1 Draw an arbitrary inflection circle I, Fig. 4. The inscribed angle subtended by arc P_1P_2 is equal to the specified angle α. The two arbitrary points P_1 and P_2 serve as the instantaneous centers of the adjustable four-bar linkage. Line "b" through points P_1 and P_2 dictates the position of link B_0B and P_2 is also the point J_{B1}, the point of intersection between link B_0B and the inflection circle I.

2 Draw an arbitrary line "a" through P_1, which represents the position of link $A_{01}A$. This line "a" intersects inflection circle I at point J_{A1}.

3 Join points J_{A1} and J_{B1}. This line is parallel to collineation axis I, according to Bobillier's construction. Thus collineation axis I can be drawn through P_1.

4 Draw an arbitrary line "l" intersecting lines "a" and "b" at points A and B; collineation axis I at point D_1, and line $J_{A1}J_{B1}$ at point M_1.

5 Join M_1P_1. This line, according to Bobillier's construction, is parallel to the fixed link $A_{01}B_0$. Thus, the fixed pivots A_{01} and B_0 can be located by drawing a line through D_1, parallel to M_1P_1, intersecting lines "a" and "b." The four-bar linkage $A_{01}ABB_0$ is thereby established.

6 Join AP_2. With point A as the center, AA_{01} as the radius, strike an arc intersecting line AP_2 at A_{02}. Thus, $A_{02}ABB_0$ is the new four-bar linkage with the fixed pivot A_{01} adjusted through an angle θ.

7 Construct the inflection circle II of the four-bar linkage $A_{02}ABB_0$ by Bobillier's construction as shown. Inflection circle II intersects inflection circle I at point C which is the required coupler point.

8 Since point C is also on inflection circle I, it sees arc P_1P_2 at angle α. Therefore the straight lines traced out by point C should be perpendicular to lines P_1C and P_2C and form an angle α.

The graphical solution presented in the foregoing satisfies the requirement of a specified α, only. Four arbitrary decisions were made:

1 Inflection circle I (size)
2 The location of points P_1 and P_2 on inflection circle I
3 Line "a" through P_1
4 Line "l"

Thus, other requirements could be satisfied by placing additional restrictions on the technique, which would eliminate one or more "arbitrary decisions." The intrinsic flexibility of this technique is most helpful in solving demanding problems.

As an example, suppose both angles α and θ are specified. Referring to Fig. 4, point A sees line P_1P_2 through an angle θ. The locus of point A is a circle arc with angle θ subtended by chord P_1P_2. Fig. 5 shows the graphical solution of an adjustable four-bar linkage with both angles α and θ specified.

1 Draw an arbitrary inflection circle I with the inscribed angle subtended by arc P_1P_2 equal to the specified angle α. Line "b" through P_1P_2 again dictates the position of link B_0B.

2 Draw another circle "O" with the inscribed angle subtended by arc P_1P_2 equal to the specified angle θ. Select point A on circle "O" as a pivot point of the four-bar linkage. Draw "a" through points A and P_1, intersecting inflection circle I at point J_{A1}.

3 Draw an arbitrary line "l" through point A intersecting line "b" at point B.

4 Line J_{A1} P_2 (J_{B1}) intersects arbitrary line "l" at points M_1 and is parallel to collineation axis I. Thus collineation axis I can be drawn through P_1 and intersects line "l" at point D_1.

Following steps 5 through 8 described before, the four-bar linkage A_0ABB_0 with adjustable link A_0A is obtained to satisfy the requirements of specified angles α and θ.

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DISCUSSION

Kurt Hain[1]

Adjustable linkages are important in varying the behavior of a linkage. The simplest way of changing this behavior is to displace a fixed point, as the authors suggest. This paper is a good example that more work should be done at the drawing board to get a simple linkage. It would mean no effort to the designer to solve the given problem when any straight-line mechanism is used and this mechanism is at all turned about the ordered turning point of the straight line. But this solution is much more expensive in practice than the authors' linkage.

The authors mention that the lengths of the "straight line" portions cannot be predicted. This is true as, in the general case, points located on the inflection circle run through an inflection point only, i.e., the center of curvature changes from one side to the other side of the coupler curve. For some applications, this result is useful; for instance, when an inverted slider represents the output link, which is then at a momentary standstill.

The authors should be asked to expand their method to the production of a better straight line. What about the following proposal: Find a four-bar linkage having a coupler point which is in two positions on two different inflection circles. This leads to an excellent straight line as two points of it are inflection points. Another way could be the use of the classical dimensional synthesis: Two straight lines are given; find a four-bar linkage of which a coupler point runs on one line and, after adjustment of a fixed point, on the other line.

Allen S. Hall, Jr.[2]

One of the more fascinating aspects of linkage mechanisms is the inherent capacity for simple adjustment or change of proportions. This is a subject which has not, in the writer's opinion, received the attention it deserves. It is hoped that the authors will continue to explore the possibilities and present additional solutions.

As the authors have demonstrated in this paper, the designing of linkages adjustable for specific objectives requires, not a new body of theory, but a large measure of ingenuity in applying basic kinematic-geometric propositions.

Delbert Tesar[3]

The paper presents a simple and straightforward graphical technique for the solution of a special problem by means of an adjustable four-bar linkage. Fig. 1 represents the general case where the angle α is unspecified in the statement of the problem. This case is far too general for further development, and the authors wisely consider the special situation where α is specified in advance. This results in a procedure having not more than four design choices:

(a) The size of the inflection circle (and therefore the linkage) is immaterial for the solution of the problem.

(b) The location of P_1 on inflection circle I. The location of P_2 is determined by the specified value of α.

(c) One parameter is associated with the direction of line a through P_1.

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(*d*) Two parameters are associated with the definition of line (*a*). Hence there are four design parameters. Generally, the designer will find the totality due to four parameters too unwieldy for a convergent trial-and-error design development. Here the authors have given one example of restricting the range of available linkages by specifying, for example, the significant design choice θ. For a graphical procedure, the designer should find it possible to control three design parameters.

The authors are to be complimented for a particularly simple and interesting solution to the given problem. The adjustable linkage is unique in its attributes among the available types of mechanisms.

Authors' Closure

The authors wish to express their gratitude to Messrs. Kurt Hain, Allen S. Hall, Jr., and Delbert Tesar for their most helpful and astute observations given in their evaluative discussion of our article.

Because of the constructive tenor of these comments, the authors have been encouraged to continue exploring this specific segment of kinematics and hope to be able to present the results from time to time.