APPENDIX

$$a_1 = \pi\left[\frac{-S_0^2 + S_0 + 1}{S_0^3(S_0+1)} - \frac{n_0(2S_0+1)A_{10}}{S_0^3(S_0+1)^2} + \frac{(-2S_0^2+2S_0+1)}{S_0^3(S_0+1)^2}A_{20}\right]$$

$$a_2 = \sin\varphi_0$$

$$a_3 = -\frac{\pi}{S_0(S_0+1)} \qquad \text{(A1)}$$

$$a_5 = \frac{\pi n_0}{S_0(S_0+1)^2}$$

$$b_1 = \pi\left[-\frac{B_{10}}{S_0(S_0+1)^2} + \frac{(2-S_0)B_{20}}{n_0 S_0(S_0+1)^2}\right]$$

For a complete list of these coefficients, see reference [9].

Acknowledgment

The authors are grateful to the University of Pennsylvania for the use of the analog-computer facilities.

References

1 W. J. Harrison, "The Hydrodynamical Theory of Lubrication With Special Reference to Air as a Lubricant," *Trans. Cambridge Phil. Soc.*, vol. 22, 1913, p. 39.

2 Y. Katto and N. Soda, "Theory of Lubrication by Compressible Fluid With Special Reference to Air Bearings," 1952 Proceedings of Second Japanese National Congress on Applied Mechanics, National Committee for Theoretical and Applied Mechanics, May, 1953, p. 267.

3 J. S. Ausman, "The Fluid Dynamic Theory of Gas Lubricated Bearings," Trans. ASME, vol. 79, no. 6, August, 1957, p. 1218.

4 B. Sternlicht and R. C. Elwell, "Theoretical and Experimental Analysis of Hydrodynamic Gas-Lubricated Journal Bearings," Trans. ASME, vol. 80, no. 4, May, 1958, p. 865.

5 H. G. Elrod and A. Burgdorfer, "Refinement of the Theory of Gas-Lubricated Journal Bearings of Infinite Length," The Franklin Institute Labs. Interim Report I-A2049-10, January, 1960.

6 B. Sternlicht, H. Poritsky, and E. Arwas, "Dynamic Stability Aspects of Cylindrical Journal Bearings Using Recompressible and Incompressible Fluids," General Electric Engineering Report Contact No. Nonr-2844(00), Task No. NR-097-348.

7 V. Castelli and H. G. Elrod, "Perturbation Analysis of the Stability of Self-Acting, Gas-Lubricated Journal Bearings," The Franklin Institute Labs. Interim Report I-A2049-11, February, 1960.

8 M. M. Reddi and P. R. Trumpler, "Stability of the High-Speed Journal Bearing Under Steady Load. 1 The Incompressible Film," Journal of Engineering for Industry, Trans. ASME, Series B, vol 84, 1962, pp. 351–358.

9 H. S. Cheng, "The Dynamics of the Infinitely Long, Self-Acting, Gas Lubricated Journal Bearing Under a Steady Load," PhD thesis, University of Pennsylvania, Philadelphia, Pa., 1961.

10 W. A. Gross, "A Gas Film Lubrication Study, Part I: Some Theoretical Analysis of Slider Bearing," *IBM Journal of Research and Development*, vol. 3, no 3, July, 1959 p. 237.

11 F. R. Gantmacher, "Applications of the Theory of Matrices," (Translated from Russian) First edition, Interscience Publishers, 1959.

DISCUSSION

E. J. Gunter[3]

I would like to compliment the authors on their ingenious use of the Galerkin method to obtain the equilibrium solution for the infinite length gas bearing. The power of this method can be seen by the comparison between the values obtained by Elrod's computer solution, and that using only the first two terms of the Galerkin expansion. The agreement is excellent except for the slight deviation at low Λ, and at high eccentricity values.

It appears that this method may be easily extended to the finite width bearing by choosing a set of mode shapes to approximate the axial pressure distribution and that also satisfy the end pressure boundary conditions. It would be of interest to know if the authors had considered this extension.

In the discussion of the whirling of the journal center, and also on the trajectories of the journal orbits shown in Fig. 5, the authors do not state what the whirl frequency is. I presume whirling refers to half-frequency whirl. Were the authors able to obtain the exact values of the whirl frequency from the analog computer solution?

It would be of interest to see the authors compare their results with the stability criterion as developed by Dr. Castelli [12] who used forward integration of the equations of motion by digital computer, and Dr. Ausman's [13] linearized *ph* solution.

In the comparison of Dr. Ausman's stability plot with that of the authors, it appears that there is favorable agreement for large Λ, but the two seem to give entirely different results for small Λ. In the Trumpler-Cheng paper, it appears that as Λ goes to 0, the bearing becomes unstable for all values of ω_1, the stability parameter, which is the case for the ideal 360 deg incompressible bearing, as demonstrated by Reddi and Trumpler [8], while Dr. Ausman's results indicate the opposite trend. Also, his equilibrium solution appears to be very similar to the Galerkin solution, only utilizing the first terms of the expansion.

It would also be of interest to note some of the discrepancies between experiment and theoretical prediction for the small ϵ case. Wildmann [14], in his experimental work on gas bearings, has observed that all bearings with clearance ratios (C/R) less than 0.0004 were stable, even for the unloaded case. Could this situation be due to the stabilizing effects such as surface irregularity, rotor flexibility, or to the neglect of the viscous shear stresses acting on the journal surface even though they are negligibly small?

Additional References

12 V. Castelli and H. G. Elrod, "Solution of the Stability Problem for 360° Self-Acting, Gas-Lubricated Bearings," PhD thesis, Columbia University, New York, N. Y., 1962.

13 J. S. Ausman, "Linearized PH Stability for Translatory Half-Speed Whirl of Long, Self-Acting, Gas-Lubricated Journal Bearings," ASME Paper No. 62—WA-185.

14 M. Wildmann, "Experiments on Gas-Lubricated Journal Bearings," ASME Paper No. 56—Lub-8, October, 1956.

15 J. S. Ausman, "An Improved Analytical Solution for Self-Acting, Gas-Lubricated Journal Bearings of Finite Length," *Journal of Basic Engineering*, Trans. ASME, Series D, vol. 83, pp. 188.

16 W. A. Gross and E. C. Zachmanoglou, "Perturbation Solutions for Gas-Lubricating Films," *Journal of Basic Engineering*, Trans. ASME, Series D, vol. 83, pp. 139.

17 R. J. Wernick and C. H. T. Pan, "Static and Dynamic Forces of Partial Arc Self-Acting Gas Journal Bearings at Moderate Compressibility Numbers," Mechanical Technology Inc. Report No. MTI-63TR4.

18 J. S. Ausman, "Linearized *ph* Stability Theory for Translatory Half-Speed Whirl of Long Self-Acting Gas-Lubricated Journal Bearings," ASME Paper No. 62—WA-185.

B. Sternlicht[4]

I find myself in an awkward position to have been requested to discuss a paper of one of my associates. The only course open to me is to try to be objective.

The Galerkin method has often been used by mathematicians in the solution of partial differential equations. In reference [19][5] it has been applied to incompressible lubrication. There the problem is linear which makes it entirely equivalent to the better-known Ritz method. To the best of my knowledge the present paper applies the method for the first time to the nonlinear gas bearing analysis. Which of these authors can claim credit for the first application of the Galerkin method to lubrication problems I am not sure. In any event, the Galerkin method is a powerful tool used to obtain the equilibrium gas film pressures, which are ex-

[3] Research Engineer, Franklin Institute, Mechanical and Nuclear Division. Philadelphia, Pa.

[4] Technical Director, Mechanical Technology, Inc., Latham, N. Y. Assoc. Mem. ASME.

[5] Numbers in brackets designate Additional References at end of discussion.

tremely nonlinear. These pressures are the starting point in any stability considerations.

The authors were also among the first to use analog computer to study the problem of gas bearing dynamics. The only other known application of analog computer to the study of journal trajectory in gas bearings is reference [20].

The method of Galerkin, which has so far been applied to infinitely long gas bearings appears to occupy the coveted position of optimum compromise between rigor and simplicity. In Table I [21], the available methods for stability analysis are listed and their relative merits are compared in Table II (reference [21]). In Fig. 1, reference [21], numerical results of various analyses are compared, and in Fig. 2, reference [21], comparison between theory and experiment is made.

Aside from the applicability of the Galerkin method to the solution of nonlinear problems, it can be readily adapted to finite length journal bearing analysis. I know that Dr. Cheng is presently extending this analysis to the finite length journal bearing stability problem.

Review of the foregoing results brings up the following questions:

1 It is curious to note that the accuracy of the steady-state results appears to be somewhat less satisfactory at small Λ (Figs. 2 and 5). Perhaps it would be better, in this range of Λ, to perform successive linearization of the Reynolds equation first before applying the method of Galerkin. Can the authors comment on this point?

2 The question of stability at small Λ's is of considerable interest, for it can be used in a number of present-day applications and it can also be compared to available incompressible theory. There seems to be considerable difference between first and second approximations (Fig. 6) and this undoubtedly is effected by the equilibrium solution. Do the authors have a better method for analyzing the region of small Λ's?

3 How much drift did the authors experience with the analog computer and did they run into complications when the trajectory passed through $\epsilon = 0$?

Additional References

19 R. I. Tanner, "The Estimation of Bearing Loads Using Galerkin's Method," *International Journal of Mechanical Science*, vol. 3, 1961, pp. 13–27.

20 T. C. Coffey, K. W. Jenkins, C. H. T. Pan, and B. Sternlicht, "Differential Analyzer Studies of Dynamic Behavior of Gas Lubricated Rigid Vertical Rotors," ONR Report NONR 2844(00), October, 1961.

21 C. H. T. Pan and B. Sternlicht, "Comparison Between Theories and Experiments for the Threshold of Instability of Rigid Rotor in Self-Acting Plain Cylindrical Journal Bearings," MTI-62TR13, August 3, 1962.

Authors' Closure

The authors would like to thank Dr. Sternlicht and Mr. Gunter for their interest and valuable comments on this paper. It appears that the truncation error in the small Λ region is only serious for high eccentricity ratios. Dr. Sternlicht's suggestion of using successive perturbations, followed by the application of the Galerkin method, is an interesting one. There are two perturbation techniques available, namely, the Ausman ϵ-ph perturbation [15] and Gross and Zachmanoglou's Λ perturbation [16]. The ϵ-ph perturbation gives satisfactory results for the equilibrium solution, but does not seem to lend itself to direct application in the stability analysis because ϵ is itself time-dependent. On the other hand, the perturbation using small Λ as parameter can be quite readily incorporated in the nonsteady Reynolds equation. Successive applications of the Λ perturbation to the partial arc gas bearing, followed by the Galerkin method, have been reported by Wernick and Pan [17].

At the present time, the authors have not found a better method to improve the stability results in the high ϵ and small Λ region besides including more terms in the Galerkin expansion.

The drift in the analog computer was controlled by adjusting the scale factor of the problem to limit the time for each run to within three or four minutes. Complications did arise when the trajectory passed near by the bearing center; however, they can be avoided by a careful selection of the initial journal position for each run.

The authors agree fully with Mr. Gunter that a complete comparison among the existing theories and experiments on the stability results is extremely important. Such a comparison has been recently reported by Pan and Sternlicht [21]. The whirl frequency was recorded, but only for a few selective runs. Most of the values fell close to half frequency, and it was only in a few high ϵ cases that frequencies as low as $^1/_4$ were also observed.

Dr. Ausman's results were revised in his recent report [18] and the trend in the small Λ region was found to agree with the present analysis.

The qualitative discrepancy between theory and experiment for small ϵ cases is very difficult to explain at the present time. It seems to call for more careful experiments in those regions.