having five infinitesimally separated positions on a straight line, the deviation and second derivative curves have a character similar to that of linkages based on curvature theory.

A symmetrical linkage of the Chebychev type is represented in Figs. 9 and 10. A Roberts type linkage is represented in Figs. 11 and 12. These linkages produce fairly inaccurate, but extended, straight-line outputs. Linkages of this group, that are based on the Chebychev minimum deviation theory and those based on curvature theory, will probably give the best results.

Figs. 13 and 14 represent a centric-rectangular Evans linkage. This linkage is closely associated with the Scott-Russell exact straight-line slider-crank mechanism and, therefore, gives very good results. There exist two other distinct configurations of the Evans linkage. Based on the results of digital computation, these configurations do not appear to give very satisfactory results.

Representative Design Procedure

Up to this point, the data have not been reduced to a form which would be most useful to the designer. In this section, the set of linkages having a Ball-double Burmester point will be analyzed to provide the designer with a technique enabling him to choose the design parameters α_a, α_b for a specified quality and length of the straight-line output. Previously it was necessary for him to judiciously choose the design parameters in a trial-and-error approach to get a satisfactory straight-line output. Furthermore, it was difficult to assess the quality of the resulting approximation to a straight line.

If, for general coplanar motion, one of the Burmester points coincides with the Ball point, the remaining three Burmester points are collinear. If, in addition, a second Burmester point coincides with the Ball point, the Ball-double Burmester point would then lie on the coupler link center line of the four-bar linkage formed by using the remaining Burmester pairs as cranks. The design equations (16) are

$$\tan \alpha_d = -\frac{V}{W + 3} \qquad (5)$$

and

$$PA = \frac{V(W - 1) \sin \alpha_a}{(W + 3) \tan \alpha_a + VW}$$
$$(6)$$
$$PB = \frac{V(W - 1) \sin \alpha_b}{(W + 3) \tan \alpha_b + VW}$$

where $V = \tan \alpha_a + \tan \alpha_b$; and $W = (\tan \alpha_a)(\tan \alpha_b)$. The remaining design equations obtained by using the Euler-Savary equation are:

$$PO_a = \frac{-PA \sin \alpha_a}{PA - \sin \alpha_a}, \qquad PO_b = \frac{-PB \sin \alpha_b}{PB - \sin \alpha_b} \qquad (7)$$

Fig. 15 is a data chart of the primary information plotted, using parameters α_a, α_b as coordinates. The length L of the approximate straight-line output for an accuracy of 0.01 was calculated. A grid of the data was formed and curves having a constant value of L were interpolated. Linkages having the ratio of the longest and shortest links greater than five are not included on the chart. Due to the symmetry of the design equations with respect to the parameters, full ranges need not be considered. Through the central region of the chart, there is a curve dividing the linkages into those that are crank and lever and those that are double lever mechanisms. Local areas that are not well defined occur when $\alpha_b < 90$ deg. Consequently, the designer can predict the type of mechanism that will result in advance. It is also possible to choose linkages that will have good proportions, since only those linkages are represented in the diagram. Other charts for transmission angles, range of motion of the cranks, and properties of the higher derivatives of the deviation curve could also assist the designer in isolating groups of linkages having properties best fulfilling specified requirements.

Assume that a linkage is needed to produce a fairly long approximate straight-line output. It must also be a crank and lever mechanism. Choosing $\alpha_a = 5$ deg and $\alpha_b = 40$ deg satisfies these requirements. The expected length of the straight-line output is given as approximately 1.30 for an accuracy of 0.01. This linkage and the corresponding curves are represented in Figs. 16 and 17. Note that the transmission angle α_a remains within very desirable limits during the production of the straight-line portion of the coupler curve.

Acknowledgments

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DISCUSSION

Lee Harrisberger[6]

I appreciate the fact that at long last we have an accurate comparative analysis of the several classical straight-line linkage mechanisms. It is the first time, to my knowledge, that we have been able to compare the range and accuracy of these several mechanisms. Even more valuable is the summary chart, Fig. 15, for the Ball-double Burmester point data. Such succinct presen-

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tations as this contribute to bringing mechanism synthesis into useful practicability. I am an avid crusader for the development of useful synthesis information from the research in mechanisms. The linkage domain is made of extremely simple, low-cost devices for which the design effort cannot afford to be tedious, time consuming, and costly.

It is noted that none of the mechanisms in the range of the chart in Fig. 15 have the length of path that is accomplished by the Centric-rectangular Evans linkage in Figs. 13 and 14.

Robert W. Mann[7]

Fig. 1 represents a solution to an analysis of the quality of straight line generated by a linkage quite similar to the studies by Drs. Tesar and Vidosic in their clearly presented, comprehensive, and useful paper. The difference between results shown in Fig. 1 and those in the authors' paper is that the linkage diagrams were sketched and analyzed in a total elapsed time of about 10 minutes using the graphical input-output and program structure of the SKETCHPAD Computer-Aided Design system. This system permits the engineer, with no knowledge of, or need to do, any computer programming, to sketch any arbitrary linkage and then animate that linkage to determine its kinematic characteristics. In the case shown, the classical linkage of Peaucellier was sketched using the light pen and a cathode-ray oscilloscope screen. By applying "constraints" by means of the light pen and push buttons, the engineer defined significant characteristics of the linkage (such as the proportions of links and the grounded pivot points) and erected the vertical line perpendicular to the line of centers of the input and output links. He then rotated the linkage to new positions by waving the light pen and asked the computer to evaluate and display on the screen the distance between the output and the vertical line. As the four figures illustrate, and as we know must be the case for this particular linkage, the distance remains constant as indicated by the repetitive 1087.

The significance of the comparison between the authors' work and this Computer-Aided Design[8] exercise is the impending release of the engineer from much of the drudgery of drawing, programming, plotting, etc., of an investigation by means of the much more intimate functional relationship of the man and the machine brought about by this Computer-Aided Design system.

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In its final embodiment, many aspects of which are already demonstrable, the system will not only analyze the graphical input-output of a problem but will help the engineer to conduct motion, stress, and other analyses, incorporate standard parts, specify tolerances, perform optimizations—in short, all those studies necessary to completely define the system to be manufactured.

The transfer of the major part of the drudgery of the engineering design process to the computer in a way which maintains the man's continuous and intimate rapport with his evolving solution will certainly enhance the creative contributions of the engineer and result in new, better, and less expensive products in less time.

Authors' Closure

The authors wish to thank the discussers for their comments on the nature of the design process as associated with mechanisms.

The comments given by Prof. L. Harrisberger are in agreement with the view of the authors in that the results of research in kinematics should be made available to the practicing engineer. As stated in the text of the paper, the limited number of mechanisms cannot provide the necessary completeness to fulfill this purpose. More complete data of this character, however, have been submitted for publication elsewhere. The authors also note the validity of Professor Harrisberger's comment with respect to the Evans mechanism. The Evans mechanism represented in Figs. 13 and 14 is one of exceptional quality and, accordingly, does not represent the general case.

Prof. W. Mann's proposal for designing mechanisms is certainly useful and perhaps the ultimate technique to be used in engineering design. The type of assistance for the designer contemplated by the authors has a sufficiently different character to make the combination of the two concepts enticing indeed. Generally, data charts are conceived by experts and not by designers. The charts, once produced, are permanent, portable, and immediately available to the designer. Also, the charts give a total instantaneous picture of the full range of the design parameters. The technique of computer-aided design allows the designer intimate knowledge of a single mechanism or closely related group of mechanisms. Unfortunately, not all designers will have the sophisticated equipment available for this purpose. If these two techniques can be used simultaneously by the designer, it should enable him to obtain an optimum solution with minimum effort.