes in the ratio, $H_{\min}/H_0$, with g_1 and ξ depend strongly on the changes in the shape of the pressure distribution. This ratio decreases with increasing g_1 except for low ξ and L_1, and decreases with increasing ξ except for low g_1 and L_1.

The maximum midlayer and surface temperatures are observed to increase with an increase in L_1, ξ, or g_1. The maximum midlayer temperature in the lubricant is significantly affected by the dependence of lubricant thermal conductivity on pressure.

The traction coefficient is observed to increase slightly with an increase of L_1 for low ξ and L_1, and to decrease with an increase of L_1 for all other conditions.

Acknowledgment

The authors wish to express their appreciation to the National Center for Supercomputing Applications and the National Science Foundation for allocating computing time for this research on the CRAY X-MP/48 supercomputer at the University of Illinois at Urbana-Champaign.

References

Bair, S., and Winer, W. O., 1979, "A Rheological Model for EHL Contacts Based on Primary Laboratory Data," ASME *Journal of Lubrication Technology*, Vol. 101, pp. 258–265.

Cheng, H. S., and Sternlicht, B., 1965, "A Numerical Solution for the Pressure, Temperature and Film Thickness Between Two Infinitely Long, Lubricated Rolling and Sliding Cylinders, Under Heavy Loads," ASME *Journal of Basic Engineering*, Vol. 87, pp. 695–707.

Conry, T. F., 1981, "Thermal Effects on Traction in EHD Lubrication," ASME Journal of Lubrication Technology, Vol. 103, pp. 533–538.

Conry, T. F., Johnson, K. L., and Owen, S., 1980, "Viscosity in the Thermal Regime of EHD Traction," *Thermal Effects in Tribology*, Proc. 6th Leeds-Lyon Symp. on Tribology, Dowson, D., Taylor, C. M., Godet, M., and Berthe, D., Editors, Mech. Engr. Pub. Ltd., London, pp. 219–227.

Conry, T. F., Wang, S., and Cusano, C., 1987, "A Reynolds-Eyring Equation for Elastohydrodynamic Lubrication in Line Contacts," ASME Journal of Tribology, Vol. 109, No. 4, pp. 648–658.

Crook, A. W., 1957, "Simulated Gear-Tooth Contact: Some Experiments Upon Their Lubrication and Sub-Surface Deformation," *Proc. I. Mech. E., London*, Vol. 171, pp. 187–199.

Crook, A. W., 1958, "The Lubrication of Rollers, Part I," *Phil. Trans. Royal Soc.*, London, Series A, Vol. 250, pp. 387–409.

Crook, A. W., 1961, "Lubrication of Rollers, Part III," *Phil. Trans. Royal Soc.*, London, Series, A, Vol. 254, pp. 237–258.

Dowson, D., and Higginson, G. R., 1959, "A Numerical Solution to the Elastohydrodynamic Problem," *J. Mech. Engr. Sci.*, Vol. 11, No. 1, pp. 7–15.

Dowson, D., and Higginson, G. R., 1977, *Elasto-hydrodynamic Lubrication*, SI Ed., International Series in Materials Sci. and Tech., Vol. 23, Pergamon Press.

Dowson, D., Toyoda, S., 1978, "A Central Film Thickness Formula for Elastohydrodynamic Line Contacts," *Elastohydrodynamics and Related Topics*, Proc. 5th Leeds-Lyon Symp. on Tribology, Mech. Engr. Pub. Ltd., pp. 60–67.

Ertel, A., Mohrenstein-, 1984, "Die Berechnung der hydrodynamischen Schmierung gekrummter oberflaschen unter hoher Belastung und Relativbewegung," *Fortschr. Ber. VDIZ*, Reihe 1, No. 115, (ISBN 3-18-141501-4).

Galvin, G. D., Naylor, H., and Wilson, A. R., 1964, "The Effect of Pressure and Temperature on Some Properties of Fluids of Importance in Elastohydrodynamic Lubrication," *Proc. I. Mech. E.*, Vol. 178, Part 3N, pp. 283–290.

Gecim, B., and Winer, W. O., 1980, "Lubricant Limiting Shear Stress Effect on EHD Film Thickness," ASME Journal of Lubrication Technology, Vol. 102, pp. 213–221.

Goglia, P. R., 1982, "The Effects of Surface Irregularities on the Elastohydrodynamic Lubrication of Sliding Line Contacts," Ph.D. thesis, Department of Mechanical and Industrial Engineering, University of Illnois at Urbana-Champaign.

Greenwood, J. A., 1969, "Presentation of Elastohydrodynamic Film-Thickness Results," *J. Mech. Engr. Science*, Vol. 11, No. 2, pp. 128–132.

Grubin, A. N., and Vinogradova, I. E., 1949, "Investigation of the Contact of Machine Components," *Central Scientific Research Institute for Technol. and Mech. Engr.*, Book No. 30, Moscow (D.S.I.R. Translation, No. 337).

Hahn, S. J., Eyring, H., Higuchi, I., and Ree, T., 1958, "Flow Properties of Lubricating Oils Under Pressure," *NLGI Spokesman*, Vol. 22, June, pp. 121–128.

Johnson, K. L., 1970, "Regimes of Elastohydrodynamic Lubrication," *J. Mech. Engr. Science*, Vol. 12, No. 1, pp. 9–16.

Johnson, K. L., and Cameron, R., 1967–1968, "Shear Behaviour of EHD Films at High Rolling Contact Pressure," *Proc. I. Mech. E.*, Vol. 182, Part 1, pp. 307–319.

Johnson, K. L., and Tevaarwerk, J. L., 1977, "Shear Behaviour of EHD Oil Films," *Proc. Royal Soc.*, London, Series A, Vol. 356, pp. 215–236.

Martin, H. M., 1916, "Lubrication of Gear Teeth," *Engineering, London*, Vol. 102, p. 119.

Ree, T., and Eyring, H., 1955, "Theory of Non-Newtonian Flow. I. Solid Plastic System," and "Theory of Non-Newtonian Flow. II. Solution System of High Polymers," *J. Appl. Physics*, Vol. 26, No. 7, pp. 793–800, (Part 1) and pp. 800–809 (Part II).

Richmond, J., Nilsson, O., and Sandberg, O., 1984, "Thermal Properties of Some Lubricants Under High Pressure," *J. Appl. Physics*, American Institute of Physics, Vol. 56, No. 7, pp. 2065–2067.

Roelands, C. J. A., Vlugter, J. C., and Waterman, H. I., 1963, "The Viscosity-Temperature-Pressure Relationship of Lubricating Oils and Its Correlation with Chemical Constitution," ASME *Journal of Basic Engineering*, Vol. 11, pp. 601–610.

Sibley, L. B., and Orcutt, F. K., 1961, "Elasto-Hydrodynamic Lubrication of Rolling Contact Surfaces," *ASLE Transactions*, Vol. 4, No. 2, pp. 234–249.

Spalding, D. B., 1972, "A Novel Finite Difference Formulation for Differential Expressions Involving Both First and Second Derivatives," *International Journal for Numerical Methods in Engineering*, Vol. 4, pp. 551–559.

Wang, S. H., and Zhang, H. H., 1987, "Combined Effects of Thermal and Non-Newtonian Character of Lubricant on Pressure, Film Profile, Temperature Rise, and Shear Stress in E.H.L.," ASME Journal of Tribology, *Transactions ASME*, Vol. 109, pp. 666–670.

Wilson, W. R. D., and Sheu, S., 1983, "Effect of Inlet Shear Heating Due to Sliding on Elastohydrodynamic Film Thickness," ASME Journal of Lubrication Technology, Vol. 105, No. 2, pp. 187–188.

DISCUSSION

Scott Bair[1] and W. O. Winer[1]

The inclusion of the energy equation in any non-Newtonian analysis of elastohydrodynamic lubrication is an important step. The rheological and thermal properties of the liquid are dependent upon local temperature. The discussers are, however, puzzled by the rheological model selected by the authors. The logarithmic shear stress nature of the Eyring model with properties averaged over the entire contact corresponds well with the average shear stress observed in EHD traction. However, when applied locally as a rheological equation of state, it fails to represent the most pronounced non-Newtonian effect observed in primary measurements—that of a rate independent limiting shear stress (Bair and Winer, 1979, 1982, 1990 and Ramesh and Clifton, 1987). It is interesting to note that the authors reference our 1979 paper showing limiting shear stress behavior for EHD lubricants.

This limiting stress has been observed in concentric cylinder rheometers (Bair and Winer, 1979, 1982, 1990) with a pressurization time on order of minutes, and impact pressure shear plate experiments (Ramesh and Clifton, 1987) for which the pressurization time was a few hundred nanoseconds. Indeed, the model oil in this paper, LVI260, has been thoroughly in-

[1]George W. Woodruff School of Mechanical Engineering, Georgia Institute of Technology, Atlanta, GA 30332-0405.

vestigated in this laboratory (Bair and Winer, 1982 and 1990). Newtonian behavior was observed to shear stresses of about 20 MPa (nearly ten times the required Eyring stress for traction curve fitting) followed by rate independent yielding at higher stresses. In the 1990 paper, lower pressures and higher shear rates were utilized in an attempt to find Eyring behavior. But, the transition from Newtonian to limiting shear was even more abrupt than at higher pressures. No primary evidence exists which supports Eyring's Sinh law behavior in lubricants at high pressure. Why then has so much time been spent incorporating it into an EHD analysis?

Additional References

Bair, S., and Winer, W. O., 1982, "Some Observations in the High Pressure Rheology of Lubricants," ASME JOURNAL OF LUBRICATION TECHNOLOGY, Vol. 104, No. 3, pp. 357-364.

Bair, S,. and Winer, W. O., 1990, "The High Shear Stress Rheology of Liquid Lubricants at Pressures of 2 to 200 MPa," ASME JOURNAL OF TRIBOLOGY, Vol. 112, No. 2.

Bair, Scott, 1990, "High Shear Stress Rheology of Liquid Lubricants," PhD thesis, Georgia Institute of Technology.

Ramesh, K. T., and Clifton, R. J., 1987, "A Pressure Shear Plate Experiment for Studying the Rheology of Lubricants at High Pressures and High Shearing Rates," ASME JOURNAL OF TRIBOLOGY, Vol. 109, No. 2.

Authors' Closure

The authors would like to thank Dr. Bair and Professor Winer for their discussion. The use of the Ree-Eyring fluid model in elastohydrodynamic (EHD) lubrication is firmly based on experimental evidence obtained on disc machines over a wide range of pressures and temperatures. The authors are confident that the results presented in this paper represent the best available estimate of the effect of heat generation on film thickness, temperature, and traction in an EHD contact. The Ree-Eyring model has a sound basis in the thermal activation theory of viscous flow. The parameters in this model, τ_o and η, can be expressed in terms of fundamental thermodynamic properties (Evans and Johnson, 1986, and Eyring, 1936), as follows:

$$\tau_o = k_B\Theta/v_\tau \tag{A-1}$$

and

$$\eta(p,\Theta) = \frac{1}{Cv_\tau}\exp\left(\frac{E+v_p p}{k_B\Theta}\right), \tag{A-2}$$

where k_B is the Boltzmann constant, Θ is the absolute temperature, C is a constant determined by the microscopic structure of the fluid, E is the thermal activation energy for the flow, and v_p and v_τ are the activation volumes for pressure and for shear stress, respectively. The Ree-Eyring model is valid when the work done by the shear stress in promoting flow, $v_\tau\tau$, is small compared to the total activation energy, $E+pv_p$. When the former quantity is comparable with the latter, the fluid behavior must be described in some other way. Experiments conducted by Imai and Brown (1976) on amorphous solid polymers suggest that the mechanism of flow changes from thermally activated motion of independent molecular segments to the formation of shear bands through the collaborative motion of adjacent segments when the shear stress reaches the order of 1/30 of the elastic shear modulus. The material can deform plastically under a constant shear stress—a limiting shear stress—when shear bands are formed.

The experiments by Bair and Winer (1979, 1982, and 1990) indicated that lubricating fluids also reach a limiting shear stress. Experimental data obtained by Evans and Johnson (1986) showed that values of the limiting shear stress were on the order of 1/45 of the elastic shear modulus of the lubricants. This suggests that the limiting shear behavior might be caused by the formation of shear bands or a similar mechanism, other than thermal activation, and that the limiting shear stress represents the ultimate strength of the material.

In Bair and Winer (1990), the limiting shear stress data for LVI-260 mineral oil indicated values of τ_L/p ranging from 0.031 to 0.040, while the value of 0.047, given in their Table 2, was used for comparison with the data of Johnson and Tevaarwerk (1977). A close examination of the models used to interpret their data from the high-pressure couette viscometer reveals a sensitivity of the experimental results to thermal effects intrinsic to the apparatus, which cannot be directly measured. This is especially true for the data obtained with very small gaps between the concentric cylinders. Moderate temperature rises at the stationary inner surface could serve to decrease the gap (thus increasing the real average shear strain rate) and reduce the viscosity in the vicinity of the inner wall (further increasing the local shear strain rate near the inner wall), while the perceived average shear strain rate would remain unchanged. This is a possible explanation for the apparent Newtonian behavior observed by the discussors for some fluids.

Bair and Winer (1979) proposed the following limiting shear stress model:

$$\dot\gamma = -\frac{\tau_L}{\eta}\ln(1-\tau/\tau_L) \quad \text{for } \tau \ge 0. \tag{A-3}$$

As the shear strain rate, $\dot\gamma$, tends to infinity, equation (A-3) shows that the shear stress, τ, approaches the limiting shear stress, τ_L. This model gives a good description of the shear behavior near the limiting shear stress for some lubricants. However, in this model, the initial non-Newtonian behavior at stress levels much lower than the limiting shear stress is determined by the value of the limiting shear stress. Since it is likely that the initial non-Newtonian behavior of a lubricant originates from a physical mechanism that is different from the limiting shear stress, the following rheological model would be more appropriate.

$$\tau = \begin{cases} \tau_i \sinh^{-1}(\eta\dot\gamma/\tau_o) & \text{for } |\eta\dot\gamma| < \tau_o \sinh(\tau_L/\tau_o), \\ \tau_L & \text{for } |\eta\dot\gamma| \ge \tau_i \sinh(\tau_L/\tau_o). \end{cases} \tag{A-4}$$

Equation (A-4) is a generalization of the model proposed by Jacobson and Hamrock (1984). Viscoelastic effects are not included under the assumption that the Deborah number is usually below one for the operating conditions usually found in machines. Equation (A-4) should only be applied when coupled with a thermal analysis and would require data on the variation of τ_L with both pressure and temperature. Unfortunately, these data are only available for a few lubricants and for a very limited temperature range.

Since the Ree-Eyring model is valid only when the shear stress is less than the limiting shear stress, it is desirable to compare the calculated shear stresses to limiting shear stress values. The experiments to obtain the limiting shear stress values of LVI-260 oil (Bair and Winer, 1982 and 1990) were conducted in a temperature range from 20 to 35°C. In order to compare the numerical solutions obtained in this paper to these experimental values of limiting shear stress, the dimensionless results presented in this paper are applied to the LVI-260 oil at different temperatures. For each case studied, with some dimensional parameters specified, an η_o value corresponding to the given dimensionless parameters, g_1 and L_1, can be found. In order to obtain this η_o value, the oil is considered to be placed at a temperature, T_o', given by a viscosity-temperature relation at ambient pressure. In this way, each case studied would correspond to an operating condition with the LVI-260 oil at a bulk temperature, T_o', and at an average contact temperature, T_a', approximately given by $T_a' = T_o' + \Delta T_{m,\max}/2$, where an estimate of the maximum midlayer temperature rise, $\Delta T_{m,\max}$, can be obtained from Fig. 7 by using