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Discussion

Complete Solution of the Nine-Point Path Synthesis Problem for Four-Bar Linkages[1]

Preben W. Jensen[2] It was with great interest I read the article by Wampler, Morgan, and Sommese about the nine-point problem because I myself have worked on the same problem for some years. The authors are to be commended for their work.

However, there are some issues that need to be clarified further. It has taken me some time to dig into the problems and this is the reason for the relatively late response. Since the article's examples were chosen from Roth and Freudenstein's paper [3], the solution of the nine-point problem as described by Wampler et al. appears to be incomplete. In the following I will try to explain the incompleteness.

It is the chosen curve-points and the way they were changed in [2] that concerns me. Let us turn our attention to an example taken from their paper, namely example 2 on page 301, here shown as Fig. 1. In this example it is stated that "the precision points were chosen by slightly displacing the corresponding locus points of an arbitrary solution." In other words, points were chosen on the coupler curve of an arbitrary starting linkage and then the positions of these points were changed slightly. To refer to these points as being arbitrarily chosen is indeed misleading.

There are two things that concern me in this context. First, in a practical and also theoretical world one would never start out with an arbitrary linkage to obtain some shape of a coupler curve and then slightly displace the curve-points. To the contrary, one would in general start out with given characteristics of the coupler curve and not take characteristics from an arbitrary starting solution. Second, by displacing the curve-points "slightly" and not give any precise definition of what is meant by slightly, and in what direction the "slight" displacements are made, leads to an ill-defined problem. Furthermore, it appears to me that in order to maintain the smoothness of the desired coupler curve the points must have been selected in such a way that ensures the smoothness of the curve. Therefore these points cannot have been selected arbitrarily. To clarify this, consider that each point had been displaced in a direction opposite from the immediately foregoing point. This would have led to a zig-zag position of the points with the result that no smooth curve could be drawn through these points. This leads to no solutions at all! In a letter to Prof. Freudenstein I asked how these points were determined, but got the answer that he would tell me some other time. Additionally the method used seems to be restricted by the fact that the final position of the fixed crank joint is very close to its starting position, which facilitates finding the solution. This condition can be found in example 1 of [2], shown here as Fig. 2.

Using the so-called arbitrary close-positioned precision points from Roth and Freudenstein's paper with built-in smoothness one certainly does not get a solution to the nine-point problem in the original sense. I suspect that it is the smoothness and the closeness of the points of the curve which can be drawn through all nine points that resulted in multiple solutions in Wampler et al. But it is too much of an artificial way to find so-called arbitrary points as described in [2]. The resulting solutions have no practical or theoretical value when points are chosen this way. One should be very careful in using the terms "arbitrary" and "slightly" without properly defining what is meant by these terms. It is misleading and gives a false impression.

What are the practical requirements that can be imposed on the desired coupler curve in order to get solutions? To take a simple example: It is known that a four-bar linkage cannot generate an ellipse but using various precision point methods, the resultant coupler curve can be made partly to look like an ellipse. The practice of prescribing the precision points in order to get a close approximation to the desired curve is really not the best one because, how, for instance, should the points be selected in order to minimize the deviation. On the other hand the least deviation probably yields a solution with worse transmission angles than a solution that is not minimizing the deviation from the ideal curve. The method only partly takes into account the shape characteristics of the desired coupler curve.

So far, to my knowledge, the method of selecting precision points has only led to a good approximation over one quarter

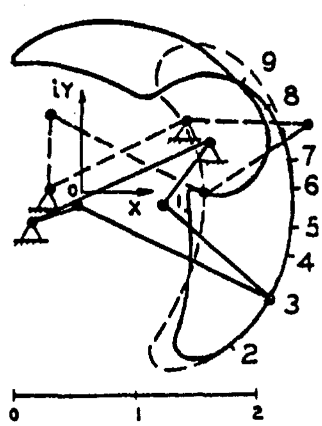

Fig. 1 Solution to Example 2. Synthesized linkage (solid), starting linkage (dashed), and their generated loci.

[1] By C. W. Wampler, A. P. Morgan, and A. J. Sommese, published in the March 1992 issue of the Journal of Mechanical Design, Vol. 114, pp. 153–159.

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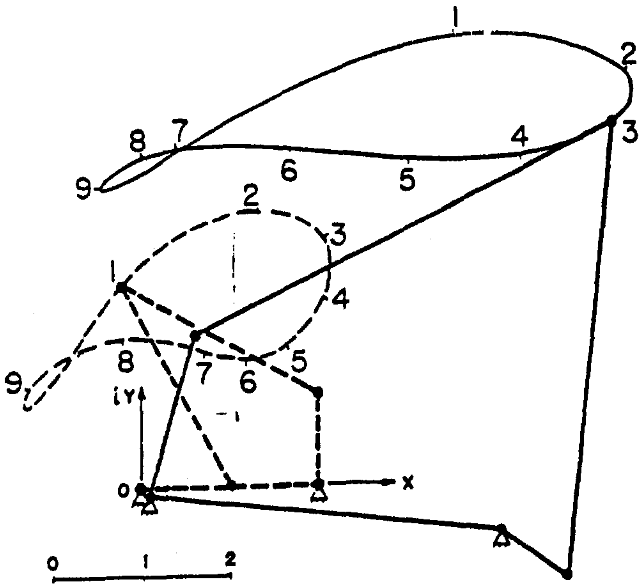

Fig. 2 Example 1, minus-one geared five-bar. The synthesized linkage and its generated path (solid lines); the starting linkage and its locus (broken lines).

of an ellipse. The resultant coupler curve deviates so much over the remaining three quarters, that one hardly can talk about an approximation to an ellipse. A better method is to prescribe as many intersections as possible with the desired curve in order to get a better approximation [4] and then select the solution out from the infinite many possible solutions. The result I have obtained is an approximation to the desired arbitrary ellipse so that it is almost impossible to separate the two curves by the eye. Even better approximations than those I have obtained by the first try are possible. (The method is not limited to specific curves.)

From my own experience with coupler curves I have found what I call the invariance of coupler curves, i.e. by changing the dimensions of the links but maintaining certain initial synthesis requirements the resultant coupler curves change very little [5]. This leads me to think that this phenomenon is causing the many multiple solutions found in Wampler et al.

Certainly there is not always a solution when the nine points are chosen arbitrarily and this was the problem originally stated by H. Alt [1]. When the points are separated along the entire curve and not too close to each other there seems to be only one solution if there is a solution (plus, of course, the Robert's cognates and the geared double cranks with a transmission ratio of +1:1).

The number of precision points can be increased considerably when the problem at hand calls for symmetrical coupler curves. Using geometrical methods a solution is obtained on a PC in no time. I have obtained 12 precision points [3] by a four-bar linkage. The number of precision points I have achieved for the centric slider-crank is 10 and for the eccentric slider-crank 8 [5]. Geometrical methods often allow one to see whether there is a solution to the problem and within what range, and the solutions are found more easily. The writing of equations after equations do not allow this kind of a deeper insight into the problems.

Summary

Obviously the conclusion to what is the general requirement to be imposed on the precision points is, that the points must lie on a curve the shape of which is characteristic for the coupler curves for the desired type of linkage.

References

1 Alt, H., "Ueber die Erzeugung gegebener ebener Kurven mit Hilfe des Gelenkvierseits," ZAMM, Vol. 3, 1923, pp. 13–19.

2 Roth, B., et al., "Synthesis of Path-Generating Mechanisms by Numerical Methods," *Transactions of the ASME*, August 1963, pp. 298–306.

3 Jensen, Preben W., "Synthesis of Four-Bar Linkages With a Coupler Point Passing Through 12 Points," *Mechanisms and Machine Theory*, Vol. 19, No. 1, 1984, pp. 149–156.

4 Jensen, Preben W., "The Polode Synthesis Method," *Engineering Research 6*, VDI-Verlag, Duesseldorf, Germany, 1992, pp. 152–163.

5 Jensen, Preben W., *Analysis, Synthesis, and Design of Mechanisms and Mechanical Devices*, to be published.

Authors' Closure

C. Wampler,[3] A. Morgan,[4] and A. Sommese[5] We appreciate the interest that Prof. Jensen has taken in our work and the opportunity to clarify the contribution made by our paper (Wampler et al., 1992). At our request, Prof. Jensen sent copies of several of his articles, so we have also gained the chance to become more familiar with his ingenious methods.

While there is an element of truth underlying some of Prof. Jensen's comments, there are also some misleading statements and some misdirected criticisms. We will attempt to sort these out and to address in brief how our methods can be applied to several of the problems mentioned in his commentary.

Prof. Jensen contends that, contrary to the claim of our title, our solution is not complete. Lest there be any confusion on the point, we do not claim to offer a comprehensive method for designing four-bar linkages for path generation. Rather, we offer a complete solution to the long-standing nine-point problem; that is, given nine points in the plane, find all four-bar linkages whose coupler curves pass through the points. We use the adjective "complete" to distinguish our method from previous approaches, which only find a partial list of solutions, such as the methods published by Roth and Freudenstein (1963) and Tsai and Lu (1990). The situation is analogous to the well-known Burmester problem of finding all centerpoint-circlepoint pairs that guide a body through five given positions in the plane. A complete solution to the Burmester problem generates a list of four centerpoint-circlepoint pairs, 0, 2, or 4 of which may be real. In the nine-point problem, our numerical evidence strongly indicates that taken over the complex numbers, a complete solution list consists of 1442 triples (Roberts cognates), some subset of which will be real. In the same manner that solution methods for the Burmester problem are useful in designing four-bars for body guidance, our solution method for the nine-point problem can aid in the design of four-bars for path generation.

Prof. Jensen's chief criticism of our methodology lies in a supposed inadequacy in our example Problem 1, drawn from Roth and Freudenstein (1963). He complains that the points have not been selected "arbitrarily." We cannot answer Prof. Jensen's questions concerning the etiology of this example, but we must stress that it has no bearing whatsoever on the validity of our numerical experiment. As we stated on page 156, our initial solution was computed using random, complex precision points. These were generated using a random number generator, which is as arbitrary as one could imagine! Roth and Freudenstein's problem is merely an example application which, being the only previously published solved example, we felt obliged to treat.

Prof. Jensen's contention that a sequence of points in a zigzag pattern will lead to no solutions at all is valid, as no four-bar coupler curve can cross a line more than six times. By finding a complete solution list in which no suitable linkages appear, our method will confirm this impossibility and the designer will

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