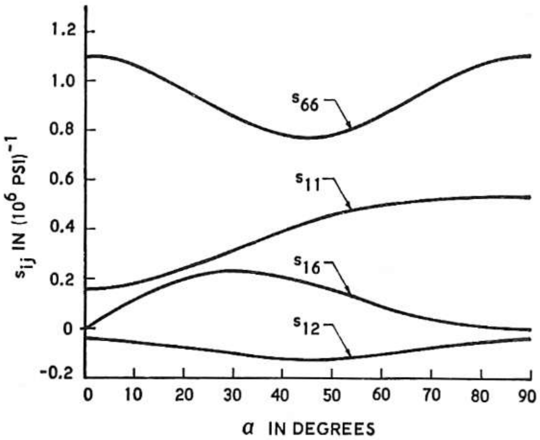

Fig. 4 Transformation of s_{ij}

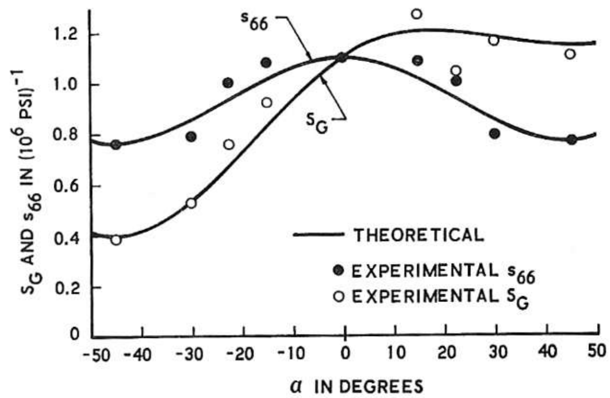

Fig. 5 S_G and S_{66}—data versus theory

epoxy resin matrix. The resin content was 23 percent by weight and 40 percent by volume. The composite material was taken to be homogeneous since there are hundreds of filaments across the plate thickness.

The beams were bent using the standard central-loading flexural test. The pure twisting test was accomplished with a very simple test fixture, which provided three fixed supports in the first, second, and third quadrants, with the directions indicated in Fig. 2. The remaining corner, in the fourth quadrant, was the loading point loaded by an Instron testing machine. The loading and supporting points for the twisting test were located about 1/4 in. from the corners of the plate specimen. The amount of overhang, which is the difference in the plate size and the loading and supporting points, was not found to be critical. Direct plotting of the applied load from the load cell and the vertical displacement of the loading point from the crosshead motion, i.e., P and w_L in (18), was obtained. The displacement of the center of the plate, w_0 in (15), was also plotted on the same recorder by means of a deflectometer. The loading fixture and the deflectometer are shown in Fig. 3. Thus from the same chart, both P/w_L, and P/w_0 were derived.

From the central-loading flexural test of three 0 deg-beams, the average compliance was:

$$S_{11} = 0.16 \times (10^6 \text{ psi})^{-1} \tag{19}$$

From the pure twisting test of 0 and 45 deg square plates, the following were obtained by using (17):

$$\begin{aligned} S_{66} &= 1.10 \times (10^6 \text{ psi})^{-1}, \\ S_{11} - S_{12} &= 0.20 \times (10^6 \text{ psi})^{-1}, \\ S_{22} - S_{12} &= 0.57 \times (10^6 \text{ psi})^{-1}. \end{aligned} \tag{20}$$

S_{12} and S_{22} were then computed by combining (19) and (20) so that,

$$\begin{aligned} S_{12} &= -0.04 \times (10^6 \text{ psi})^{-1} \\ S_{22} &= 0.53 \times (10^6 \text{ psi})^{-1} \end{aligned} \tag{21}$$

By bending 90 deg-beams, the resulted S_{22} was $0.50 \times (10^6 \text{ psi})^{-1}$. This was a good independent check of the S_{22} derived in (21).

Knowing the four independent elastic constants, one can readily compute from (3) through (8) the theoretical elastic constants for other orientations. These constants, of which there are six, are plotted in Fig. 4. Note that s_{22} and s_{26} curves are not drawn explicitly because they are the mirror image of s_{11} and s_{16}, respectively. The plane of reflection is at $\alpha = 45$ deg.

The present test procedure furnishes readily an independent check of the results obtained above. Three additional square plates with orientations 15, 22.5, and 30 deg were made and tested. From these plates, s_{66} and S_G for six different orientations were obtained. These data, as shown in Fig. 5, compared favorably with the theoretical results computed from the four independent constants.

Conclusions

It has been shown that the proposed method of determining the orthotropic elastic constants is easy to pursue and yields reliable data. The special test fixture for the pure twisting test is extremely simple. Beyond this, no additional equipment, instrumentation, or strain gages are required. If a deflectometer is not available, a dial gage can be mounted above the center of the plate and discrete dial readings can be recorded as the applied load increases. This obviously is more time-consuming and less accurate than the continuous recording of load-versus-deflection. It has also been shown that independent check on the data can be readily obtained, e.g., bending of 90 deg-beams, and twisting of plates with orientations other than 0 and 45 deg.

In place of the bending test, one can use uniaxial tension tests of 0 or 90 deg rods from which S_{11} or S_{22} can be obtained, respectively.

References

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DISCUSSION

R. E. Beckett,[2] R. J. Dohrmann,[3] and K. D. Ives[4]

The author is to be congratulated for his work described in this technical paper in that a unique method has been developed whereby the elastic constants of orthotropic plates can be determined. The method described, requiring one bending test and two twisting tests at two prescribed angles of 0 and 45 deg, materially reduces the amount of work required. In using the method, an assumed solution to the differential equation for an anisotropic plate was given by equation (9). When the plate is loaded similarly to Fig. 2 for the twisting test, certain terms in the equation drop out, yielding equation (11), which is then evaluated by solving for the constants a, b, and c, using the boundary conditions.

[2] Professor, Department of Mechanics and Hydraulics, State University of Iowa, Iowa City, Iowa. Mem. ASME.

[3] Chief, Applied Mechanics Section, The Babcock & Wilcox Company Research Center, Alliance, Ohio. Mem. ASME.

[4] Senior Engineer, The Babcock & Wilcox Company Research Center, Alliance, Ohio.

In a recent paper[5] it was noted that the existing procedures for determining the constants were inadequate in various respects. We also pointed out some of the pitfalls of using an assumed equation such as given by equation (9) in the present paper. If the plate should not bend into the particular surface described by equation (9) or equation (11), then large errors in determining the elastic constants can be introduced. Such a possibility is a real danger if the experimental setup is not carefully controlled to insure that all boundaries are satisfied exactly. To avoid this danger, a method was presented in the earlier paper[5] that made use of the relative positions of points on the surface in the immediate vicinity of the test region and subsequent measurement of curvature. The so-called method of "curvatures" provides additional checks concerning the validity of the elastic constants determined, such as how well the experimental setup duplicated the application of the desired pure moment and pure twisting conditions.

It would be interesting to have the author's comments on the adaptability of his method to orthogonally stiffened plates of a repeating-section type that are not necessarily of constant thickness but vary in thickness as a function of the repeating section.

Author's Closure

The author wishes to thank the discussers for pointing out their method of testing orthotropic plates. In an unpublished dissertation by Dr. E. R. Scheyhing, submitted to Yale University in 1965, entitled "Model Analysis of Elastic Shells Using Glass-Reinforced Plastics," a critical evaluation of most existing test methods of orthotropic plates was reported. The technique proposed by the discussers, i.e., to take measurements at various points far removed from the loading points, was found to be an improvement over most other methods. The determination of Poisson's ratio, however, was still very difficult because it depended on the difference between two nearly equal deflections. An increase in accuracy can be obtained by using strain gages, from which Poisson's ratio is calculated from the ratio of strains rather than the difference in deflections.

Dr. Scheyhing also evaluated the method proposed by the author and found "the specimen dimensions are convenient and testing is accomplished quickly and easily. The automatic recording of load-deflection curves is felt to be the greatest advantage, since it largely eliminates the subjective nature of deflection reading and interpretation which is characteristic of discrete observations. This approach is highly recommended to other investigators"; (p. 87 of the dissertation).

In the case of a stiffened plate of a repeating section type as described in the discussers' paper, the use of strain gages will not be satisfactory. The method proposed by the author should be applicable and yield accurate data, provided that the stiffened plate contains many repeating sections, i.e., is quasi-homogeneous.

As a concluding remark, the key technical issue here is the ability to duplicate experimentally the mathematical boundary conditions. A homogeneous plate subjected to homogeneous bending and twisting moments will become a quadratic surface. This is exact within the framework of strength-of-materials; equations (9) and (11) require no additional assumptions. The problem is that of satisfying the boundary conditions of equation (10), or other loading conditions, adequately. It is believed that the author's method provides an answer to this seemingly trivial problem.

[5] R. E. Beckett, R. J. Dohrmann, and K. D. Ives, "An Experimental Method for Determining the Elastic Constants of Orthogonally Stiffened Plates," *Experimental Mechanics*, Pergamon Press, 1963.