to investigate the behavior of asperities on real surfaces under combined normal and tangential loading to gain an understanding of the frictional behavior of real surfaces.

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DISCUSSION

J. H. Tripp[1]

The calculations presented in this paper provide a useful extension of results available hitherto for contact of bodies exhibiting a strain-hardening behavior. The combined effects of tangential displacement and interfacial friction have been added to the stresses previously calculated for purely normal indentation of a half-space by a rigid smooth cylindrical punch. A point to keep in mind is that the friction coefficient is chosen sufficiently large to preclude any slip at the interface, so that once in contact, the surface layer simply adheres to the rigid indenter. The contact is thus "ideally rough." The load/compliance relationships deduced for this model of an isolated asperity, Figs. 11 and 13, should prove useful in those cases where a statistical treatment of extended surface contact through independent asperity events is appropriate.

One effect of roughness is seen in Fig. 5 which shows normal pressure distributions in the contact zone. These are considerably flatter at the center compared to the frictionless elastic (Hertz) semielliptic distributions. Since the contact widths shown in Fig. 9 are approximately Hertzian, the total load for a given central maximum pressure should be *higher* for the elastoplastic than for the elastic material. The result presented in Fig. 6 however is just the opposite. This Figure makes better physical sense if in fact the upper curve represents the frictionless elasto*plastic* case of Dumas and Baronet [5] which an examination of their own Fig. 7 shows could well be the case. The frictionless elastic parabola would then lie lower than both of these elastoplastic curves. Flattening of the pressure distributions due to strain-hardening is thereby reduced by friction and the combination of roughness with strain-hardening renders the contact more closely Hertzian in behavior. An explanation of this lies in the ability of the "dead" adherent material to decrease the local strain, bringing it nearer to the elastic range. Johnson [13] has discussed a related situation in which the strain field under a sharp indenter (wedge) begins to resemble that beneath a blunt indenter (cylinder) when friction is introduced. A finer network of elements might help to clarify this problem of the ordering of the three load curves.

While a thin elastic zone is always present under the cylinder even in the smooth case, the presence of roughness is expected to enlarge this region and may be the reason for the persistence of elastic elements under the cylinder even at the highest normal load applied here. Again it would be useful to trace the elastic-plastic boundary more accurately by using a finer discretization of the region containing plastified elements. It would also be interesting to compute grid distortion patterns as indentation proceeds, particularly to compare the loading to the unloading phase. The incremental formulation used here is well suited to construct such diagrams which would reveal where the displaced material has gone. Historically this has been a matter of some controversy in assessing the plausibility of variouis asperity models and the diagrams could contribute usefully to resolving this question.

In the frictionless normal approach of elastoplastic bodies as orginally reported by Dumas and Baronet [5], the shape of the elastic-plastic boundary at any load deviates significantly from that found in the present work, which indicates a more nearly semi-circular plastic zone. Since the fixed boundary is placed at about the same distance in both calculations, the deviations appear to be attributable only to imposition of the rough boundary condition on the cylinder itself. Here, however, the restraining effect of friction would seem to narrow the breadth of the plastic region compared to its depth, whereas the opposite has occurred.

Finally a question arises concerning the rotational equilibrium of the cylinder. As noted, the asymmetry of the normal stress distribution across the contact interface when tangential displacement occurs is consistent with the need to prevent rotation of the cylinder under the action of the frictional torque. Quantitatively the balance of the two torques requires that the equilibrating tangential force be applied to the cylinder at a height of about 10^{-2} R above the surface of the half-space. Intuitively it seems reasonable that the point of application of this force should not be very high, yet it is surprising to find it as much as two orders of magnitude lower than the value R suggested by the author.

Additional Reference

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[1] NASA Lewis Research Center, Cleveland, Ohio 44135.