

Fig. 6 The limit curve for the maximum velocity occurring in the diagonal space between the tubes for staggered tube banks

Puchir based his criticism primarily on the assertion that the velocities given in my paper should be measured near the tube so that higher values would result than the average. This would lead to the false consideration of my paper according to his interpretation. As a matter of fact I must stress here that the velocities for the minimum gap between two neighboring tubes of one row given in my paper are average values, calculated from the measurement far downstream in the section free of tubes. Thus, a further foundation of his criticism proved to be untrue.

The final argument of Puchir is based on his assessment that the curious variation of Strouhal numbers for staggered tubes given in my paper should result from a shift of the point of maximum velocity from the lateral space between tubes to the diagonal space between tubes. He seems to explain that the quoted curious variation does not suit his concept and the reason for that should lie in the choice of the parameter for the maximum velocity in the Strouhal number.

Let us examine the position of the maximum velocity in the staggered tube bank. The maximum velocity will arise in the diagonal space between tubes only if the following relation is valid:

$$(l^2 + l^2)^{1/2} - 2d < t - d$$

or

$$(l/d) < [(2t/d) + 1]^{1/2}$$

(see sketch, Fig. 6). The limit curve for $(l/d) = [(2t/d) + 1]^{1/2}$ is shown in Fig. 6. The measured cases given in my paper belong in the main to the field for the maximum velocity lying in the lateral space between tubes, but not in the diagonal one as claimed by Puchir. Thus, his last point for proving his theory fails once more to reflect any sound foundation, just like those mentioned previously.

However, a more serious investigation was carried out recently by Jaudet and Hutzler [10]. It shows the relationship between the vortex shedding frequency ($S = fd/V$) and the longitudinal tube spacing ($x_l = l/d$). They wanted to improve upon my first approach for the vortex frequency shown in my paper and derive an equation as follows:

$$S = [1 \pm \{1 - \epsilon\beta^2[\tanh \pi\alpha/x_l - \tanh \pi(x_l - \alpha)/x_l]\}^{1/2}]/2x_l$$

where ϵ is the vortex formation factor, β is the ratio of the separation velocity to the free-stream velocity, and $\alpha = h/d$ ($h =$ width of the vortex street). They base their derivation on the equation

$$u_R = (\Gamma/2l)(\tanh \pi h/l - \tanh \pi h'/l)$$

given in my paper, where u_R is the translation velocity of the vortex, Γ is the circulation of a single vortex, and $h' = t - h$.

However, the latter equation was derived at the time under great simplification, as clearly stated in my paper. It is only sufficient to allow a rough approach to be made, as in my paper, but is not adequate as a basis for deriving any further accurate theory. The equation given by Jaudet and Hutzler seems there-

fore to be questionable. The support they find in my chart (see Fig. 3) for their theory is too weak for justifying their accuracy. There is only one point in this chart obeying their equation, whereas all the other points show no such behavior. Even in the recent investigation carried out by the writer [11], no such evidence can be traced.

References

- 1 Puchir, M., "Letters & Comment," *Mechanical Engineering*, Vol. 93, No. 5, May 1971, pp. 60-61.
- 2 Chen, Y. N., "Flow-Induced Vibration and Noise in Tube-Bank Heat Exchangers Due to von Karman Streets," *JOURNAL OF ENGINEERING FOR INDUSTRY*, TRANS. ASME, Series B, Vol. 90, No. 1, Feb. 1968, pp. 134-146.
- 3 Roshko, A., "On the Drag and Shedding Frequency of Two-Dimensional Bluff Bodies," NACA TN 3169, 1954.
- 4 Grotz, B. J., and Arnold, F. R., "Flow-Induced Vibrations in Heat-Exchangers," TN No. 31 to Office of Naval Research from Stanford, AD 104568, Aug. 1956.
- 5 Owen, P. R., "Buffeting Excitation of Boiler Tube Vibration," *Journal of Mechanical Engineering Science*, Vol. 7, 1965, pp. 431-439.
- 6 Wallis, R. P., "Photographic Study of Fluid Flow between Banks of Tubes," *Engineering*, Vol. 148, 1939, p. 423.
- 7 Meyer, E., Mechel, F., and Kurtze, G., "Experiments on the Influence of Flow on Sound Attenuation in Absorbing Ducts," *Journal of the Acoustical Society of America*, Vol. 30, 1958, pp. 165-174.
- 8 Chen, Y. N., "Lateral Helmholtz Resonator Silencer with Turbulence Absorption," *Proceedings of the Institution of Mechanical Engineers* (London), Vol. 182, Part I, No. 3, 1967/68, pp. 60-72.
- 9 Ising, H., "Biegewellenfeld mit überlagerter Strömung als Modell eines 'erstarrten Turbulenzmusters,'" Technical Note of the Acoustic Institute, The Technical University, Berlin, 1971. Private communication.
- 10 Jaudet, A., and Hutzler, D., "Pulsations acoustiques et contraintes vibratoires dans les échangeurs de chaleur" (Acoustic pulsations and vibratory stresses in heat exchangers), *Proceedings of the Heat Exchangers Conference of the Institut Français des Combustibles et de L'énergie*, 1971, paper 13.
- 11 Chen, Y. N., "Ursache und Vermeidung Rauchgasseitiger Schwingungserscheinungen in Kesselanlagen infolge Brenngasdrall-Instabilität und Karman-Wirbelstrassen," *Mitteilungen der Vereinigung der Grosskesselbetreiber*, Vol. 51, 1971, pp. 113-123.

Discussion

Michael Puchir. Dr. Chen seems to believe he is countering my criticism. He is, however, trying to refute some things he imagines are implied by the comments in my letter. His technical brief does not alter my conviction that his experiment dealt with periodic compression and expansion of a gas stream flowing through tube banks and not with Karman vortices. The "so-called" Strouhal numbers which he determined by laborious experiment are mathematically predictable on the basis of periodic compression and expansion of a gas flow through a tube bank.

The following terminology will be used in further discussion on this score:

- V_a = average velocity of gas flow within the tube bank
- V_m = maximum velocity of gas flow in the tube bank; in the case of the staggered tube arrangement this may sometimes occur in the diagonal rather than the lateral space between tubes
- l = longitudinal spacing between tube centerlines
- d = tube diameter
- S = Strouhal number
- f = the noise or forcing frequency generated within the tube bank

Any consistent units of measure may be used.

Assuming that the aerodynamic phenomenon does result from periodic compression and expansion, we can say that

$$f = \frac{V_a}{l} \quad (1)$$

Even though the phenomenon may not be due to Karman vortices, we can specify the frequency as

$$f = S \frac{V_m}{d} \quad (2)$$

The Strouhal number in the above may be specified as that necessary to satisfy the requirement that the frequency determined by equation (2) equal that of equation (1). Hence

$$S \frac{V_m}{d} = \frac{V_a}{l}$$

and

$$S = \left(\frac{V_a}{V_m} \right) \left(\frac{d}{l} \right) \quad (3)$$

The Strouhal numbers calculated from equation (3) correspond closely to the experimental results obtained by Dr. Chen. Those who would care to check me on this point might find it more convenient to do some further mathematical manipulation to express the ratio V_a/V_m as a function of the physical parameters of the tube bank.

It may be noted that as the lateral spacing is increased the difference between V_a and V_m decreases so that eventually V_a/V_m becomes practically unity. From equation (3) it may be noted that for this reason the Strouhal numbers could be expected to peak at a value of d/l . Dr. Chen's article presents a graph of Strouhal numbers for the staggered tube arrangement. It may be noted in this graph that the Strouhal numbers for some longitudinal spacings peak at approximately d/l and level off while for others there is an indicated rise to approximately d/l followed by a decline for lateral spacings exceeding approximately two tube diameters. This decline of the Strouhal number stems from the fact that the Strouhal number is related to the velocity in the diagonal space between tubes rather than that through the lateral space between tubes for such lateral spacings.

The Strouhal numbers determined from equation (3) will not agree exactly with Dr. Chen's numbers for the same reason that variation is shown in the numbers determined by different experimenters; the Strouhal numbers do not relate to precisely the same velocities. The location of the velocity measuring probe is extremely important. There is a very significant range between maximum and minimum velocity when dealing with close lateral spacing of tubes and, therefore, increased possibility for wide variations in measured velocity. Hence, the results of different experimenters could be expected to show good correlation only with relatively wide lateral spacing of tubes.

My letter which prompted Dr. Chen's technical brief jocularly hinted that I might someday offer some thoughts on aerodynamic vibration of smoke-stacks. This seems an appropriate opportunity to do so.

In my view, instability is a likely cause of smoke-stack vibration. The condition of instability can be determined from a knowledge of the longitudinal stress in the windward-side extreme fiber. To permit of a mathematical expression, consider

s = longitudinal stress in the windward-side extreme fiber, psi
 x = longitudinal distance along the windward-side extreme fiber, in.

Instability develops when a steady wind force develops the condition $ds/dx = 0$ at any elevation on the windward-side extreme fiber. The most extreme instability occurs when $ds/dx = 0$ at the base of the stack.

The condition described here is a point of transition between inverted elastic pendulum and spring behavior. It is comparable to the condition achieved by tilting a vertical cylinder so that its top center is directly above the kern circle radius of its base.

In the case of conical-base stacks, instability can develop at upper elevations with lower wind velocities than required for instability at the base. A conical-base stack may, therefore, be unstable over a range of steady wind velocities. This may account for the more common occurrence of such vibration in conical-base stacks.

Author's Closure

Mr. Michael Puchir formulated his flow model in the present discussion much more precisely than in his original letter published in *Mechanical Engineering* [1]. He described his model with the gas stream experiencing periodic compression and expansion during flow through the tube bank. This would cause the generation of the vibration in tube banks, the frequency of which should obey his equation (1), according to his hypothesis. His model may be interpreted as the flow pattern sketched in Fig. 7. The stream would be compressed when it enters the gap between two neighboring tubes of one row (region 1), followed by an expansion interval when it comes out of the gap (region 2). What the flow looks like inside the wake (region 3) would not be essential for his model, if we still follow his thought. Therefore all my consideration in the present technical brief about the vortex formation in the wake is superfluous for the discussion on his flow model, as can be clearly read from his statement.

If we consider only the flow path along regions 1 and 2 of the Puchir model, any gas particle would certainly experience periodic compression and expansion with a frequency according to his equation (1). But if we really could neglect the vortices in region 3, the flow path along regions 1 and 2 would remain unchanged with time. Then we would be dealing with a steady flow along this path. Such a flow is then comparable with a flow streaming through a channel with the same variation of its sections (see Fig. 8). It is clear that no vibration with a frequency of $f = V_a/l$, equation (1) of Puchir, can be generated where the mean flow velocity V_a is the main governing factor. Rather, a vibration with one of the natural frequencies of the gas column in the channel may be generated, where the velocity of sound is the primary factor. Even this vibration cannot be generated by the steady flow as sketched with respect to the image of Mr. Puchir. Such a vibration can only be generated by

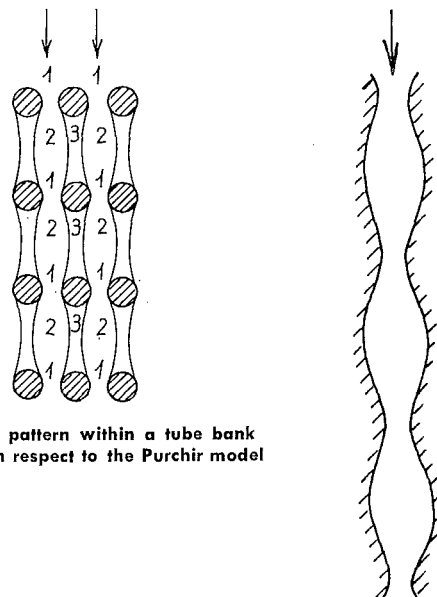


Fig. 7 Flow pattern within a tube bank sketched with respect to the Puchir model

Fig. 8 Flow through a channel with the same streaming-section variation as in the Puchir model

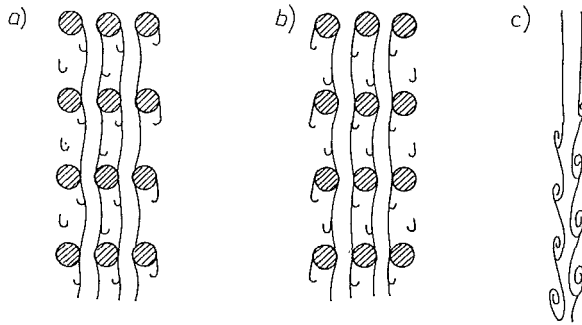


Fig. 9 Swing of the main flow in the tube bank; (a) swing to the right; (b) swing to the left at half a period later; (c) vortex formation along a free jet

the turbulence and/or the vortices possibly formed in the flow due to the section variation.

If we further follow the idea of Mr. Puchir for the flow in the tube bank (Fig. 7), the pressure around the tube periphery would remain constant due to the steady behavior of the flow. Such a flow cannot cause the tube to vibrate. Analogously, no vibration can be generated in the gas column perpendicular to the flow. The inadequacy of the Puchir model is therefore clear. It can be concluded that any neglect of the vortex phenomenon within the wake is not permissible for the consideration of flow-induced vibration in tube banks.

As a matter of fact, the periodic compression and expansion of the gas stream along the path cannot be pronounced at all. The flow pattern is rather the one as sketched in Fig. 9. It is well known that the wake behind a tube will swing during the shed-

ding of the Karman vortex. The main flow must be forced to swing at the same rhythm also. As shown in Fig. 9(a), the vortices are supposed to be just formed on the right-hand shoulder of each tube. The main flow is swinging to the right accordingly. Fig. 9(b) shows the flow pattern half a period later. The main flow with the asymmetrical vortices on its flanks corresponds very well to a free jet at moderate Reynolds number as sketched in Fig. 9(c). The middle line of this jet possesses a curved shape also. The vortices must represent a mechanism which tends to keep the main flow in the tube bank at a minimum change of its streaming section. This must be the cause of keeping the pressure drop within the tube bank as low as possible. This seems to be a law of nature that the resistance in the flow would always remain at a minimum level, as proposed by Kronauer [12] for a cross flow past a cylinder and verified by Bearman [13] and Chen [14] subsequently. The swing of the main flow in the tube bank, accompanied by the shedding of the Karman vortices, will generate a fluctuating pressure on the tube and on the gas column in the direction perpendicular to the flow. This pressure fluctuation is then responsible for the excitation of the vibration on the tube and in the gas column.

Additional References

- 12 Kronauer, R. E., "Predicting Eddy Frequency in Separated Wakes," paper presented at the IUTAM symposium on concentrated vortex motions in fluids, University of Michigan, Ann Arbor, Mich., July 6-11, 1964 (quoted by Bearman [13]).
- 13 Bearman, P. W., "On Vortex Street Wakes," National Physical Laboratory, Teddington, England, Aero Report No. 1199, 1966.
- 14 Chen, Y. N., "Fluctuating Lift Forces of the Karman Vortex Streets on Single Circular Cylinders and in Tube Bundles, Part 1, The Vortex Street Geometry of the Single Circular Cylinder," JOURNAL OF ENGINEERING FOR INDUSTRY, TRANS. ASME, Series B, Vol. 94, No. 2, May 1972, pp. 603-612.

The Relative Angular Velocity Between Links

MAHMOUD A. MOUSTAFA¹

The magnitude of the relative angular velocity between two links is equal to the component of the relative velocity between any two points, one on each link, along the line joining them divided by the normal to this line from the instantaneous center of the two links. Its direction is the same as the moment of this component about the instantaneous center.

Introduction

In the static force analysis of plane mechanisms the reaction force of one link on another connected by a turning joint, when friction is considered, is tangent to a small circle called the friction circle [1].² The position of this tangent force depends upon the direction of the relative angular velocity of the links.

Usually the relative angular velocity of the two links is determined by obtaining the angular velocity of each link separately from the velocity polygon or by any other means. In this type of analysis only the sense of direction has to be determined. The following hypothesis suggests an easy method for determining the magnitude and the direction of the relative angular velocity.

¹ Mechanical Engineering Department, University of Alexandria, Alexandria, Egypt.

Contributed by the Mechanisms Division of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS. Manuscript received at ASME Headquarters, September 23, 1971.

² Numbers in brackets designate References at end of Note.

Hypothesis

The magnitude of the relative angular velocity of two links is equal to the component of the relative velocity of any two points, one on each link, along the line joining them divided by the normal to this line from the instantaneous center of the two links. Its direction is the same as the moment of this component about the instantaneous center.

Proof

Suppose that link 2 rotates about point O , while link 3 rotates about Q as shown in Fig. 1. The angular velocity of links 2 and 3 are represented by the vectors ω_2 and ω_3 , respectively, which are normal to the plane of the page and are considered positive outwards. Point I is the instantaneous center of links 2 and 3. From the definition of the instantaneous center of a pair of links [2]

$$\omega_2 \times OI = \omega_3 \times QI \quad (1)$$

If A is any point on link 2, and B is any point on link 3, the velocity of A relative to B is given by

$$V_{AB} = V_A - V_B \quad (2)$$

But

$$\left. \begin{aligned} V_A &= \omega_2 \times OA \\ V_B &= \omega_3 \times QB \end{aligned} \right\} \quad (3)$$

Substituting equations (3) into equation (2), then

$$V_{AB} = \omega_2 \times OA - \omega_3 \times QB \quad (4)$$