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Technology Evolution Prediction Using Lotka–Volterra Equations

During the development planning of a new product, designers and entrepreneurs rely on the prediction of product performance to make business investment and design strategy decisions. Moore's law and the logistic S-curve model help make such predictions but suffer several drawbacks. In this paper, Lotka–Volterra equations are used to describe the interaction between a product (system technology) and the components and elements (component technologies) that are combined to form the product. The equations are simplified by a relationship table and maturation evaluation in a two-step process. The performance data of the system and its components over time are modeled by simplified Lotka–Volterra equations. The methods developed here allow designers, entrepreneurs, and policy makers to predict the performances of a product and its components quantitatively using the simplified Lotka–Volterra equations. The methods also shed light on the extent of performance impact from a specific module (component technology) on a product (system technology), which is valuable for identifying the key features of a product and for making outsourcing decisions. Smartphones are used as an example to demonstrate the two-step simplification process. The Lotka–Volterra model of technology evolution is validated by a case study of passenger airplanes and turbofan aero-engines. The case study shows that the data fitting and predictive performances of Lotka–Volterra equations exceed those of extant models. [DOI: 10.1115/1.4039448]

Keywords: product performance, Lotka–Volterra equations, technology evolution, technology prediction, technological forecasting, product development planning

1 Introduction

Technology evolution describes a change in a technology performance over time. Technology evolution prediction interests designers, entrepreneurs, and policy makers. Specifically, designers rely on the technology evolution prediction to estimate the life cycle of their product in order to make informed R&D decisions [1,2]. Entrepreneurs adopt technology evolution prediction result to classify the development phase of their product in order to select appropriate business strategies such as performance maximization, sales maximization, or cost minimization [3,4]. Policy makers compare the evolution curves of different technologies in order to allocate the resources wisely through technology evolution prediction [5]. Of note, technology evolution prediction is also called product performance prediction because any product can be regarded as a technology [6].

The logistic S-curve model and the simple exponential model (Moore's Law) are two commonly used models in technology evolution prediction [1,2,7,8], where technology evolution is modeled as a standard S-curve and a simple exponential function, respectively. The parameters in the models are estimated based on past technology performance data. Future technology performance is predicted by mathematical extrapolation.

Despite several successful applications, these models do not fit technology evolution data in many cases [9,10]. Moreover, most of the parameters in those two models are hard to associate with the causal factors of technology evolution, such as R&D investment. Thus, the logistic model and Moore's Law cannot identify the factors shaping future technology performance and do not provide guidelines on how practitioners can influence performance. Importantly, the models focus on how a technology evolves in isolation and do not consider how it interacts with other technologies. In practice, a single standalone technology is rare. Most technologies are supported by other technologies [11]. If technology interaction is not considered in a prediction model, the prediction result produced by the model may suffer significant error [12].

In this paper, a product is referred to as a system technology. The system technology is realized through the integration and support of hardware and software [6], which are referred to as component technologies. Here, the interactions between the system technology and the component technologies and their impact on performance are modeled by Lotka–Volterra equations.

To our knowledge, the successful application of Lotka–Volterra equations in this manner is a new contribution to technology evolution modeling and prediction. Unlike the logistic S-curve model and Moore's Law, Lotka–Volterra equations cover a variety of curve shapes, which include the simple exponential curve and the standard S-curve [13]. The parameters in Lotka–Volterra equations can be associated with causal factors (e.g., R&D investment, government policy) of technology evolution, offering practitioners useful insights to manipulate the future of system and component technologies. Lotka–Volterra equations have interaction terms, which represent the effect of technology interaction. The interaction terms reveal the extent of the impact of different component technologies on the performance of the system technology. They allow designers and entrepreneurs to identify the components critical to system performance improvement and make important outsourcing decisions. The comparison between Moore's Law, the logistic S-curve model, and Lotka–Volterra equations is illustrated in Table 1.

This paper presents methods to apply Lotka–Volterra equations in technology evolution prediction. It is organized as follows: First, the background of Lotka–Volterra equations is introduced. Then, the equations are validated as a powerful tool in technology evolution prediction through parameter interpretation and a discussion of functional equivalence with accepted technology evolution models. In the remainder of this paper, guidelines are provided for practitioners to decompose a product and to model the interaction between the system technology and the component

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Table 1 Comparison among Moore's law, logistic S-curve model, and Lotka–Volterra equations

Technology evolution model	Curve shape	Parameter association	Technology interaction
Moore's law	Simple exponential curve	No	No
Logistic S-curve model	Standard S-curve	No	No
Lotka–volterra equations	Flexible	Yes	Yes

technologies using a full set of Lotka–Volterra equations. The equations are simplified by a relationship table and a maturation evaluation process. Smart phones are used as an example to illustrate the two-step simplification. Subsequently, the data pretreatment and data fitting technique for the simplified Lotka–Volterra equations are presented and demonstrated through a case study of passenger airplanes and turbofan aero-engines. The six steps to apply Lotka–Volterra equations in technology evolution prediction are summarized. The paper concludes with a brief discussion of the contribution of the work presented here and future research directions.

2 Lotka–Volterra Equations

Lotka–Volterra equations were first introduced by Vito Volterra in the early 20th century to model population changes of sharks and fish in the Adriatic Sea. The model has been expanded and successfully applied in the fields of demography and ecology during last century [14].

Pistorius and Utterback suggested studying the interaction between two technologies by generalized Lotka–Volterra equations in 1997 [15]. The equations they introduced are

$$\frac{dN}{dt} = a_n N - b_n N^2 + C_{nm} NM \tag{1}$$

$$\frac{dM}{dt} = a_m M - b_m M^2 + C_{mn} MN \tag{2}$$

where $N(t)$ and $M(t)$ denote two technology performances. The derivatives dN/dt and dM/dt represent the performance change rates of the two technologies respectively.

There are three terms on the right-hand side of the equal sign in the Lotka–Volterra Eqs. (1) and (2). Each of these terms has an interpretation in community ecology [16] and corresponding analogies in technology interaction [17]. These interpretations help us to associate each term with the causal factors of technology evolution.

The first term in Eqs. (1) and (2) represents unlimited growth rate in community ecology. It originates from the breeding instinct of species. In technology interaction, the term indicates the technology performance independent growth rate, which covers all the stimulation factors except the influence from other technologies. The stimulation factors include, but are not limited to, R&D investment and government policy encouragement.

The second term in Eqs. (1) and (2) describes the self-crowding effect in a biological population. It is a negative term and arises from ecosystem resource limitations (e.g., food and water). The second term's analogy in technology interaction is technical difficulty. The term may establish a technical barrier (upper limit value) for the technology performance when it compensates the performance improvement effect arises from the first and the third terms. The derivatives dN/dt equals zero in this case.

The third term in the equations is an interaction term. It denotes the beneficial or detrimental effect from the other species in community ecology, and from the other technologies in technology interaction. The term can be positive or negative. The value of the parameters C_{nm} and C_{mn} captures the dependency between the two species or, in our case, technologies.

The interpretations outlined earlier differentiate Lotka–Volterra equations from other technology evolution models. Lotka–Volterra equations not only mark the position of a technology on its life cycle but also offer practitioners useful insights to manipulate the future of the system technology through changes in component technology performance. Moreover, Lotka–Volterra equations can be reduced to a variety of mathematical functions in simplified cases. The logistic model and Moore's Law are special cases of Lotka–Volterra equations [14,16]. The details of reducing Lotka–Volterra equations to the logistic model and Moore's law are provided in the Appendix. Lotka–Volterra equations also cover Gompertz, Bass, nonsymmetrical responding logistic, and Sharif-Kabir models [13]. Further discussion on the functional equivalence for Lotka–Volterra equations can be found in a book chapter written by Cunningham [14]. The passenger airplane case study in Sec. 4.3 also reveals that the fit and prediction performance of Lotka–Volterra equations exceed those of extant models. These results validate Lotka–Volterra equations as a powerful tool in technology evolution prediction.

Researchers have applied Lotka–Volterra equations in product and firm competition [18–20], where the interaction terms are negative (i.e., C_{nm} and C_{mn} in Eqs. (1) and (2) have negative values), and the performance metrics are sales revenue or market share. Based on an extensive review of the state of the art, the successful application of Lotka–Volterra equations in the manner presented here is a new contribution to system design and integration, technology evolution modeling, and technology evolution prediction. In system design, technology performance is measured by technical parameters such as speed, capacity, and efficiency or combined parameters such as capacity per unit cost. The interaction terms of Lotka–Volterra equations are positive in common cases.

Pistorius and Utterback pioneered the application and solution of Lotka–Volterra equations with positive interaction terms to better understand technology interaction [17,21]. Due to the restriction of computational power at that time, they used a numerical method proposed by Pielou [22] in 1969 to solve Lotka–Volterra equations and set the step length $h=1$. Their solution shows oscillatory behavior in the mature phase of technology evolution. This instability precluded this research stream from moving forward. Fortunately, this instability problem could be solved by setting smaller step length (e.g., $h=0.1$) with the computational power today. In addition, Pielou's method is a first-order difference scheme for which the truncation error is $O(h^2)$. High order Runge–Kutta methods (e.g., Dormand–Prince method [23]) are recommended here to improve the efficiency and accuracy for solving Lotka–Volterra equations.

3 Two-Step Simplifications for Lotka–Volterra Equations

The majority of modern engineered products are complex systems. For example, an aircraft consists of thousands of modules, components, and elements. Solving the full set of Lotka–Volterra equations for thousands of components is unnecessary in many cases because several component technologies have negligible impact on system technology performance growth. The two-step simplification presented in this section helps users to identify those negligible impacts and reduce the equations accordingly.

The product of interest is called a system technology here. As a generic problem, the system can be viewed as the integration of n different component technologies. Each technology performance is modeled with its own equation. Thus, the full set of Lotka–

Volterra equations includes $n+1$ equations (one equation for system technology, and n equations for n component technologies). The equation for the system technology has the form

$$\frac{dy_0}{dt} = a_0 y_0 - b_0 y_0^2 + \sum_{i=1}^{n} C_{0i} y_0 y_i \tag{3}$$

In Eq. (3), y_0 is the performance of the system technology, and y_i is the performance of the component technology i ($1 \leq i \leq n$), where i and n are positive integers. a_0, b_0, and C_{0i} are the constant parameters for the independent growth term, the technical difficulty term, and the interaction term(s), respectively. Each component technology also has a corresponding equation similar to Eq. (3). For example, the first component technology has the equation as

$$\frac{dy_1}{dt} = a_1 y_1 - b_1 y_1^2 + \sum_{i=1}^{n} C_{1i} y_1 y_i \tag{4}$$

3.1 Step One: Relationship Table. More than one performance metric always exists for any system technology [24]. A table can be used to illustrate the relationships between the system technology performance metrics and the component technologies. The system performance metrics of interest appear in the first column of the table, and those of the different component technologies appear in the first row. The table is similar to the adjacency matrix in graph theory [25]. An "X" is marked in a cell of the table if the corresponding component technology has a significant impact on the performance metric of the system technology. On the contrary, the cell is left blank if the impact is assumed negligible.

Because the practitioner focuses only on one system technology performance, a specific performance metric is chosen from the column of the relationship table, and the X symbols on that row are noted. If m X symbols appear on that row ($1 \leq m \leq n$), it implies that m component technologies affect the system technology performance. In this case, Eq. (3) is reduced to

$$\frac{dy_0}{dt} = a_0 y_0 - b_0 y_0^2 + \sum_{i}^{m} C_{0i} y_0 y_i \tag{5}$$

where y_0 is the system technology performance of interest.

3.2 Step Two: Maturation Evaluation. The number of interaction terms for the system technology is reduced from n to m after the relationship table simplification. To simplify Lotka–Volterra equations further, the evolution curves of the m component technologies are evaluated to determine whether a component technology is mature when the system technology starts to grow. The component technology performance y_i has a constant value during the evolution time range if it is mature. The interaction C_{0i} term can be combined into the independent growth a_0 term in this circumstance, so the corresponding interaction C_{0i} term is removed from Eq. (5), reducing it to

$$\frac{dy_0}{dt} = a_0 y_0 - b_0 y_0^2 + \sum_{i}^{m-p} C_{0i} y_0 y_i \tag{6}$$

If there are p mature component technologies, only $m-p$ interaction terms are left in Eq. (6). Thus, the simplified Lotka–Volterra equations include only $m-p+1$ equations rather than the original $n+1$ equations after the two-step simplification.

3.3 Smart Phones Example. To illustrate the two-step simplification method outlined earlier, smart phones are taken as an example of system technology. The smart phone is decomposed into five key component technologies: touch screen, CPU, integrated circuit (IC), battery, and operating system. This example is used only to demonstrate the simplification method. The technology evolution model is not developed because technology performance data are unavailable. A detailed case study of passenger airplanes that includes data fitting and analysis is provided in Sec. 4.3.

The full set of Lotka–Volterra equations includes six equations with the equation for the smart phone (system technology) having the form

$$\frac{dy_0}{dt} = a_0 y_0 - b_0 y_0^2 + \sum_{i=1}^{5} C_{0i} y_0 y_i \tag{7}$$

The performance metrics of a smart phone include, but are not limited to, *speed to open an app*, *image resolution*, *heat dissipation*, *battery endurance*, and *weight*. If desired, combined metrics also can be considered, such as *speed to open an app divided* by *phone weight*. A relationship table for a smart phone is shown in Table 2. A significant impact from a component technology on the system technology's (smart phone) performance metric is denoted by X in the corresponding cell.

If the *speed to open an app* is of interest, only the CPU and the operating system have significant impact on this performance metric. Through this process, the five interaction terms in Eq. (7) are reduced to two terms by approximating C_{01}, C_{03}, and C_{04} to zero.

The performance of the operating system is constant if it is assumed to be mature during smart phone evolution. This assumption is used to demonstrate the simplification process. In this case, y_5 is a constant, and the interaction C_{05} term is combined into the independent growth a_0 term after the maturation evaluation process.

Using the *speed of a smart phone to open an app* as an example, the two-step simplification of Lotka–Volterra equations for the system technology performance is illustrated in Fig. 1. The simplified Lotka–Volterra equations include only the following two equations:

$$\frac{dy_0}{dt} = a_0 y_0 - b_0 y_0^2 + C_{02} y_0 y_2 \tag{8}$$

$$\frac{dy_2}{dt} = a_2 y_2 - b_2 y_2^2 + C_{20} y_2 y_0 \tag{9}$$

In Eqs. (8) and (9), y_0 is the *speed of a smart phone to open an app*, y_2 is the CPU *performance* (e.g., clock speed), C_{02} term represents the impact of the component technology (CPU) on the system technology (smart phone), and C_{20} term represents the impact of the system technology (smart phone) on the component technology (CPU). Of note, the C_{20} term may not equal the C_{02} term. These C terms are generally positive, but they may have negative values in rare cases. For example, fast development of CPU leads to more compact CPU design, thereby causing CPU to produce more heat per unit area. This heat production has a detrimental effect on the heat dissipation performance of a smart phone, leading to a negative C term in the system technology equation.

4 Data Fitting for the Simplified Lotka–Volterra Equations

A structured method is presented in Sec. 3 to allow practitioners to simplify the full set of Lotka–Volterra equations. This simplification method becomes important as more detailed explorations of complex multicomponent products are performed.

In this section, the simplified system–component interactions are analyzed. The specifics of fitting the simplified Lotka–Volterra equations to performance data of system and component technologies over time are discussed. A comparison between the abilities of Lotka–Volterra equations and typical technology evolution models to fit technology performance data and to predict future system technology performance is explored through a case study of passenger airplanes.

Table 2 Relationship table for smart phone

Smart phone performance	Touch screen	CPU	Integrated circuit	Battery	Operating system
Speed to open an app		X			X
Image resolution	X				X
Heat dissipation	X	X	X	X	X
Battery endurance	X	X		X	X
Weight	X		X	X	
Speed to open an app/weight	X	X	X	X	X

4.1 Dimensionless Treatment of Time History Data. Practitioners may use different units for the same technical parameter (e.g., psi and MPa for pressure), which leads to different data fitting results for the same problem. To avoid confusion, technology performance data should be nondimensionalized before substituting into the simplified Lotka–Volterra equations for data fitting. This treatment also helps practitioners compare the evolution curves of different technologies. The nondimensionalized method is borrowed from fluid mechanics [26]. Component and system technology performances y_i and y_0 are divided by corresponding characteristic values Y_i and Y_0 as

$$y_i^* = \frac{y_i}{Y_i} \tag{10}$$

$$y_0^* = \frac{y_0}{Y_0} \tag{11}$$

The characteristic value can be the start or maximum technology performance, or the technology performance at a specific time. It is recommended to choose the maximum performance or the performance at current time as characteristic values Y_i and Y_0. In that way, system and component technologies performance data are normalized within the same range (0, 1]. They will have the same weight in the following data fitting process.

Dimensionless component and system technology performances y_i^* and y_0^* replace y_i and y_0 in the simplified Lotka–Volterra equations. For example, Eq. (6) is replaced by

$$\frac{dy_0^*}{dt} = a_0 y_0^* - b_0 y_0^{*2} + \sum_i^{m-p} C_{0i} y_0^* y_i^* \tag{12}$$

The only unit in the simplified Lotka–Volterra equations is time t after the dimensionless treatment above.

4.2 Data Fitting Process. The simplified Lotka–Volterra equations model the interactions between one system technology and $m-p$ component technologies. They include $m-p+1$ equations. Each equation has $m-p+2$ unknown parameters (e.g., a_0, b_0, and C_{0i} for the system technology performance equation). There are $(m-p+1)*(m-p+2)$ unknown parameters in total. The initial value of each technology performance is also required to solve differential equations. The first performance data points (the initial or earliest known technology performance) of each technology are used as the initial values of the simplified Lotka–Volterra equations. The initial values also can be treated as unknown parameters to improve the accuracy of data fitting. This treatment leads to $(m-p+1)*(m-p+3)$ unknown parameters for the simplified Lotka–Volterra equations.

The goal of the data fitting process is to search for the various a, b, and C parameter values that minimize the sum of squared errors between technology performance time history data and the solutions of the simplified Lotka–Volterra equations. A trust region reflective algorithm [27] or other optimization algorithms [28] may be used for this purpose. As we know, the simplified

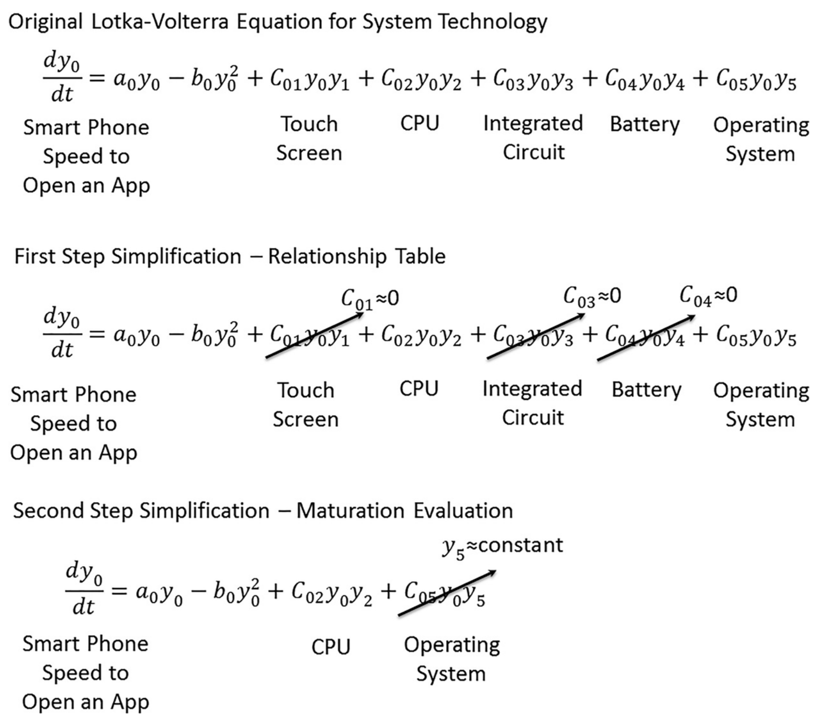

Fig. 1 Two-step simplification of Lotka–Volterra equations for smart phone speed to open an app

Lotka–Volterra equations are a set of differential equations. The analytic solutions for the equations are not available in a general case. As stated above, Dormand–Prince method [23], which belongs to Runge–Kutta formulae family, can be employed to solve the equations numerically here.

The data fitting process above yields the optimum values of parameters in the simplified Lotka–Volterra equations. These optimum values are plugged into the simplified Lotka–Volterra equations, and performance predictions for the system technology and each component technology are made by mathematical extrapolation. These performance predictions enable designers to track the product along its technology life cycle. In particular, practitioners can identify when product performance reaches an upper limit and estimate the remaining life of the technology. Importantly, Lotka–Volterra equations also indicate the relationship between the system technology and the component technologies. In the equation for system technology (Eq. (12)), the values of the parameters in the interaction terms (C_{0i}) denote the degree of impact of the specific component technology i on the system technology. A higher positive value of parameter C_{0i} reflects a greater importance of the component technology i to the system technology evolution.

From the viewpoint of product design and manufacture, the component technologies with the highest C_{0i} values are identified as the key features of the product (system technology). Designers would likely develop and manufacture these key component technologies in-house, or at least establish a more synergistic relationship (e.g., through joint R&D or cross-shareholding) with the suppliers of these component technologies. Similarly, the component technologies with the smallest C_{0i} values would be candidates for outsourcing. Designers would likely choose the commercial component technologies in the market with competitive prices for these less impactful components. The continued R&D for such low impact component technologies would also likely be reduced or even terminated in the company. A good example of such outsourcing strategy is from Apple. Apple develops only the key components (e.g., IOS software and A6 chip) of iPhone in its California plant. A great number of other components are outsourced [29]. The outsourcing strategy saves time and money for Apple (Cupertino, CA) during product development and manufacture. Of note, these strategic design decisions are typically made on the basis of designers' qualitative analysis. The model developed here allows designers to measure the significance of each component technology through the parameter C_{0i} values and make more informed decisions.

4.3 Case Study: Passenger Airplane and Turbofan Aero-Engine. The dimensionless treatment and data fitting process are demonstrated by the interaction between passenger airplane and turbofan aero-engine as a case study in this section. The case study also illustrates the ability of Lotka–Volterra equations to model technology evolution and interaction. The model is compared to currently accepted technology evolution models to demonstrate its improved accuracy for technology evolution prediction. Of note, the passenger airplane is supported by many different component technologies (e.g., electronics, aerodynamics) other than the aero-engine. Due to data limitation, only the interaction between the passenger airplane and the turbofan aero-engine is considered. This simplification may involve some prediction error.

The passenger airplane is treated as the system technology. A performance metric of passenger capacity·speed·range (km^2/h) is used. A turbofan aero-engine is the component technology. For the engine, take-off thrust (kN) is the component technology performance metric.

The time history performance data for passenger airplanes and turbofan aero-engines during 1960–2010 are collected[2] [30–32]. The highest performance value of each data set is chosen as the characteristic value for the passenger airplane and the turbofan aero-engine performance in the given time period. To create a dimensionless metric of performance value for system and component technologies, the actual performance metric values are divided by its associated technology's highest value of the performance metric.

In this case, the simplified Lotka–Volterra equations are

$$\frac{dy_0^*}{dt} = a_0 y_0^* - b_0 y_0^{*2} + C_{01} y_0^* y_1^* \tag{13}$$

$$\frac{dy_1^*}{dt} = a_1 y_1^* - b_1 y_1^{*2} + C_{10} y_1^* y_0^* \tag{14}$$

where y_0^* is the dimensionless passenger airplane performance, and y_1^* is the dimensionless turbofan aero-engine performance.

The initial values of y_0^* and y_1^* are treated as unknown parameters. Thus, there are eight parameters for the simplified Lotka–Volterra equations Eqs. (13) and (14). The trust region reflective algorithm [27] is used to search for the optimum values of the parameters. The optimization algorithm gives the equations and initial conditions with optimum values of parameters as

$$\frac{dy_0^*}{dt} = 0.303 y_0^* - 0.557 y_0^{*2} + 0.260 y_0^* y_1^* \tag{15}$$

$$\frac{dy_1^*}{dt} = 0.0345 y_1^* - 1.18 \cdot 10^{-7} y_1^{*2} + 3.55 \cdot 10^{-14} y_1^* y_0^* \tag{16}$$

$$y_0^*(t=0) = 0.0718 \tag{17}$$

$$y_1^*(t=0) = 0.225 \tag{18}$$

where $t = 0$ represents the start year of 1960.

The final data fitting result for the system technology performance (passenger capacity·speed·range of passenger airplane) is illustrated in Fig. 2. The coefficient of determination R^2 for the system technology time history data fit is 0.9742. By comparison, the coefficient values of determination R^2 are 0.7307 and 0.9389, respectively, if the time history performance data are modeled by Moore's Law (Eq. (A4) in the Appendix) and the logistic model (Eq. (A2) with $b_n/a_n = 1$ in the Appendix). Thus, using R^2 as a measure of model fit, Lotka–Volterra equations have a greater accuracy in modeling the technology performance evolution than those of the existing and commonly accepted technology evolution models.

The prediction for passenger airplane performance is made by mathematical extrapolation. Eqs. (15)–(18) predict that the passenger airplane performance will reach 1.68×10^{10} km^2/h in 2040. This performance level is around 2.15 times of the Airbus A380-800 performance. The double bubble D8 [33] is a candidate to meet this prediction. Of note, the underlying assumption of this prediction is that the environmental conditions for the system technology evolution stay the same in the following years. In reality, the values of parameters a_0, b_0, and C_{01} are likely to be functions of time rather than remaining as constants. For example, a governmental policy stimulation may be able to increase the value of a_0 and improve the system technology performance. An economic crisis may lead to less R&D investment and a reduced value of a_0.

As highlighted in Sec. 4.2, the simplified Lotka–Volterra equations also shed light on the relationship between the system technology and the component technologies. In Eq. (15), C_{01} is in the same order with a_0, which indicates that the component technology (turbofan aero-engine) has a significant impact on the evolution of the system technology (passenger airplane).

The data fitting result for the component technology performance (turbofan aero-engine take-off thrust) is illustrated in Fig. 3. The coefficient of determination R^2 for the component technology time history data fitting is 0.9410. The parameter values for the

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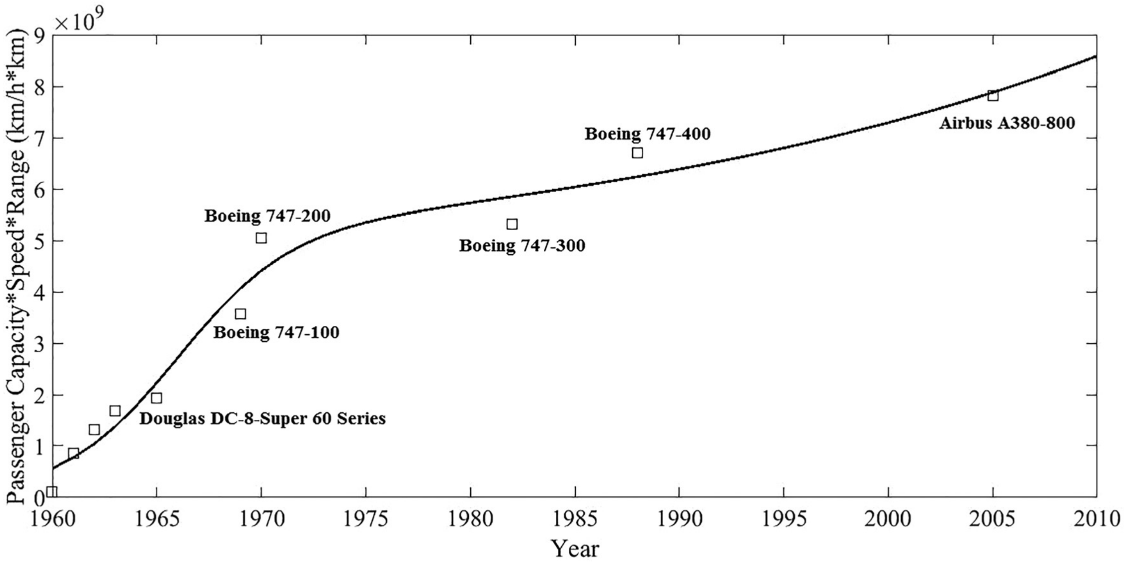

Fig. 2 Data fitting result for system technology performance (Passenger capacity*speed*ranger of passenger airplane) [30]

second term b_1 and the third term C_{10} in Eq. (16) have very small values. These values indicate that the evolution of turbofan aero-engine almost follows Moore's Law. The system technology-passenger airplane has little impact on the aero-engine performance growth. The reason for the exponential growth may be that the modern aero-engine performance improvement also relies on the advances in its component technologies such as materials and combustion science. Only the interaction between passenger airplanes and turbofan aero-engines is considered in this case study due to data limitation. The beneficial influence from the component technologies of the aero-engine is included in a_1 term in Eq. (14) as an approximation. The impact from the passenger airplane may be small compared to the impact from component technologies (materials and combustion science). The development of these component technologies has not confronted significant technical barriers up to now. These possible reasons may lead to small b_1 and C_{10} values in Eq. (16).

The prediction capability of Lotka–Volterra equations is validated and compared with the extant models through a holdout sample test. The time history performance data for passenger airplanes and turbofan aero-engines during 1960–1998 are modeled using Lotka–Volterra equations (Eqs. (13) and (14)), the logistic S-curve model (Eq. (A2) with $b_n/a_n = 1$ in the Appendix), and Moore's law (Eq. (A4) in the Appendix), respectively. The data fitting results and the extrapolation prediction for the passenger airplane (system technology) performance are illustrated in Fig. 4. Both the logistic model and Moore's law fail to predict the appearance of the Airbus A380-800 in 2005. The logistic S-curve model predicts the passenger airplane performance reaches an upper limit value around 1985, leading to a 22% prediction error in 2005. Moore's law assumes the passenger airplane performance continues increasing exponentially, which results in a 122% prediction error in 2005. In contrast, the 7 year prediction error of Lotka–Volterra equations is only 5% in this case. The successful prediction of Lotka–Volterra equation is a result of modeling the interaction between technologies. Our model takes into account the impact of component technology (turbofan aero-engine) on system technology (passenger airplane).

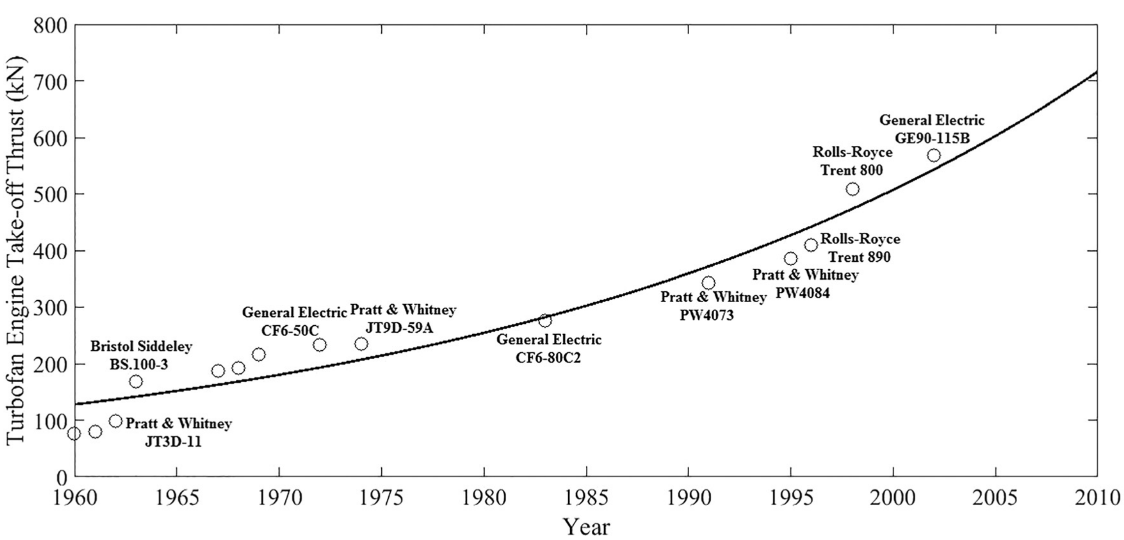

Fig. 3 Data fitting result for component technology performance (take-off thrust of turbofan aero-engine) [31,32]

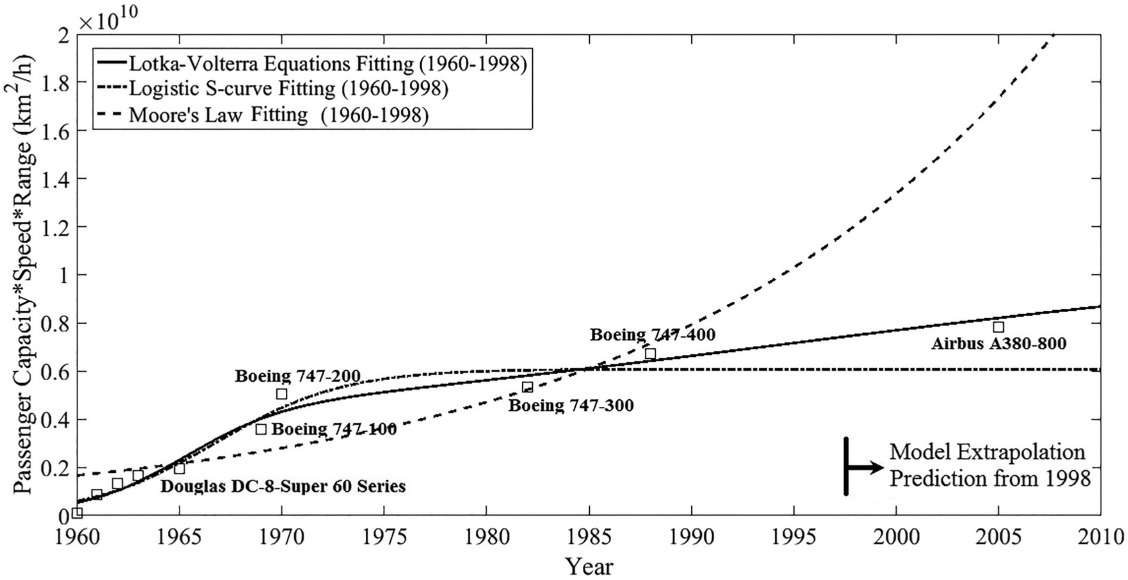

Fig. 4 Predicted system technology performance on historical data (1960–1998)

5 Easy Steps to Apply Lotka–Volterra Equations in Technology Evolution Prediction

Sections 3 and 4 present the methods to apply Lotka–Volterra equations in technology evolution prediction. These methods are summarized into six simplified steps below. The six steps are also illustrated in Fig. 5. Practitioners could follow these guidelines for technology evolution prediction.

Step 1—System decomposition. Practitioners first define the system technology performance of interest. Through system decomposition, any component technology that may have an impact on the system technology performance should be identified. The component technology can be tangible (e.g., CPU of a smart phone) or intangible (e.g., operating system of a smart phone). Practitioners should list as many component technologies as possible in this step.

Step 2—Component technology selection. From the list developed in step 1, practitioners choose the key component technologies that have a significant impact on the system technology performance. The relationship table and maturation evaluation provided in Secs. 3.1 and 3.2 are helpful for this process. The key component technologies also can be selected by practitioners' experience or industry experts' advice. Once the key component technologies are chosen, the performance metric for each component technology should be determined. Typically, the performance metric is an indicator of the technology evolution (e.g., clock speed of CPU).

Step 3—Data collection and pretreatment. The time history technology performance evolution data for the system and the component technologies selected in step 2 are collected. These data are divided by corresponding characteristic values as dimensionless treatment. It is recommended to choose the maximum performance or the performance at current time as characteristic value for each technology.

Step 4—Parameter space definition. The Lotka–Volterra equation (Eq. (12)) is developed for each system and component technology. The parameter space (range) for a, b, and C in each equation is determined for the optimization search in step 5. Parameters a and b have positive values for common cases, so their parameter space is $(0, +\infty)$. Depending on the relationship between the two technologies, parameter C may have a positive or negative value. Therefore, the parameter space of C is $(0, +\infty)$ or $(-\infty, 0)$. In some cases, the interactions between component technologies could be neglected. Lotka–Volterra equations could be further simplified in that case.

Step 5—Data fitting. The values of parameters a, b, and C in each Lotka–Volterra equation are searched through parameter spaces defined in step 4. Optimum parameter values are found that minimize the sum of squared errors between the technology performance time history data and the solutions of Lotka–Volterra equations. A trust region reflective algorithm [27] is suggested for optimization. A high order Runge–Kutta numerical method (e.g., Dormand–Prince method [23]) is suggested to solve Lotka–Volterra equations in each search step.

Step 6—Prediction and analysis. Using the optimum parameter values derived in step 5, future technology performance of system and component technologies are derived by mathematical extrapolation from Lotka–Volterra equations. The prediction results should multiply the characteristic values in step 3. The degree of impact of the component technology i on the system technology could be measured from parameter C_{0i} in the Lotka–Volterra equation for the system technology (e.g., Eq. (15)). A higher positive value of parameter C_{0i} shows greater importance of the component technology i to the system technology.

6 Conclusions

A product can be viewed as a system technology. A product is an integration of many elements, which are called component technologies. The interactions between the system technology and the component technologies are described by a full set of Lotka–Volterra equations. The full equation set is simplified by a relationship table and a maturation evaluation process. Smart phones are used as an example of this two-step simplification. The time history data of technology performance evolution are fit by the simplified Lotka–Volterra equations. The data fitting process is demonstrated by the interaction between passenger airplanes and turbofan aero-engines.

The methods developed in this paper allow designers, entrepreneurs, and policy makers to predict the performance of a product (system technology) and its modules (component technologies) quantitatively through Lotka–Volterra equations. The impact of a specific module (component technology) on the product's (system technology) performance is also derived from the equations. These results aid R&D investment and component outsourcing strategy decisions.

This work also offers opportunities for future research. First, more quantitative technology evolution case studies could be carried out using Lotka–Volterra equations, in particular, complex cases in which a system technology interacts with two or more

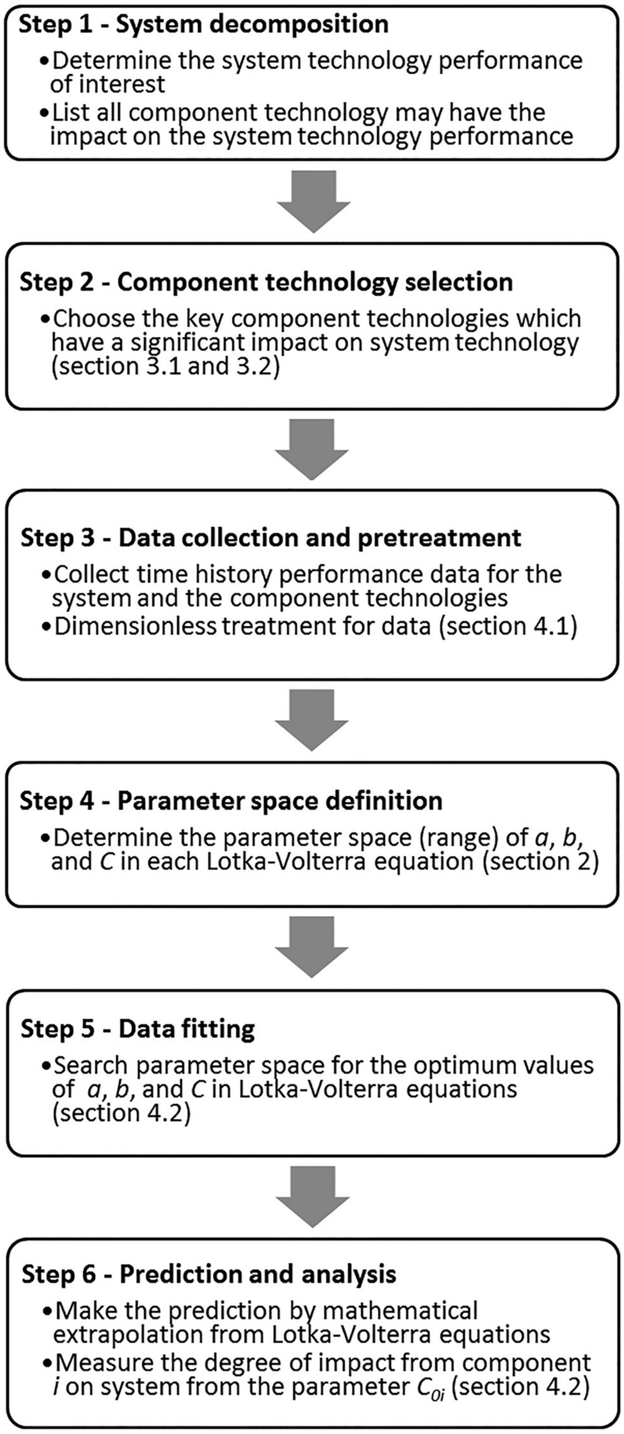

Fig. 5 Six steps to apply Lotka–Volterra equations in technology evolution prediction

component technologies (e.g., passenger airplanes with aero-engine and primary material of composition). Second, the relationship between the parameters (e.g., a_0) in Lotka–Volterra equations and the decision variables (e.g., R&D investment) could be explored further for generalizable business strategies. Third, only one system technology performance metric (e.g., speed of a smart phone to open an app) is taken into account in this paper. Future work can consider several different system technology performance measures, leading to multi-objective optimization [28,34]. Finally, it may be helpful to introduce probability theory [10,35] into Lotka–Volterra equations. Through this approach, practitioners can derive the probabilistic prediction of product performance and make more informed decisions.

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Appendix: Reduction of Lotka–Volterra Equations to Logistic Model and Moore's Law

When the interaction term is neglected ($C_{nm}=0$), Eq. (1) is reduced to

$$\frac{dN}{dt} = a_n N - b_n N^2 \tag{A1}$$

The general solution of Eq. (A1) has the form of the generalized logistic model

$$N = \frac{A}{\frac{b_n}{a_n} + Be^{-a_n t}} \tag{A2}$$

where A and B are integral constants, and a typical logistic S-curve model is obtained when $b_n/a_n = 1$.

Further simplification can be made for Eq. (A1) if the technical difficulty term also equals zero ($b_n = 0$ and $C_{nm} = 0$). In this case, Eq. (A1) is reduced to

$$\frac{dN}{dt} = a_n N \tag{A3}$$

Solving Eq. (A3) yields

$$N = De^{a_n t} \tag{A4}$$

where D is an integral constant. Eq. (A4) gives exponential growth for technology performance N, which is Moore's Law.

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