$$\sigma = (82.32) - (8.7055)N - (4.81)F + (8.56)C.F. - (4.09)W \quad (20)$$

where:

σ = completely reversed bending fatigue strength at room temperature
N = cutter rpm (1 = 100 rpm, 1.5 = 150 rpm)
F = feed; (1 = .004 in. (.102 mm), 2 = .008 in. (.20 mm))
$C.F.$ = cutting fluid (0 = none, 1 = Hocut)
W = wear land (1 = .003 in. (.076 mm), (6 = .018 in. (.46 mm))

For the peripheral milling, the cutting rate equation (8) is repeated below:

$$R = (\frac{12V}{\pi D})(F \times t)(d_a \times d_r) \quad (21)$$

where:

D = 4 in. (101.6 mm), t = 12, d_a = 2 in. (50.8 mm),
d_r = .03 in. (.76 mm)

When the lines of constant stress level and constant cutting rate were plotted (Fig. 14), the points of tangency defined the R-T characteristic curve in the F-V plane. As can be seen from this figure, any given level of surface integrity as measured by the reversed bending stress, σ, the highest cutting rate, can be achieved by choosing machining conditions along the R-T characteristic curve.

Conclusions

1 The existence of cutting rate-tool life *(R-T)* characteristic curves was experimentally verified. These experiments demonstrated that the R-T characteristic curves for circular sawing and for peripheral end milling occur in the economic working region. Furthermore, the R-T characteristic curves showed that while feed is the critical variable for the sawing tests, speed is the critical variable for the milling experiments.

2 The analytical and geometrical interpretation of the R-T characteristic curves led to the establishment of a linear equation for logarithms of cutting variables and a quadratic equation for logarithms of cutting rate and tool life. Mathematically, R-T characteristic curves have an optimum in the R-T domain; whether this optimum will lie in a practical working range depends on the specific operation.

3 The concept of R-T characteristic curves is capable of providing solutions to several material removal problems. *(a)* It is possible to approach economic optima by selecting machining conditions as close to the R-T characteristic curve as possible; this can be accomplished even when limited cost data are available. A measure of the closeness to the R-T characteristic curve is described as the material removal efficiency factor. *(b)* Tool life tests and other machining responses can be established more efficiently and economically using cutting rates and R-T characteristic curves in planning experimental strategies. *(c)* A comparison of machining reponses can be made more rigorous by comparing the R-T characteristic curves. When there are optima in the R-T characteristic curves, the machining responses can be uniquely compared. *(d)* The R-T characteristic curves provide an effective objective function for adaptive control. *(e)* The material removal rate can be maximized at any desired level of surface integrity.

4 The concept of R-T characteristic functions is valid for many machining responses and performance characteristics that are influenced by cutting variables. The concept is applicable to conventional as well as nontraditional material removal processes.

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DISCUSSION

D. G. Lee[4] and J. E. Mayer, Jr.[5]

These papers are an important addition to the literature on economics of machining. The concept of the R-T characteristic function which contains both the minimum cost and maximum productivity machining conditions provides a new insight into the principles of machining economics.

The authors correctly point out that one of the principal obstacles to the application of machining economics to shop practice is the scatter in tool life data. It is not clear whether the techniques presented in these papers would reduce this obstacle, since accurate tool life data remain an important requirement. For high production manufacturing operations, each tool life data point would require machining a minimum of 50 and possibly several thousand parts. Thus, it is easy to envision several weeks of machining being required to establish the$R$-T characteristic function for an operation.

An important feature of machining economics analyses based on tool life equations such as Taylor's equation is that the tool life equation can be established in the laboratory for a class of operations and made specific for a particular plant operation through incorporation of existing plant data obtained at the present operating conditions.[6] Unless R-T characteristic functions can be established in a similar manner, their use in optimization of existing high-production operations will be almost impossible due to the difficulty of making a succession of speed or feed changes on a production machine. It is for new operations, for which there are no existing plant data, that these techniques may offer their greatest advantage over tool life equation-based analyses. In these cases, the operations could be simulated on a machine with variable speed and feed to establish the optimum operating conditions. The production machine could then be constructed to operate at the optimum conditions.

An important requirement of a machining economics analysis for high production operations is the ability to optimize multi-tool operations. It is not clear how the R-T characteristic function approach would be used if several tools were cutting simultaneously. An extension of the analysis to include multi-tool capabilities would be a welcome addition to this most interesting approach to machining economics.

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Author's Closure

It is gratifying to read that the R-T characteristic functions introduced by the late Dr. Moshe Friedman and I have been recognized by Drs. David Lee and John Mayer, Jr. as providing new insights into the principles of machining economics. In response to the discussers' comments, I would like to state the following:

1 For shop floor determination, the approximate location of the R-T characteristic function can be established through five to six different combinations of speeds and feeds as illustrated in Fig. 9, Part 2 [6]. This procedure has been recently successfully implemented on the shop floor in a major aerospace company.

2 The discussers rightfully point out that the R-T characteristic fuctions offer their greatest advantage over Taylor's equation for identifying economic combinations of machining conditions for new operations. Recent experience in the aerospace industry shows that the existing operations can also be improved by the R-T characteristic functions.

3 The application of the R-T characteristic functions to multi-tool, multi-station machining systems was presented at the SME International Tool and Manufacturing Conference, Detroit, April 7–10, 1975. Further work in this area is in progress. The R-T characteristic functions provide a rigorous means for the development and use of machinability data for computerized process planning [7]. By introducing more variables into the R-T characteristic function analysis, substantially improved optimum can be obtained [8]. Recently, R-T characteristic functions were being used for economic generation of tool life and other machining response data [9].

In conclusion, it should be mentioned that the Taylor and extended Taylor equations are suboptimal special cases of the R-T characteristic functions.

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