

A J -Integral Approach to Crack Resistance for Aluminum, Steel, and Titanium Alloys¹

P. S. Theocaris.² Paper presents crack growth resistance data performed on the crack line wedge loaded (CLWL) specimens in some aluminum, steel, and titanium alloys. The J -integral was used for the expression of the obtained data and its values were determined analytically by using the elastic fracture theory, the Dugdale model and the Prandtl-Reuss theory. The author of the present discussion would like to add some further comments on this paper related to the use of the above-referred three models for predicting the behavior of the CLWL specimens in the elastic and especially in the plastic region of deformation.

First of all, the discussor would like to observe that, as it is established in the paper, the Bueckner-Hayes energy approach and original Dugdale type elastic-plastic analysis showed excellent agreement. However, only below five percent for the ratio of applied stress to yield stress the elastic approach and Dugdale approach yield similar results. At higher ratios large differences are noticed in $J^{1/2}$ values for aluminum alloys 7075-T6 and 2024-T3. Furthermore a comparison of analytical plastic-zone sizes indicated another unexpected trend in plastic zone behavior, where an extremely large plastic zone was calculated at an applied stress of more than five percent of the yield stress. This indicates that the Dugdale-plastic zone assumption did not conform to the expected trends. Further, authors proved that a more rigorous and accurate approach of the problem based on a Prandtl-Reuss elastic-plastic analysis of the CLWL specimen, yields more reasonable results where the deviation from the elastic case is progressive.

Relatively to these results of the paper discussor would like to remark that the unrealistic values for $J^{1/2}$ integral in the CLWL specimen configuration from the Dugdale analysis is expected and what is surprising, is the coincidence for the center cracked tension (CCT) specimen geometry, where the two first authors, in a previous publication [1],³ found that the Dugdale and the Prandtl-Reuss elastic plastic analysis were similar up to sixty percent of yield. One possible explanation is that these results were derived from stiffened panel geometries. Since we do not possess this publication, we cannot go any further in the discussion, but it seems worthwhile that the authors look more closely to this and other effects contributing to such an unusual coincidence. Indeed, Dugdale developed his model for elastic-perfectly plastic materials and corroborated the results of the model for a plain carbon steel which presents insignificant amount

of strain-hardening. Even for this material, the experimentally determined lengths of the plastic zones at the tip of the crack measured by Dugdale were smaller after a certain load than their corresponding theoretical values calculated from the model. In order to get smaller crack lengths as in the experiments the distribution of the stresses inside the plastic zone must be variable and not constant as predicted by the model. However, most of the engineering materials present appreciable amount of strain-hardening, which has to be taken into account when plastic stress distributions are studied. Thus, the stress distribution in the plastic zone of a cracked plate made of a strain-hardening material is magnified through strain-hardening. Indeed, if we assume a simple strain-hardening stress-strain curve of the form $\sigma = \sigma_0(\epsilon/\epsilon_0)^n$ the stress is magnified because of strain hardening by a factor $(\epsilon/\epsilon_0)^n$, when compared with the perfectly plastic behavior. The flow stress in the plastic enclave at the vicinity of the crack tip of a mode I crack for elastic-perfectly plastic as well as strain-hardening material is not constant, but it changes, presenting its maximum value at the interior of the plastic enclave, depending on the amount of strain-hardening. This result was established in previous publications of the discussor and his co-workers [2, 3] and it is shown in Fig. 1(a) for a cracked specimen, made of a USS-T₁ steel which presents an almost elastic-perfectly plastic behavior, and in Fig. 1(b) for a perforated plate in tension made of an aluminum alloy which is a strain-hardening material. It can be observed from these figures that the stress distribution in the plastic zone is variable even when the material is elastic-perfectly plastic (Fig. 1(a)) and that the maximum of the stress lies in the interior of the plastic zone.

In order to alleviate this discrepancy between the typical Dugdale model and the experience Hahn and Rosenfield modified this model by introducing a variable stress distribution in the plastic zone, instead of a uniform one, as in the case of the Dugdale model. In the same line the discussor and his co-workers [4, 5] have studied further the laws of this discrepancy and gave some obvious reasons for this difference. They established that the region of validity of the simple Dugdale model depends not only on the amount of the strain-hardening of the material and the triaxiality of the stress field at the crack tip, but also on its strength, and that modified Dugdale models must be used when all these factors increase.

Then although it is obvious that for any elastic plastic analysis the Prandtl-Reuss method when combined with the actual stress-strain curve of the material is the only accurate, this method has the drawback to be time consuming, since it necessitates lengthy computer courses, which must be executed for each particular material and for each particular loading history.

Instead of this method the authors recommended a procedure which uses experimental data combined with the elastic $J^{1/2}$ analysis for the CLWL specimen for defining the $J_R^{1/2}$ crack growth resistance curve by taking into account plasticity. Discussor suggests that a simple and versatile method for computing the values of the J -integral is by using the modified Dugdale model, which as it was proved in references [4] and [5] gives reliable results for many engineering materials, in combination with the optical method of caustics developed by the discussor and used

¹By D. P. Wilhem, M. M. Ratwani, and G. F. Zielsdorff, published in the April 1977 issue of the JOURNAL OF ENGINEERING MATERIALS AND TECHNOLOGY, TRANS. ASME, Series H, Vol. 99, pp. 97-104.

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³Numbers in brackets designate additional References at end of discussion.

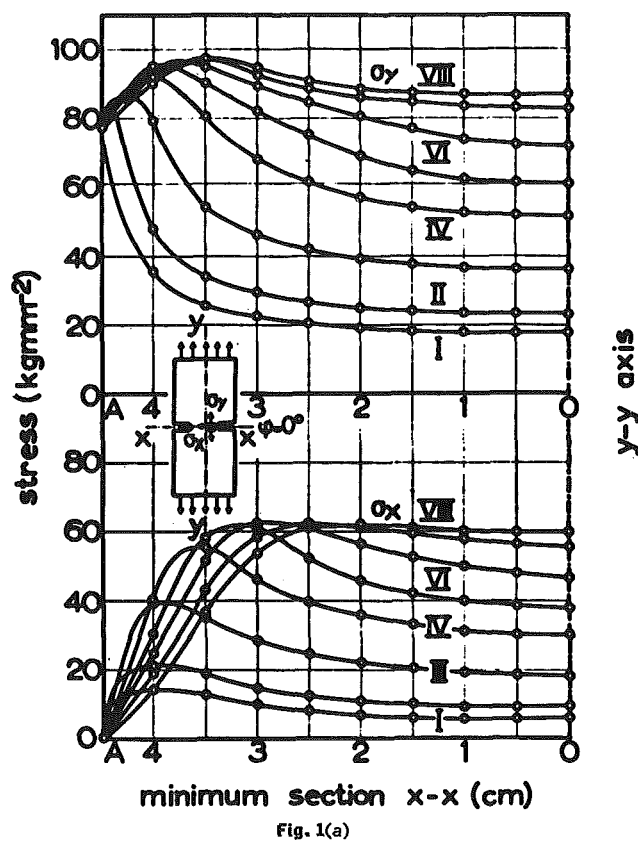


Fig. 1(a)

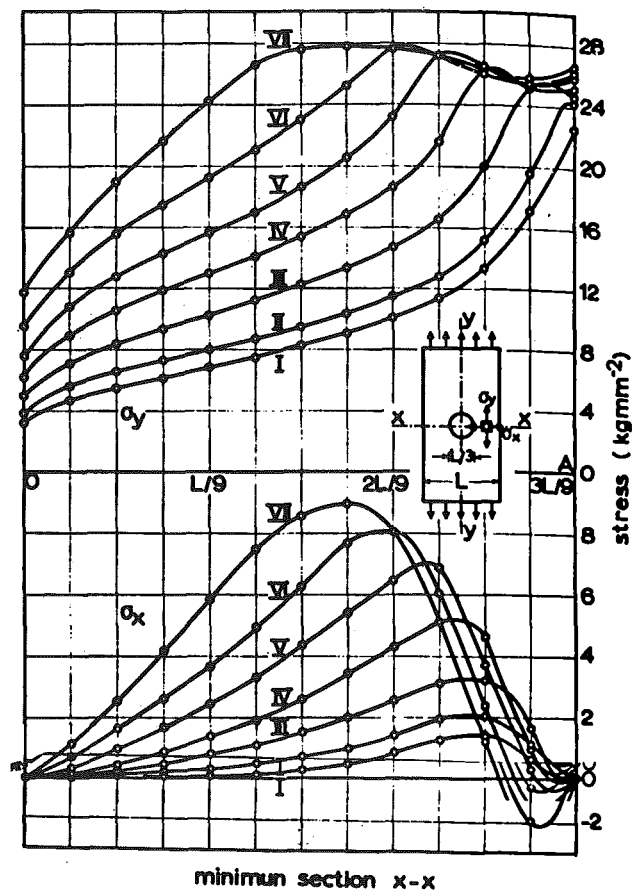


Fig. 1(b)

Fig. 1 Stress distribution along the minimum section of a cracked specimen made of a USS-T₁ with yield stress $\sigma_y = 770 \text{ Nmm}^{-2}$ and an ultimate stress $\sigma_u = 840 \text{ Nmm}^{-2}$ (a) and along the minimum section of a perforated strip made of an aluminum alloy with yield stress $\sigma_y = 243 \text{ Nmm}^{-2}$ and ultimate stress $\sigma_u = 280 \text{ Nmm}^{-2}$ (b). Stresses are given for eight and six loading steps for the cracked plate and the perforated strip respectively, corresponding to the elastic up to the fully plastic deformation of the specimens.

for the solution of many elastic and plastic stress fields.

According to this method the plastic zone at the tip of the crack is transformed to a highly illuminated curve, the caustic, on a reference screen placed at some distance from the specimen. This is achieved by illuminating the polished neighborhood of the crack-tip of the specimen with a laser light beam and receiving the reflected light rays on a screen. By comparing the theoretically obtained caustics by assuming various types of variable stress distribution in the plastic zones with those obtained experimentally, the most probable stress distribution within the plastic zone for each loading step can be readily found, as well as the characteristic properties of the plastic zone and the region of validity of each separate stress profile assumed for the modified Dugdale model. This method was successfully used in references [4] and [5] for finding the region of validity of the simple, as well as the various versions of the modified Dugdale model in steel alloys.

From these investigations it was proved that caustics are very sensitive to the variation of the stress distribution inside the plastic zone and therefore they provide accurate means for the determination of the stress distribution in the plastic zone from the corresponding caustics. It was also concluded that three separate regions of deformation may be distinguished in cracked metallic plates. The first is the region corresponding

to elastic or small plastic deformations at the crack tip, the second is that where appreciable amounts of plasticity are introduced, but they can be studied by using the various versions of the modified Dugdale model, and the third region is that which corresponds at the neighborhood of fracture, where plasticity is established all over the plate. This third region cannot be attacked by the modified Dugdale model and necessitates a different mode of confrontation.

The extra work for simply polishing the strip of the probable front of the propagating crack combined with the use of a laser light for detecting caustics does not add to the cost of each particular experiment. On the contrary, the caustic is the most suitable contour for evaluating J -integrals since its generatrix curve (initial curve) is a circle for elasticity and an oblong curve for plasticity lying in the closest possible position around the crack-tip singularity. Thus the use of the method of caustics circumvents the crack-tip singularity. It is therefore reasonable to accept that the evaluation of the J -integral by this method is the most accurate and therefore the most suitable.

Additional References

- 1 Ratwani, M. M., and Wilhem, D. P., "Development and Evaluation of Methods of Plane Stress Fracture Analysis-Part II, Volume I, A Technique for Predicting Residual Strength

of Structure," AFFDL-TR-73-42, Part II, Vol. I, Apr. 1975.

2 Theocaris, P. S., and Marketos, E., "Elastic-Plastic Strain and Stress Distribution in Notched Plates Under Plane Stress," *J. Mech. Phys. Solids*, Vol. 11, 1963, pp. 411-428.

3 Theocaris, P. S., and Marketos, E., "Elastic-Plastic Analysis of Perforated Thin Strips of a Strain-Hardening Material," *J. Mech. Phys. Solids*, Vol. 12, 1964, pp. 377-390.

4 Theocaris, P. S., and Gdoutos, E., "Verification of the Validity of the Dugdale-Barenblatt Model by the Method of Caustics," *Engineering Fracture Mechanics*, Vol. 6, 1974, pp. 523-535.

5 Theocaris, P. S., and Gdoutos, E., "The Modified Dugdale-Barenblatt Model Adapted to Various Fracture Configurations in Metals," *Int. J. Fracture*, Vol. 10, 1974, pp. 549-564.

Authors' Closure

As mentioned in the discussion, the results of \sqrt{J} values obtained for stiffened, center cracked panels gave excellent correlation with experimental residual strength data (reference [1]). The analytical values obtained using Dugdale plastic zone assumptions predicted failure stresses within 10 percent (80 percent of the total within ± 5 percent) of those observed experimentally for very complex skin/stiffener geometries for a variety of materials and skin thicknesses. Since these panel fracture strengths and other pressure vessel results (reference [2]) have been predicted with a high degree of success based on Dugdale model calculations, it is our opinion that reliance on a modified Dugdale model, although agreeably more refined, is apparently not necessary in the use of the proposed elastic-plastic fracture criterion.

Those effects presented by the discussor regarding the plastic zone description are well known and have been shown analytically in reference [1]⁴ as well as elsewhere in the fracture me-

chanics literature. As the authors have noted in reference [3], the use of the Prandtl-Reuss elastic-plastic analysis method is both time-consuming and expensive, even with the use of high speed computers. Since the Dugdale model provided the necessary predictive accuracy in our research, any further correlation based on Prandtl-Reuss Behavior was not pursued.

The experimental details of determining K_R values for the crack line wedge loaded (CLWL) specimen geometry are described in detail in reference [4] (reference [8] of our paper). This method of test is currently a tentative recommended practice of the American Society for Testing and Materials (ASTM). Thus the data obtained from these tests can be readily compared with those obtained from other investigators. The authors have used the experimental procedure of the ASTM practice to obtain an elastic-plastic curve, good for all materials and thicknesses as noted in the paper. If the method of caustics had been an integral part of a standard practice, then it too could have been utilized in a similar manner. The authors do not know of the existence of such a standard practice for determining crack growth resistance using caustic methodology.

Additional References

1 Ratwani, M. M., and Wilhem, D. P., "Development and Evaluation of Methods of Plane Stress Fracture Analysis - Part III, Application of Residual Strength Prediction Technique to Complex Structure," AFFDL TR-73-42, Part III, Oct. 1975.

2 Ratwani, M. M., and Wilhem, D. P., "Application of Resistance Curves to Residual Strength Prediction," paper submitted to ASME, JEMT.

3 Ratwani, M. M., and Wilhem, D. P., "Development and Evaluation of Methods of Plane Stress Fracture Analysis," Part II, Volume I, "A Technique for Predicting Residual Strength of Structure," AFFDL TR-73-42, Part II, Vol. I, Apr. 1975.

4 "Tentative Recommended Practice for R-Curve Determination," E561-76T, 1976 Annual Book of ASTM Standards, Part 10, American Society for Testing & Materials, pp. 539-557.

⁴Numbers in brackets designate References at end of closure.