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Experimental Oscillation-Assisted Cylindrical Plunge Grinding

The paper presents experimental studies of the influence of amplitude, frequency, time of oscillation, and average value of the rotational speed of the workpiece on the waviness of the ground surface during cylindrical outer diameter plunge grinding on a mass-produced cylindrical grinder. Oscillations were introduced in order to counteract the development of self-excited vibrations during spark-out. The waviness of the surface was determined by a laser distance measurement system. The results of the experiments indicate the important positive influence of the oscillations applied during spark-out on the waviness of the ground surface. The conclusions of the study regarding the process as well as the technology of cylindrical plunge grinding are presented. [DOI: 10.1115/1.4051341]

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Introduction

In modern grinding technology, it is particularly important to maintain processing conditions that ensure that the desired surface quality can be achieved while maintaining economically feasible process conditions. The grinding process itself is the source of many dynamic processes that make it difficult to formalize the evaluation and description of the condition of the surface being ground. These phenomena include, among others, self-excited vibrations, which are a cause of chatter. The difficulty of a mathematical description of this phenomenon results from the complexity of conditions created by numerous, randomly scattered and oriented cutting grains with irregular cutting edges bonded together, irregular void spaces resulting from the porosity of the grinding wheel, the cutting capacity of the grinding wheel changing over time as a result of wear, etc. All this takes place in conditions defined by the mechanical system of the headstock of the grinding wheel and headstock of the workpiece mounted on the machine tool with specified stiffness and dynamic properties. For this reason, the problem of modeling grinding process and chatter has been the subject of deliberations of many scientists [1–11], and it is rather difficult to find any useful general principles or observations in these studies that would be directly applicable to any specific dynamic system including the grinding process.

In order to improve grinding performance and counteract chatter, scientists have developed methods for selecting such cutting conditions so as to avoid dynamic instability regions that give rise to chatter. This can be done based on the developed models of the grinding process or on the basis of experimental studies. The latter method is much more accurate however, it requires that numerous grinding tests with different sets of parameters be performed, which is highly time-consuming and work-intensive. The aim of this theoretical and experimental study is to plot a stability lobe diagram.

Regardless of the above goal, an attempt was made to actively counteract chatter with deliberately introduced oscillations of the workpiece. The crux of this method is to use a time-varying rotational speed of the workpiece, where a low-frequency harmonic variable is added to the constant component of the rotational speed value, whose amplitude is selected so as to ensure that the direction of the peripheral speed of the workpiece remains unchanged. This kinematics of the plunge grinding process reduces chatter by interfering with one of its causes, namely workpiece regeneration. The active methods of chatter counteract were also considered for the other machining processes [12–14]. The authors of publications [15–21] analyzed similar issues for grinding. However, the approach they took consisted mainly in the flexible torsional support of the workpiece that resulted in time-varying rotational speed of the workpiece caused by the torsional compliance of the grinding process system.

In the method proposed by the author of this paper, kinematic excitation is independent of process loads. The parameters of the kinematic excitation are easily controllable and can be precisely selected for cylindrical grinding [22,23] and surface grinding [24,25]. This method, proposed and tested by the author [22], was the subject of considerations aimed at developing a model of the grinding process in which the oscillations introduced could be taken into account. The obtained results should allow determining the influence of these oscillations on grinding performance expressed as the waviness of the machined surface. These analyses

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led to the design of a model whose presentation and simulation results were published in the papers: in the case of cylindrical grinding [23] and surface grinding [25]. The results of the simulation carried out on the identified model of the cylindrical grinder were presented therein [23]. Simulations were carried out whose results confirmed the possibility of eliminating chatter or reducing the amplitude of self-excited vibrations in any such conditions where this phenomenon occurs without the introduced oscillations. The theoretically analyzed method requires experimental studies in order to verify the positive effect of deliberately introduced oscillations of the workpiece on the quality of the machined surface (reduction of surface waviness).

Experimental Setup

Experimental tests were performed with a mass-produced cylindrical grinder SWF-25 on which a drive system was mounted to introduce oscillations of the rotational speed of the workpiece. The laser distance measuring system was used to measure the surface profile of the machined shaft at the test bench.

Experimental tests included the execution of the planned grinding tests with the defined kinematic parameters in order to determine the effect of intentionally introduced oscillations of the rotational speed of the workpiece spindle on the quality of the processed surface. It was decided that the quality of the surface was to be indicated by the parameter of the arithmetic mean of the ordinates of the waviness profile Wa as specified in the PN-EN ISO 4287 standard.

The introduction of the workpiece speed oscillation was achieved through a control program created especially for this purpose, which enabled the setting of a time waveform of the control signal for the motor in the drive system of the workpiece spindle. The control program, built-in LabView, also makes it possible to introduce other kinematic parameters of grinding and to monitor the implementation of the machining cycle. The control card and motor drivers used in the grinder drive were manufactured by National Instruments. In the axis of the workpiece, an encoder was placed with a resolution of 6000 counts per revolution, whose task was to track the nature of the time waveform of the rotational speed of the workpiece and to introduce feedback in the speed control system of the workpiece spindle.

In the experiment, a cycle of finishing plunge grinding was carried out, involving the following stages:

- plunge grinding with the following parameters:
  - total radial reduction of the workpiece during the grinding cycle: 25 μm,
  - preset grinding depth: $a_{e0} = 5$ μm,
  - grinding wheel speed: $n_s = 24.7$ rps,
  - workpiece spindle speed: $n_w = 1$ rps,
  - width of cut: $b = 40$ mm,
  - grinding wheel diameter: $d_s = 450$ mm,
  - workpiece diameter: $d_w = 50$ mm,
- 10-s spark-out at the workpiece spindle speed of $n_w = 1$ rps (preset grinding depth $a_{e0} = 0$ μm),
- an oscillation stage completed during spark-out with the following variable parameters:
  - frequency of the workpiece rotational speed oscillation f_w,
  - duration of the workpiece speed oscillation t_w,
  - amplitude of the workpiece rotational speed oscillation A_{nw},
  - average value of the workpiece speed oscillation n_{wav},
  - the onset of oscillation introduction was synchronized with the onset of the spark-out.

The machined sample was made of C55 steel hardened and tempered to 35 ± 3 HRC (Hardness Rockwell C scale). Each shaft was pre-ground to achieve an initial diameter of $d_w = 50$ mm. The spark-out time during pre-grinding was 5 s. To ensure reproducible and comparable conditions for experimental testing, each shaft was used in the tests until the reduction in diameter reached 1 mm (maximum 20 tests). Plunge grinding was carried out with the use

of a 3% oil–water emulsion as a cutting liquid. Taking into account the grinding conditions used in the experiments, the permissible operating time of the grinding wheel (until the next dressing) is approximately 50 grinding cycles according to the technological guidelines. The decision was made to dress the grinding wheel after every ten cycles to avoid changing conditions during subsequent plunge grinding cycles and to keep grinding conditions comparable. Based on the technological experience of grinding instructors, dressings were performed with a single-point diamond dresser while maintaining constant dressing parameters after every ten grinding cycles instead of each grinding cycle. Following each plunge grinding cycle, the samples were laser-measured for the waviness of the workpiece surface.

Laser System of Measuring the Ground Surface Waviness

In order to analyze the profile of the workpiece surface, a measuring system was built using a laser position measuring system. The laser system can be used to perform non-contact measurements at the test bench. The laser system is designed to measure the waviness of the machined surface.

The measuring system of the ground surface waviness profile used at the test bench includes a control unit for the laser head and a power supply unit cooperating with the laser head. Micro-Epsilon laser head type ILD 2220-3 with the following technical parameters was used:

- measurement range: 2 mm,
- measurement resolution: 0.03 μm,
- measurement frequency: 20 kHz.

The waviness measurement setup is shown schematically in Fig. 1. The measurement head (item 1), conditioning amplifier (item 2), and the computer (item 3) together with properly selected connecting and coupling cables are the components of the system for measuring the waviness of the machined surface. The laser head (item 1) is mounted on a special bracket (item 4) attached to the grinding table so that measurements can be achieved in a normal plane to the ground surface in the radial direction. This mounting of the laser head to the grinding table was aimed to reduce the effect of forced external vibrations (vibrations of the devices operating in the vicinity of the grinding machine and vibrations of the machine-tool base) on the measurement.

Photographs of the laser measurement head mounted on the test bench for measuring the waviness of the workpiece surface are shown in Fig. 2.

Measurement duration was set as the duration of one revolution of the workpiece, which ensured that the entire circumference of the ground surface was measured. The laser measurement method was used to analyze the profile of the ground surface after each grinding cycle. This ensured a high accuracy of measurement since the influence of a possible contact of the measurement sensors on the measured variable was eliminated.

According to the current PN-EN ISO 4287 standard, one of the surface waveform parameters is the arithmetic mean of the ordinates of the Wa profile, calculated according to the formula:

$$Wa = \frac{1}{n} \sum_{i=1}^{n} |Z_i| \qquad (1)$$

where n is the number of measurements made on the perimeter of the ground object and Z_i is the value of the ordinate of the measured profile.

In order to use the signal set during the profile measurement to determine the waviness of the analyzed surface, as described in relation (1), a software-based filtering of the collected data using a band-pass filter with limit frequencies of 15 Hz and 500 Hz was used. The filter needed to extract the measurement signal for the surface waviness profile was programmed in the Matlab system.
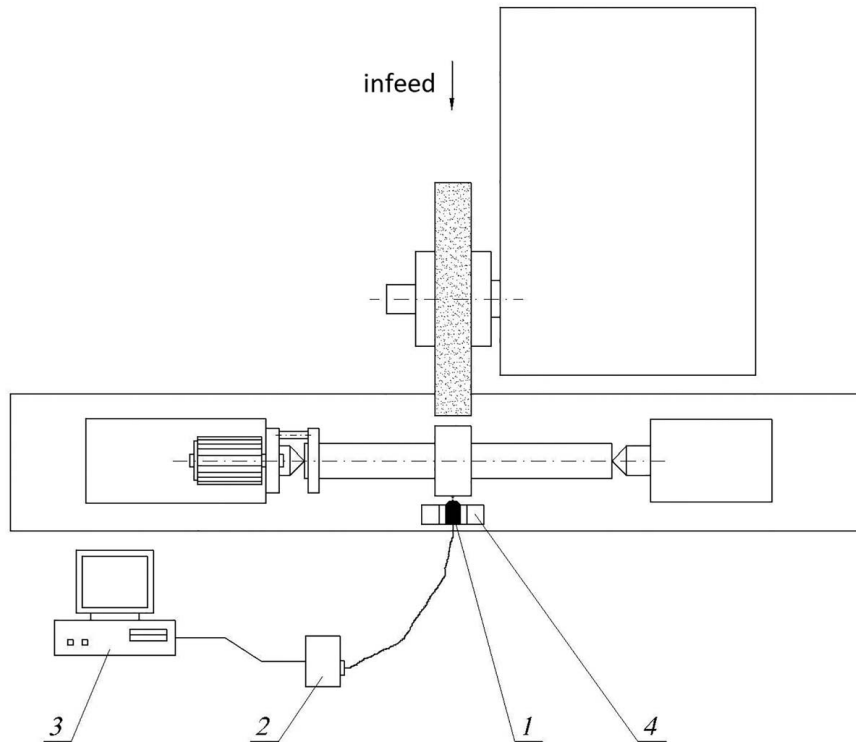
Fig. 1 Schematic diagram of the setup for measuring the waviness of the workpiece surface: 1, laser head; 2, amplifier; 3, computer; and 4, bracket

Profilometer Verification of Surface Waviness Measurement

To verify the results obtained from measurements taken directly on the test bench using an optical measuring system, a certain batch of items was measured with a TALYROND 3 profiler from Rank Taylor Hobson. The measurement on the TALYROND 3 profiler may have a measurement error with the following maximum values:

- roundness error for objects with a diameter of $3.2 \div 100$ mm $0.025\ \mu$m,
- maximum straightness error $0.05\ \mu$m.

Methodology and Experimental Design

In line with the theory of research design, an appropriate schedule for conducting the experiment was adopted in accordance with the number of the grinding process input variables to be analyzed, their range, and the nature of their impact on the effect of machining.

The following input variables were set for the process of plunge grinding of shafts:

- amplitude of the workpiece rotational speed oscillation A_{nw},
- frequency of the workpiece rotational speed oscillation f_w,
- the average value of the workpiece speed oscillation n_{wav}, and
- duration of the workpiece rotational speed oscillation t_w.

The choice of input variables from among those that characterize the introduced speed oscillations of the workpiece is justified by the fact that it was decided to introduce oscillations only during the spark-out process. This limitation was introduced once the results of simulations carried out based on the grinder model, and the preliminary experimental tests had been analyzed. During the simulations, an improvement in the shape of the profile of the ground surface was achieved with the introduction of oscillations during the spark-out. This observation was confirmed by the results of the preliminary experimental tests. During the initial research, oscillations of the rotational speed of the workpiece during plunge

grinding were introduced and an increase in the waviness of the surface of the ground shaft was observed compared to grinding carried out without the oscillation. For this reason, it did not seem useful to include in the analysis the effect of variation in all parameters of the plunge cutting process such as grinding depth or grinding duration on the shape variation in the profile of the workpiece surface.

The grinding process was treated as a source of interference in the profile of the ground object, the impact of which needed to be minimized as effectively as possible during sparking out. The spark-out was carried out at the rotational speed of the workpiece $n_w = 1$ rps. Therefore, taking into account the workpiece diameter $d_w = 50$ mm, the peripheral speed of the workpiece is $v_w = 0.157$ m/s. In the period of time allocated for sparking out, the stage of the workpiece rotational speed oscillation with the following values of oscillation parameters was completed:

- the amplitudes of the workpiece rotational speed oscillation $A_{nw} = 0.25$ rps and $A_{nw} = 0.5$ rps, whereas the respective amplitudes of the workpiece peripheral speed oscillation $A_w = 0.039$ m/s and $A_w = 0.078$ m/s;
- workpiece rotational speed oscillation frequency f_w from 5 Hz to 50 Hz;
- average values of the workpiece rotational speed oscillations n_{wav} ranging from 0.25 rps to 1 rps, whereas their respective average values of the workpiece peripheral speed oscillations, v_{wav} ranging from 0.039 m/s to 0.157 m/s; and
- duration of the workpiece rotational speed oscillation t_w ranging from 0 s (no oscillation) to 10 s (oscillation occurs throughout the spark-out).

The assumed upper limit of the oscillation frequency depends on the capability of the workpiece spindle drive system which at frequencies higher than the set one does not afford the possibility of inducing oscillations with the amplitude value within the range assumed in the study. This restriction is conditioned, among other things, by the use of a toothed drive belt in the chain drive of the
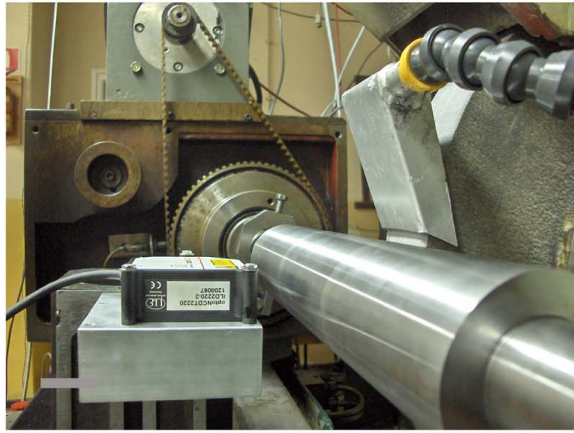
Fig. 2 Laser measurement head mounted on the test bench for measuring the waviness of the workpiece surface: (a) side view and (b) front view

workpiece whose compliance eliminates the possibility of introducing frequencies greater than those specified in the experiment.

The assumed lower limit average value of the workpiece rotational speed oscillation may be tested only if the lower limit value of the adopted amplitude range of the introduced vibrations is applied simultaneously. The purpose of this is to prevent the possibility of periodically driving the workpiece in the reverse direction.

During the preliminary experimental tests, the effect of the parameters of the workpiece rotational speed oscillation on the roughness of the ground surface was assessed by determining the Ra parameter (arithmetic mean of the ordinates of the roughness profile). The measurements were made with the Surftest profiler from Mitutoyo. The grinding process during each experiment was carried out in accordance with the conditions described previously. The results of the Ra parameter of surface roughness for all the grinding tests ranged from 0.5 to 0.6 μm. Based on the results of the measurements of the Ra parameter of the ground surface roughness, it can be concluded that the influence of the parameters of the workpiece rotational speed oscillation on the roughness of the ground surface is very small. For this reason, in further experimental tests, analysis of the roughness of the ground surface was abandoned.

The result of the experimental studies of the plunge grinding process is a profile of the workpiece surface analyzed after each grinding test, which can be used to determine the waviness of the ground surface as the arithmetic mean of the ordinates of the Wa waviness profile. The Wa waviness parameter of the workpiece surface is the output value of the studied plunge grinding process.

The presented number and assumed ranges of input values and the grinding process output value were used to design the

experiment. Due to the predicted nonlinear dependence of the output variable on the input variables and because it was necessary to adjust the respective values of the input parameters to the assumed limitations, a multifactorial central composite plan was chosen. Its goal was to determine the relationship between the output value and the input values as a response surface, i.e., a mathematical description of the relation between the output value and the selected pairs of input values.

In view of the planned performance of experiments within the scope and limits specified earlier, it was decided to carry out two variants of the experiments:

- variant I: with an A_{nw} oscillation amplitude of 0.25 rps, and
- variant II: with an A_{nw} oscillation amplitude of 0.50 rps.

The two scenarios resulted in a more accurate representation of the correlation and effect of the input values on the result of grinding since it is not possible to do the experiments for the given amplitudes within the same range of variability of the other input values. Combining these two schemes into a single global experimental plan could have resulted in a less accurate estimation of the response surface parameters.

According to the experimental plan, a set of 32 tests should be carried out in order to achieve the possibility of generating a response surface and to describe on this basis the influence of the input variables on the grinding effect expressed as an arithmetic mean of the ordinates of the waviness profile Wa. Each planned experiment was repeated five times to confirm the repeatability of the obtained measurement results. Additional experiments were also carried out to complement and verify the results obtained according to the said plans. In total, more than 160 tests were carried out, including plunge grinding of shafts with different input values.

Measurement Results

After each grinding test with the kinematic parameters resulting from the experimental design, the profile of the machined surface was measured using a laser measuring system.

The outline of each examined profile, which was acquired with a personal computer (PC), was statistically processed to determine the Wa waviness parameter. An example of a recorded profile measurement signal is shown in Fig. 3.

The waviness of the ground surface without the workpiece rotational speed oscillation obtained by measuring the waviness profile (Fig. 3) is $Wa = 2.9$ μm.

Figure 4 shows the signal waveform of the measurement of the profile of the machined surface as a function of the angle of rotation of the workpiece with oscillations with the following parameters:
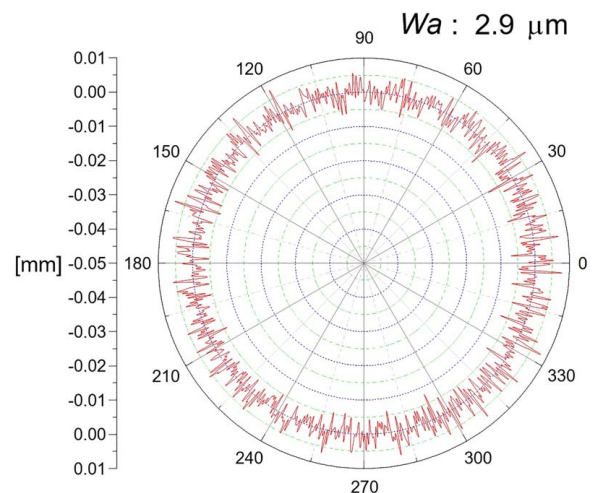

Fig. 3 Signal waveform of the measurement of the profile of a ground surface with 10-s spark-out without oscillation
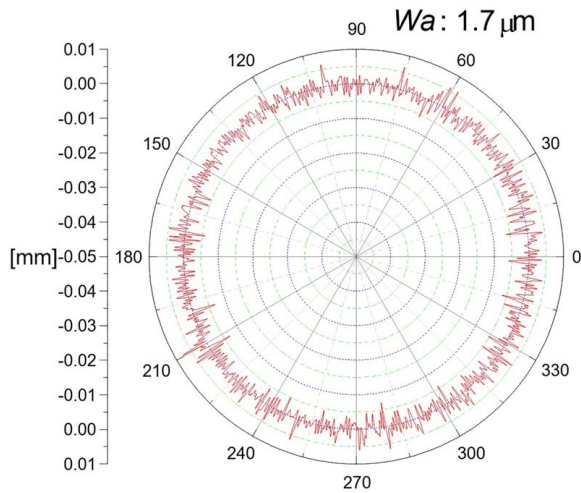
Fig. 4 Signal waveform of the measurement of the profile of a ground surface with a 10-s spark-out and oscillations with $A_{nw} = 0.25$ rps, $f_w = 5$ Hz, $n_{wav} = 0.75$ rps, and $t_w = 5$ s

- workpiece rotational speed oscillation amplitude, $A_{nw} = 0.25$ rps, equivalent to workpiece peripheral speed oscillation amplitude—$A_w = 0.039$ m/s,
- workpiece rotational speed oscillation frequency—$f_w = 5$ Hz,
- average value of the workpiece rotational speed oscillation—$n_{wav} = 0.75$ rps, converted to the average value of the workpiece peripheral speed oscillation $v_{wav} = 0.118$ m/s,
- duration of the workpiece rotational speed oscillation—$t_w = 5$ s.

The waviness of the ground surface with workpiece rotational speed oscillations (Fig. 4) is $Wa = 1.7$ μm.

Based on the results of the laser measurements, the Wa waviness parameter was determined. Values of Wa as a function of kinematic parameters of the introduced oscillations were obtained for variants I and II.

The results of the variant I experiments are presented in Table 1 and as response surfaces, i.e., approximated surfaces, which are a graphical representation of the relation between the Wa waviness parameter and the two selected oscillation parameters, which are illustrated in Figs. 5–7.

Figure 5 shows the surface of the value of waviness Wa as a function of the duration of oscillation t_w and the average value of the workpiece rotational speed oscillation n_{wav} with the amplitude of the workpiece speed oscillation $A_{nw} = 0.25$ rps. Figure 6 presents a contour diagram of the waviness Wa as a function of the duration of oscillation t_w and the average value of the workpiece rotational speed oscillation n_{wav} at $A_{nw} = 0.25$ rps. Figure 7 shows the surface concerning the value of waviness Wa as a function of the duration of the workpiece oscillation t_w and the frequency of oscillation f_w with $A_{nw} = 0.25$ rps.

The results of variant II experiments are presented in Table 2 and in Figs. 8–10 as response surfaces. Figure 8 illustrates the surface related to the value of waviness Wa as a function of the duration of oscillation t_w and the average value of the workpiece speed oscillation n_{wav} with the amplitude of the workpiece speed oscillation $A_{nw} = 0.50$ rps. Figure 9 presents the contour diagram of the
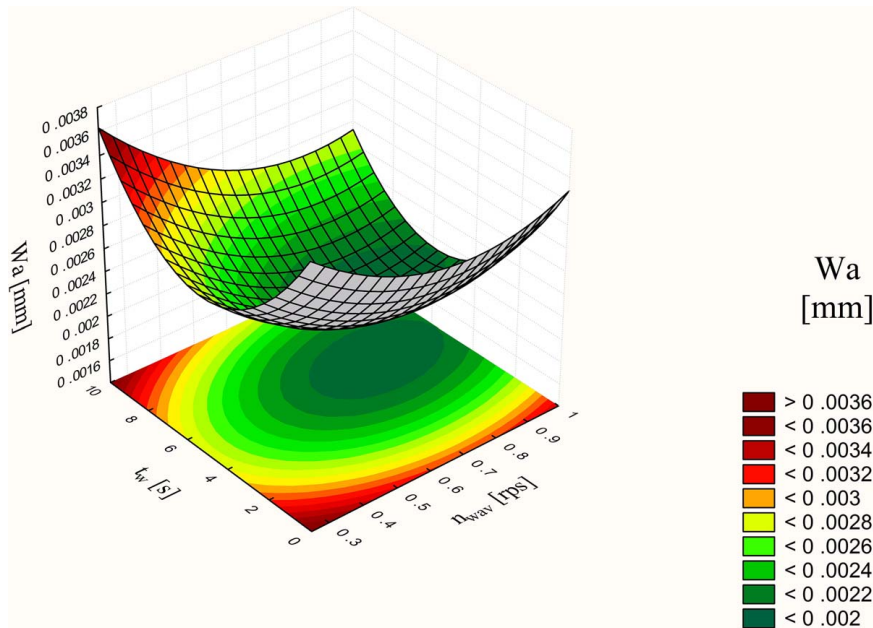
Table 1 The waviness results obtained for variant I (the amplitude of the workpiece speed oscillation $A_{nw} = 0.25$ rps)

Oscillation conditions	f_w (Hz)	n_{wav} (rps)	t_w (s)	Wa (mm)
1	5	0.25	0	0.0029
2	5	0.25	10	0.0021
3	5	1.00	0	0.0029
4	5	1.00	10	0.0019
5	50	0.25	0	0.0029
6	50	0.25	10	0.0028
7	50	1.00	0	0.0029
8	50	1.00	10	0.0027
9	5	0.75	5	0.0017
10	50	0.75	5	0.0020
11	30	0.25	5	0.0021
12	30	1.00	5	0.0023
13	30	0.75	0	0.0029
14	30	0.75	10	0.0028
15	30	0.25	10	0.0027
16	30	0.75	5	0.0019

Fig. 5 Three-dimensional (3D) graph of waviness Wa as a function of the oscillation duration t_w and the average value of the workpiece rotational speed oscillation n_{wav} with the amplitude of the workpiece speed oscillation $A_{nw} = 0.25$ rps
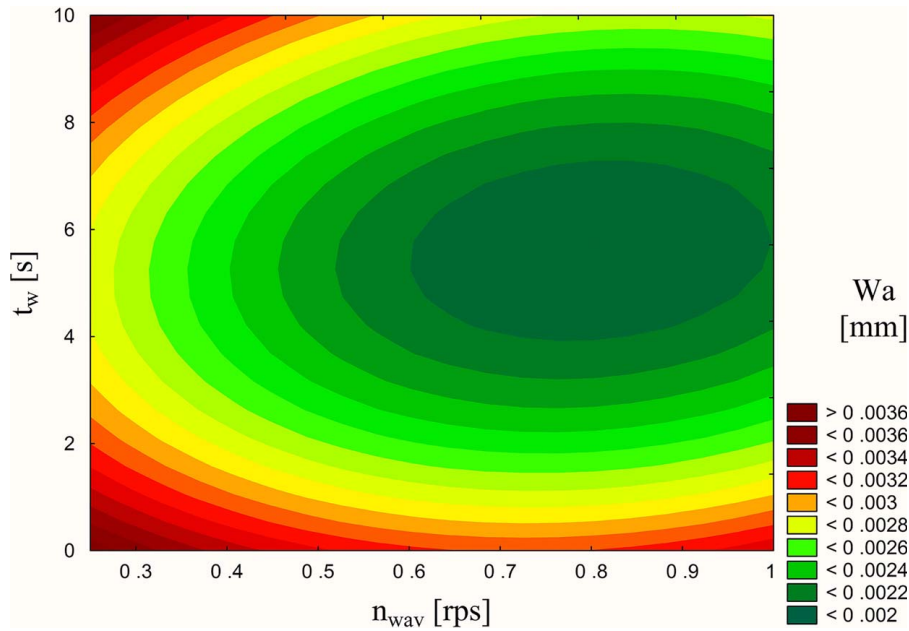
Fig. 6 Contour diagram of waviness *Wa* as a function of the oscillation duration t_w and the average value of the workpiece rotational speed oscillation n_{wav} with the amplitude of the workpiece speed oscillation $A_{nw} = 0.25$ rps

waviness *Wa* as a function of the duration of oscillation t_w and the average value of the workpiece rotational speed oscillation n_{wav} at $A_{nw} = 0.50$ rps. The chart of the waviness *Wa* as a function of the workpiece oscillation duration t_w and the frequency of oscillation f_w with $A_{nw} = 0.50$ rps is the surface shown in Fig. 10.

The character of the relations between the waviness of a ground surface and the parameters of the workpiece rotational speed oscillations (Figs. 5–10) established in the experiments derives from the dynamic properties of the machine-tool system. The form of these relations presented as response surfaces also involves simplifications resulting from the applied mathematical modeling. An indicator of the accuracy of this model representation of the studied

relations is the correlation coefficient (0.92 for variant I and 0.93 for variant II).

High values of waviness *Wa* at high values of the duration of the workpiece speed oscillation t_w, presented in Figs. 5–10, can be explained based on the analysis of the spindle drive system of the workpiece. The workpiece spindle drive system is equipped with a toothed belt drive. The introduction of the workpiece rotational speed oscillation by means of a toothed belt causes a time-varying tension force of the belt, which generates forced vibrations in the machine-tool system. These vibrations are transmitted to the grinding wheel spindle via the workpiece spindle bearing system. With a long duration of the workpiece speed oscillation, the spark-out time
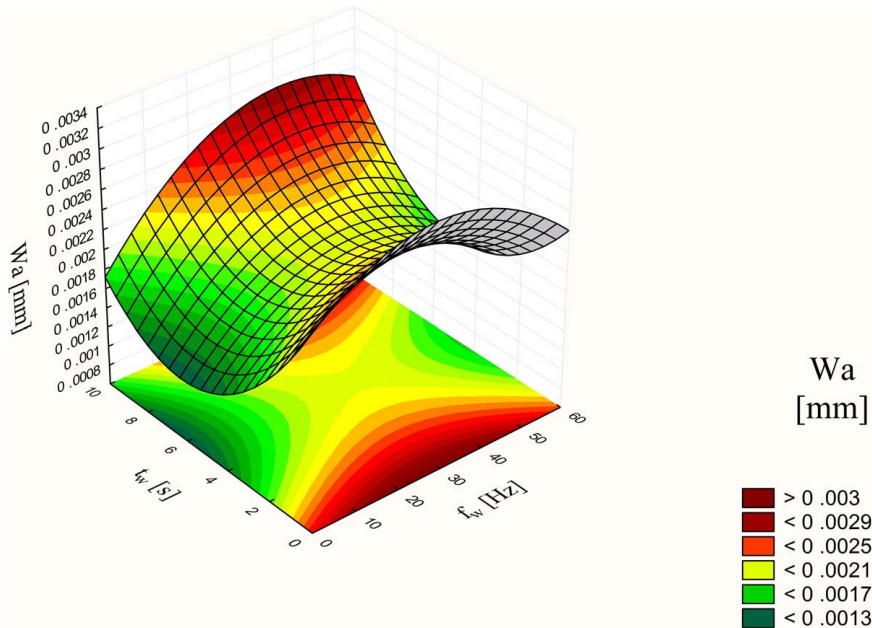

Fig. 7 3D graph of waviness *Wa* as a function of the workpiece oscillation duration t_w and oscillation frequency f_w with the amplitude of the workpiece speed oscillation $A_{nw} = 0.25$ rps

Table 2 The waviness results obtained for variant II (the amplitude of the workpiece speed oscillation $A_{nw} = 0.50$ rps)

Oscillation conditions	f_w (Hz)	n_{wav} (rps)	t_w (s)	Wa (mm)
1	5	0.50	0	0.0029
2	5	0.50	10	0.0023
3	5	1.00	0	0.0029
4	5	1.00	10	0.0021
5	50	0.50	0	0.0029
6	50	0.50	10	0.0032
7	50	1.00	0	0.0029
8	50	1.00	10	0.0028
9	5	0.75	5	0.0019
10	50	0.75	5	0.0024
11	20	0.50	5	0.0020
12	20	1.00	5	0.0026
13	20	0.75	0	0.0029
14	20	0.75	10	0.0024
15	20	0.50	10	0.0025
16	20	0.75	5	0.0021

without forced vibrations is short. The effect of grinding with a long duration of the workpiece speed oscillation t_w during sparking out is a deterioration in the waviness of the ground surface, which may be caused by the forced vibration introduced in the drive system of the workpiece. Whereas with a very short duration of the workpiece speed oscillation, the waviness of the ground surface is high due to too weak effect of interference with the process of self-induced vibrations development in the dynamic machine-tool system. Based on the relations between the waviness of the ground surface Wa and the duration of the workpiece speed oscillation t_w (Figs. 6 and 9) observed in the experimental tests, the value of the oscillation duration t_w at which the waviness of the ground surface Wa is the lowest can be determined.

During the experimental tests, high values of the waviness of the ground surface Wa were obtained at high average values of the workpiece speed oscillations n_{wav} (Figs. 5, 6, 8, and 9). The reason for this may be that with a high value of the average speed oscillation of the workpiece n_{wav}, the ratio of the speed oscillation amplitude (with the constant value $A_{nw} = 0.25$ rps in variant I and $A_{nw} = 0.50$ rps in variant II) to the average speed of the oscillation

n_{wav} is small, and thus, the effectiveness of the impact on the development of self-excited vibrations is low. On the other hand, a low average value of the workpiece speed oscillation n_{wav} resulted in an increase in the waviness of the ground surface due to the low number of revolutions of the workpiece during sparking out. Based on the relation of the waviness of the ground surface Wa to the average speed oscillation of the workpiece n_{wav} (Figs. 6 and 9) recorded during the experimental tests, the average value of n_{wav} oscillation at which the waviness of the ground surface Wa is the lowest can be determined.

Based on the relation of the waviness of the ground surface Wa to the frequency of oscillation of the rotational speed of the workpiece f_w obtained in experiments conducted according to variant I (Fig. 7), one can observe that as the frequency of oscillation increases, so does the waviness of the ground surface. This is due to a high amplitude of the force generated in the spindle drive system of the workpiece when high-frequency velocity oscillations are induced. The occurrence of a maximum value of the ground surface waviness (Fig. 7) at an oscillation frequency of approx. 36 Hz may be related to the natural frequency of the dynamic machine-tool system being of a similar value.

The results of measurements obtained in the individual experimental plans were used to estimate the parameters of the mathematical description of the following relations:

- waviness of the ground surface Wa as a function of the average value of the rotational speed oscillation of the workpiece n_{wav} and the oscillation duration t_w with $A_{nw} = 0.25$ rps,
- waviness of the ground surface Wa as a function of the frequency of oscillation of the rotational speed of the workpiece f_w and the oscillation duration t_w with $A_{nw} = 0.25$ rps,
- waviness of the ground surface Wa as a function of the average value of the speed oscillation of the workpiece n_{wav} and the duration of the oscillation t_w at $A_{nw} = 0.50$ rps,
- waviness of the ground surface Wa as a function of the frequency of oscillation of the rotational speed of the workpiece f_w and the oscillation duration t_w at $A_{nw} = 0.50$ rps.

Taking advantage of the STATISTICA computation capabilities, a mathematical model of these relations was developed that can be useful for determining the predicted waviness of a ground surface under certain conditions of the workpiece speed oscillation.
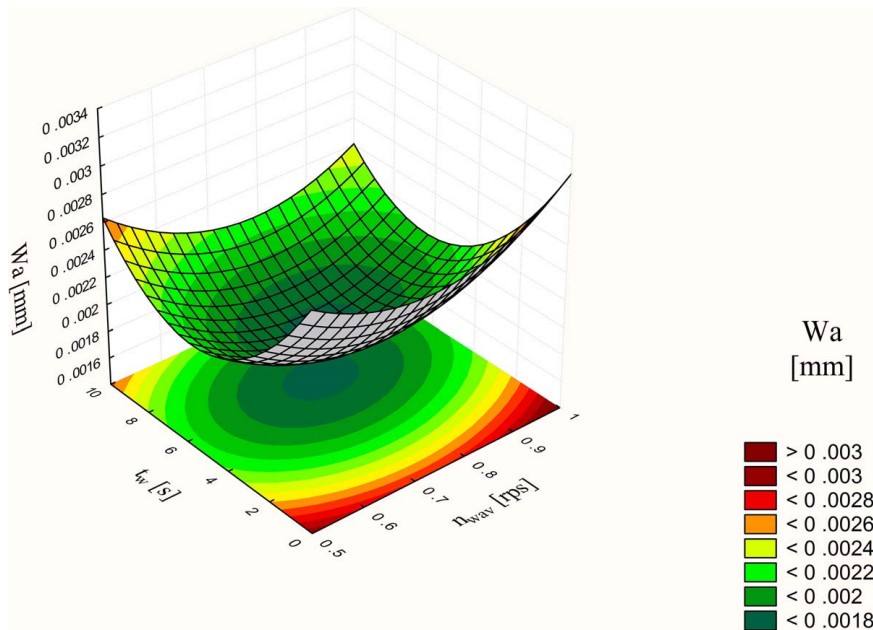

Fig. 8 3D graph of waviness Wa as a function of the duration of oscillation t_w and the average value of the workpiece speed oscillation n_{wav} with the amplitude of the workpiece speed oscillation $A_{nw} = 0.50$ rps
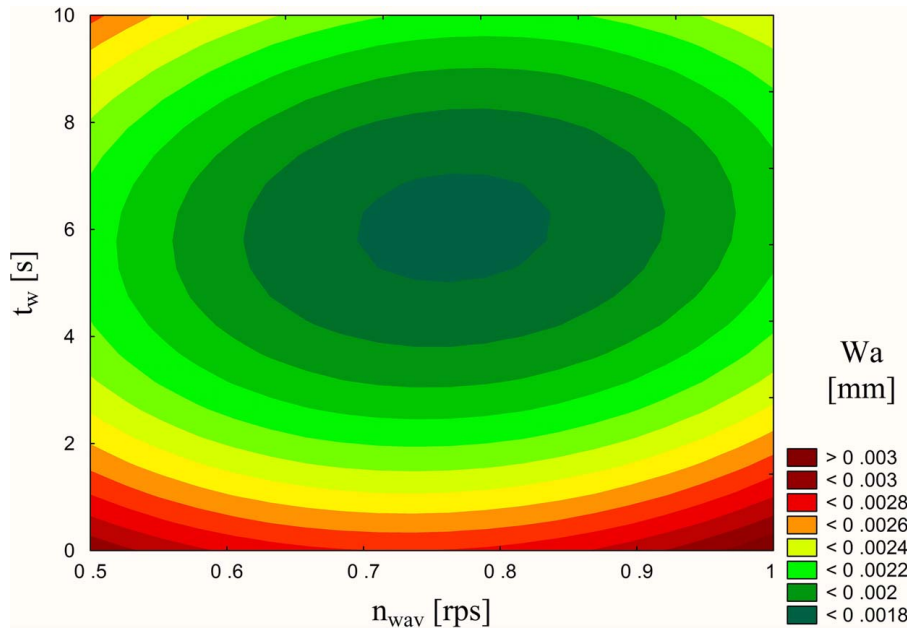
Fig. 9 Contour diagram of waviness Wa as a function of the oscillation duration t_w and the average value of the workpiece speed oscillation n_{wav} with the amplitude of the workpiece speed oscillation $A_{nw} = 0.50$ rps

On the basis of the experiments performed in accordance with the first variant, the mathematical expression of the relation between the waviness of the ground surface Wa and the average value of the speed oscillation of the workpiece n_{wav} and the oscillation duration t_w at $A_{nw} = 0.25$ rps can be presented in the following way:

$$Wa(n_{wav}, t_w) = 4.44 \cdot 10^{-6} - 3.74 \cdot 10^{-6}\, n_{wav} + 2.59 \cdot 10^{-6}\, n_{wav}^2 +$$
$$- 0.34 \cdot 10^{-6}\, t_w + 0.04 \cdot 10^{-6}\, t_w^2 - 0.07 \cdot 10^{-6}\, n_{wav}\, t_w$$

$$(2)$$

where Wa is the waviness of the ground surface in m, n_{wav} is the average value of rotational speed oscillations in rps, and t_w is the duration of oscillation in s.

The calculated minimum predicted value of the ground surface waviness is **$Wa = 1.7\,\mu$m**. This minimum waviness value occurs with the following values of the oscillation parameters:

- amplitude of the rotational speed oscillation of the workpiece $A_{nw} = 0.25$ rps ($A_w = 0.039$ m/s),
- average value of the rotational speed oscillation of the workpiece $n_{wav} = 0.81$ rps ($v_{wav} = 0.127$ m/s),
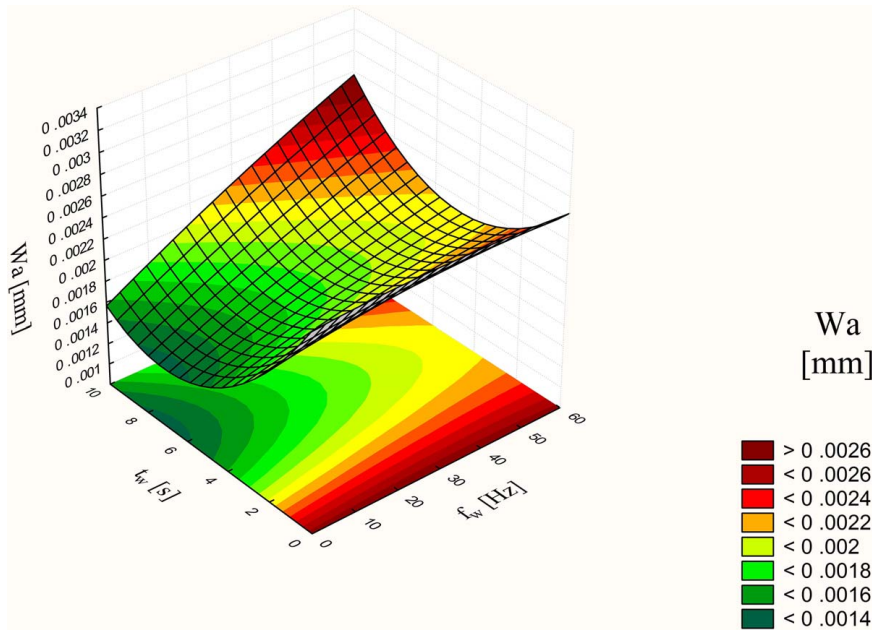- duration of oscillations $t_w = 5.3$ s.

Fig. 10   3D graph of waviness *Wa* as a function of the workpiece oscillation duration t_w and oscillation frequency f_w with the amplitude of the workpiece speed oscillation $A_{nw} = 0.50$ rps

For the first experimental plan, at $A_{nw} = 0.25$ rps, the mathematical form of the dependence of the waviness of the ground surface Wa on the frequency of the workpiece rotational speed oscillation f_w and on the duration of oscillation t_w can be expressed as follows:

$$Wa(f_w, t_w) = 2.58 \cdot 10^{-6} + 0.04 \cdot 10^{-6} f_w - 0.01 \cdot 10^{-6} f_w^2 +$$
$$- 0.44 \cdot 10^{-6} t_w + 0.04 \cdot 10^{-6} t_w^2 + 0.01 \cdot 10^{-6} f_w t_w$$

(3)

where Wa is the waviness of the ground surface in m, f_w is the frequency of the rotational speed oscillation in Hz, and t_w is the duration of oscillation in s.

Based on the dependence of the surface waviness on the frequency of oscillation, expressed by the Eq. (3), it follows that the lowest effectiveness of reducing the waviness of the ground surface is observed for the value of the frequency of oscillation at $f_w = 35.2$ Hz. This value is similar to one of the natural frequencies of the headstock of the grinding wheel (38.5 Hz) determined experimentally during the test of amplitude–frequency characteristics of the headstock of the grinding wheel. For experiments carried out according to the second experimental plan, at $A_{nw} = 0.50$ rps, the mathematical form of the dependence of the waviness of the ground surface Wa on the average value of the rotational speed oscillation of the workpiece n_{wav} and on the duration of oscillations t_w can be expressed in the following way:

$$Wa(n_{wav}, t_w) = 5.55 \cdot 10^{-6} - 7.86 \cdot 10^{-6} n_{wav} + 5.40 \cdot 10^{-6} n_{wav}^2 +$$
$$- 0.26 \cdot 10^{-6} t_w + 0.026 \cdot 10^{-6} t_w^2 - 0.07 \cdot 10^{-6} n_{wav} t_w$$

(4)

where Wa—waviness of the ground surface in m, n_{wav} is the average value of the workpiece rotational speed oscillation in rps, t_w is the duration of oscillation in s.

Similarly, as was the case for the first experimental variant, the minimum dependence (Eq. (4)), i.e., the lowest predicted value of the waviness Wa and the corresponding average value of the rotational speed oscillation of the workpiece n_{wav} and the time of oscillation t_w were calculated.

The determined minimum predicted value of the ground surface waviness is $Wa = 2.1 \, \mu m$. This waviness value occurs for the following values of the oscillation parameters:

- amplitude of the workpiece rotational speed oscillation $A_{nw} = 0.50$ rps ($A_w = 0.078$ m/s),
- average value of the rotational speed oscillation of the workpiece $n_{wav} = 0.79$ rps ($v_{wav} = 0.124$ m/s),
- duration of oscillations $t_w = 5.9$ s.

For the second experimental variant, the mathematical form of the dependence of the waviness of the ground surface Wa on the frequency of the rotational speed oscillation of the workpiece f_w and on the duration of the oscillation t_w at $A_{nw} = 0.50$ rps is formulated as follows:

$$Wa(f_w, t_w) = 2.66 \cdot 10^{-6} + 0.01 \cdot 10^{-6} f_w - 0.01 \cdot 10^{-6} f_w^2 +$$
$$- 0.37 \cdot 10^{-6} t_w + 0.02 \cdot 10^{-6} t_w^2 + 0.01 \cdot 10^{-6} f_w t_w$$

(5)

where Wa is the waviness of the ground surface in m, f_w is the frequency of the rotational speed oscillation in Hz, t_w is the duration of oscillation in s.

On the basis of the dependence of the surface waviness on the frequency of oscillation, expressed by Eq. (5), it follows that the effectiveness of reducing the waviness of the ground surface decreases with an increase in the frequency of oscillation f_w.

For so determined conditions of the speed oscillation of the workpiece, most advantageous in terms of the quality of the ground surface, further experiments were carried out to verify the predicted values and the waviness of the ground surface was measured. The

verification tests were repeated five times and brought the following average results:

- for the parameters most advantageous in terms of the quality of the ground surface, specified in variant I, i.e.:
  - amplitude of the rotational speed oscillation of the workpiece $A_{nw} = 0.25$ rps ($A_w = 0.039$ m/s),
  - frequency of the workpiece rotational speed oscillation $f_w = 5$ Hz,
  - average value of the rotational speed oscillation of the workpiece $n_{wav} = 0.81$ rps ($v_{wav} = 0.127$ m/s),
  - duration of oscillations $t_w = 5.3$ s (spark-out duration 10 s), the waviness of the ground surface is $Wa = 1.6 \, \mu m$,
- for the parameters most advantageous in terms of the quality of the ground surface, specified in variant II, i.e.:
  - amplitude of the workpiece rotational speed oscillation $A_{nw} = 0.50$ rps ($A_w = 0.078$ m/s),
  - frequency of the workpiece rotational speed oscillation $f_w = 5$ Hz,
  - average value of the workpiece rotational speed oscillation $n_{wav} = 0.79$ rps ($v_{wav} = 0.124$ m/s),
  - duration of oscillations $t_w = 5.9$ s (spark-out duration 10 s), the waviness of the ground surface is $Wa = 1.9 \, \mu m$.

In the experimental tests, the average value of the waviness of the plunge ground surface with no workpiece speed oscillation introduced (with 10-s spark-out) was determined as $Wa = 2.9 \, \mu m$.

The obtained result of the waviness of the ground surface without oscillation and the predicted and experimentally verified results of the waviness of the ground surface with rotational speed oscillations are graphically presented in Fig. 11. The results of grinding with the most advantageous values of oscillation parameters as determined in the experimental tests in variants I and II are presented. The graph also shows the established standard deviations.

Based on the results of waviness measurements performed for the most beneficial oscillation parameters as regards the quality of the ground surface, it follows that the actual values of Wa are lower than the predicted values. The reason for the discrepancy (6.25% in the case of variant I and 10.5% in variant II) is that the predicted values are determined on the basis of a mathematical model of the relation between the waviness Wa and the parameters of oscillation, which contains some simplifications resulting from the assumption to map the variability in the dependent variables using second-degree polynomial functions. The results of the verification experiments confirmed that the most advantageous values of the parameters of the workpiece speed oscillations in terms of the quality of the ground surface were identified correctly.

Selected samples of plunge ground shafts were analyzed with the TALYROND 3 profiler from Rank Taylor Hobson. Sample results
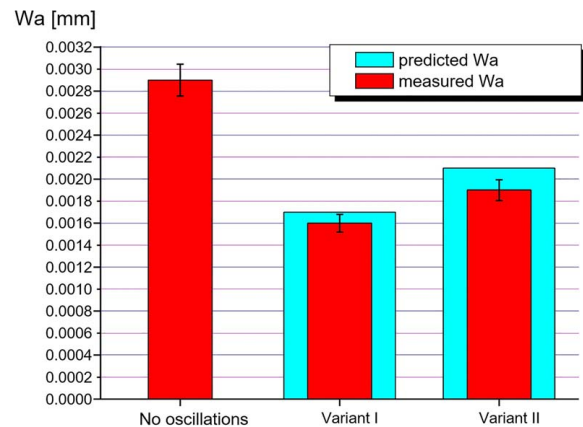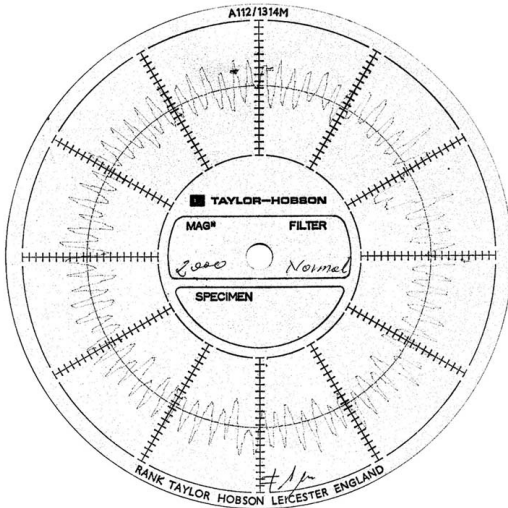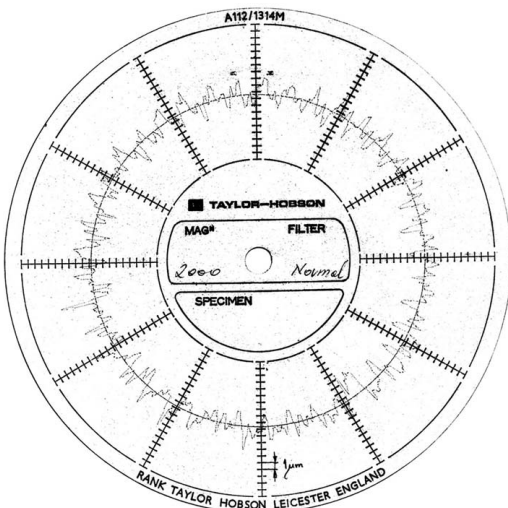

Fig. 11 Predicted and measured waviness Wa of the ground surface without oscillation and with the most advantageous parameters of the workpiece speed oscillation

Wa: 2.9 μm

Fig. 12 Profilogram of the surface ground without oscillations ($f_w = 0$ Hz)

Wa: 1.6 μm

Fig. 13 Profilogram of the surface ground with oscillations with $A_{nw} = 0.25$ rps, $f_w = 5$ Hz, $n_{wav} = 0.81$ rps, and $t_w = 5.3$ s

of these measurements are presented as profilograms, respectively, in Fig. 12, the case of the surface ground without the introduced workpiece speed oscillations, whereas in Fig. 13, the case of the surface ground with oscillations with the following parameters:

- workpiece speed oscillation amplitude $A_{nw} = 0.25$ rps, equivalent to workpiece peripheral speed oscillation amplitude $A_w = 0.039$ m/s,
- workpiece rotational speed oscillation frequency $f_w = 5$ Hz,
- average value of the workpiece rotational speed oscillation $n_{wav} = 0.81$ rps, (average value of the workpiece peripheral speed oscillation $v_{wav} = 0.127$ m/s),
- duration of the workpiece speed oscillation $t_w = 5.3$ s.

The results of measuring waviness of the ground surface with the TALYROND 3 profiler from Rank Taylor Hobson are almost the same as the results of the laser measurement of the ground surface profile. The difference between the results of the measurements carried out with these two methods compared to the results

of the measurement on the profilometer did not exceed 2% for all measured samples. Therefore, it can be concluded that extensive laser measurements, less labor-intensive and more convenient, are also correct and can be successfully used in research and formulating conclusions.

Conclusions

Using the conducted experimental research, a method of counteracting the development of self-excited vibrations during plunge grinding of shafts has been developed. The aim of experimental studies was to select: amplitude, frequency, average rotational speed, and duration of the introduced oscillations of the rotational speed of the workpiece during sparking out to obtain a reduction in waviness of the ground surface. Although the results can be related to the particular machine tool, the procedure of finding the optimal conditions of machining in order to solve the vibration problems can be applied to other machine-workpiece-process systems.

Based on the conducted experimental tests, the following conclusions were formulated:

- The highest quality (lowest waviness) of the ground surface is achieved at a speed oscillation amplitude of approximately 25% of the workpiece speed during sparking out. This value should eliminate the possibility of momentary stopping or changing the direction of the rotational speed of the workpiece, and therefore, it should be selected based on the average rotational speed oscillation of the workpiece.
- The most advantageous oscillation frequency should be lower than the natural frequency of the dynamic system of the machine tool.
- The best quality of the ground surface is achieved when the duration of the introduced workpiece rotational speed oscillations is about half the duration of the spark-out. If the oscillation is switched off too late, the waviness of the ground surface may deteriorate due to forced vibrations in the drive system of the workpiece.
- With the aid of software that enables the experiments planning and analysis of results, one can identify the parameters of the workpiece rotational speed oscillation to obtain the lowest waviness of the workpiece surface during plunge grinding of the shafts.

The oscillation-assisted cylindrical plunge grinding method does not solve all problems related to the dynamics of the grinding process, but it can be taken into account when choosing a technological scenario in order to ensure high quality of the ground surface shape. A major constraint for the application of this method is the possibility of introducing oscillation of the rotational speed of the workpiece to the grinder's control program or by means of a specially designed mechanical system. The latter method is dedicated to older type conventional grinders, for which the method can bring satisfactory grinding results despite the wear of the machine tool. Overall, application of the oscillation-assisted cylindrical plunge grinding method may result in obtaining a lower waviness of the ground surface compared to grinding without oscillation as long as the parameters of the oscillation and the grinding process are properly selected.

Conflict of Interest

There are no conflicts of interest.

Data Availability Statement

The data sets generated and supporting the findings of this article are obtainable from the corresponding author upon reasonable request. The authors attest that all data for this study are included

Funding Data

List of Abbreviations

b = width of cut (mm)
a_{e0} = preset grinding depth (μm)
d_s = grinding wheel diameter (mm)
d_w = workpiece diameter (mm)
f_w = frequency of the workpiece rotational speed oscillation (Hz)
n_s = grinding wheel rotational speed (rps)
n_w = workpiece rotational speed (rps)
n_{wav} = average value of the workpiece rotational speed oscillation (rps)
t_w = duration of the workpiece speed oscillation (s)
v_{wav} = average value of the workpiece peripheral speed oscillation (m/s)
A_{nw} = amplitude of the workpiece rotational speed oscillation (rps)
A_w = amplitude of the workpiece peripheral speed oscillation (m/s)

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