the thermocouple in the medium is less in the case of high-conductivity material because of the similar physical properties, neglecting the bead size may be reasonable.

Conclusions

1 The technique presented in this paper may be used to predict the correct surface temperature in a deflagrating insulator.

2 There is an "optimum" ratio of the thermocouple bead diameter to the lead wire diameter when the thermocouple bead is spherical. However, no matter what the bead size is, correction for the thermocouple response and lead loss must be made. The error is more sensitive to the change in wire diameter.

3 When a part of the thermocouple bead is exposed to a gas stream, especially when the thermocouple bead is large, the measurement can be quite erroneous unless the temperature gradient in the gas and the heat transfer coefficients are known.

4 The surface temperature of deflagrating M-2 double-base propellants is about 300–330 deg C at 5–10 psia.

Acknowledgment

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DISCUSSION

Robert C. Pfahl, Jr.[3]

Suh and Tsai have tackled part of a very difficult problem: the study of the response of thermocouples embedded in an ablating or deflagrating material. They have extended the work of Nydick [6] to include the effect of a thermocouple bead. Both Nydick and Suh and Tsai endeavor to model realistically the effect of a surface burning at a constant rate and temperature. Both works have modeled the moving surface by sacrificing a complete transient two-dimensional model of the conduction heat transfer between the material and the thermocouple. Transient conduction between the material and the thermocouple is modeled using heat transfer coefficients.

There are two heat transfer coefficients in Suh and Tsai's model: one to describe heat transfer between the thermocouple wire and the material, and one to describe heat transfer between the thermocouple bead and the material. The coefficients, equation (7), were developed by Nydick [6] from Beck's report [9]. Beck derived his heat transfer coefficient for a steady-state problem, for a linear temperature distribution in the material, and for a thermocouple oriented normal to the heated surface. The equation included an effective length of the thermocouple, l, which was less than the total thermocouple length, L. Beck determined the validity of his approximate method of estimating thermocouple temperature perturbations by comparing his solution with a two-dimensional finite-difference solution to the problem; he concluded that while the results showed substantial agreement, the numerical results would be recommended in general.

Nydick used Beck's formula for the heat transfer coefficient, but changed the effective length to an undefined length, l_2, without indicating the change or offering comment. He applied the formula to a transient rather than steady-state problem in which the temperature distribution was exponential instead of linear. He did not demonstrate the validity of Beck's formula under these new conditions.

Nydick extended Beck's formula to a wire located parallel to the heated surface. In his extension he assumed that the heat transfer coefficient was the sum of two terms: (1) Beck's formula for a wire normal to the heated surface and (2) a term which he described as "the gradient at that point in an undisturbed material." It is not clear to this discusser why this second term should be present. Nydick further modified this model to the case of a wire at an angle to the heated surface; it is this coefficient which appears as equation (7) in Suh and Tsai's article.

The following paragraphs discuss the suitability of equation (7) for describing heat transfer between the thermocouple wire and the material for the two limiting cases:

$\alpha = 0$, the thermocouple oriented normal to the heated surface

$\alpha = \pi/2$, the thermocouple oriented parallel to the heated surface.

For the $\alpha = 0$ case equation (7) should, but does not, reduce to Beck's original form. In addition, the ability of Beck's formula to describe transient conduction in a body with exponential temperature distribution should be demonstrated before it is used to describe this new situation.

The $\alpha = \pi/2$ case has been studied in analytical [11] and experimental [12] papers. The paper by Pfahl and Dropkin [11] models the transient two-dimensional conduction for a step in surface heat flux using finite-differences. The conclusions of the study are that the disturbances can be severe if the sensor is located within four radii of the heated surface, or if the volumetric specific heat, ρc_p, of the thermocouple is greater than twice that of the material. The disturbances are relatively insensitive to the ratio of thermal conductivities in contrast to the $\alpha = 0$ case [5].

The study predicts that there are two transient phases to the temperature disturbance. The thermocouple initially over-responds because the temperature "wave" as it progresses into the material crosses the thermocouple faster than the neighboring material due to the thermocouple's higher diffusivity. After this brief initial transient, the thermocouple begins to lag the undisturbed temperature response. This delay in response is caused by the volumetric specific heat of the thermocouple being greater than that of the material; hence, the thermocouple acts as a heat sink.

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The above effects were observed experimentally by Brewer [12]. Brewer instrumented samples with thermocouples oriented both normal and parallel to the heated surface and compared them to a reference thermocouple. He exposed his samples to a plasma arc so that his experimental boundary condition is essentially an isotherm moving at constant velocity. His results support the predictions for the $\alpha = 0$ case [5] and $\alpha = \pi/2$ case [11].

The above discussion indicates that a heat transfer coefficient for the $\alpha = \pi/2$ case should depend on the ratio of volumetric specific heats and on the depth of the thermocouple; equation (7) depends on neither. The transient change in the sign of the perturbation suggest that the coefficient is a strong function of time.

For the heat transfer coefficient between the bead and the material, Suh and Tsai again use equation (7). A possible alternative expression which has been developed to describe transient conduction from a sphere which is suddenly placed in a low-conductivity material at a different temperature [13] is

$$h = 2k/D \tag{15}$$

For a bead diameter three times the wire diameter this alternative expression becomes

$$h = 2k/3R \tag{16}$$

which agrees with the first term of equation (7) except that the $\ln(2L/R)$ term does not appear in the denominator. For Suh and Tsai's experiments with a $^1/_2$-mil wire, $\ln(2L/R) = 6.93$; thus, the new coefficient would be 593 percent larger.

I emphasize that the heat transfer coefficients used by Suh and Tsai were developed by previous authors. Suh and Tsai clearly question the validity of the coefficients which they use; they simply are using the best information available to them. They have demonstrated in Figs. 3 to 5 that an error of 100 percent in the coefficient will not strongly affect their predictions. The purpose of the preceding discussion is to indicate that these coefficients could be in error by more than 500 percent and that they could be strongly time-dependent. Until the accuracy of the coefficients is verified experimentally or by a more complete analytical model, it is premature for Suh and Tsai to use their model to reach conclusions on secondary effects such as the existence of an optimum bead diameter; the effect of changing the bead diameter by a factor of two is approximately the same as that produced by changing the heat transfer coefficient by the same factor.

The effect of bead diameter on temperature measurements from thermocouples embedded in deflagrating materials also has been studied analytically by Strittmater, Holmes, and Watermeier [14]. They develop a model which approximates what they consider to be the dominant mechanisms of heat transfer. They conclude that, "Although some important factors in the complete heat-transfer problem have been neglected in the foregoing analysis, qualitative estimates of these factors lead the writers to believe that the implications (of their results) will remain essentially unchanged in a more complete analysis." Their results do not reveal an optimum bead size, but rather that the ratio of D/d should be as large as possible. This prediction results from only considering the bead's ability to store thermal energy and neglecting the temperature perturbation which the bead causes. An unrealistic prediction from this model is that the disturbance is independent of wire diameter. Suh and Tsai's model, being more complete than that of reference [14], does predict that the disturbance depends primarily on wire diameter.

Brewer [12] indicates that a serious source of error when there is a steep temperature gradient is not knowing the exact location of the sensor. Suh and Tsai's experimental results have been presented in a manner which masks any uncertainties in thermocouple location. This masking occurs because the authors shift the time scale of their curves (time and location relative to the moving surface are linearly related) so that all thermocouples reach the surface at time zero. The experimental results offer no support for the conclusion that there is an optimum bead diameter, but the results do illustrate the severity of temperature perturbations and show the strong dependency on wire diameter. It is to be hoped that these results will serve to point out to the uninitiated the severity of temperature perturbations produced by embedded sensors.

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James V. Beck[4]

The authors are to be commended on their choice of a method of analysis which utilizes the concept of a heat transfer coefficient in a solid. This concept was originally proposed in my report for AVCO [9]. While a paper based on this work was never produced, one on the transient, two-dimensional analysis of a thermocouple normal to the heated surface has been published [5].

The concept of a heat transfer coefficient in a solid is valid, but one must be careful to use it properly. An implicit premise in my derivation of h is that it is a *correction* to a measurement that is being sought. Thus, say, a 10 percent error in estimating the correction results in a smaller error in the actual corrected temperature. Average coefficients found by relatively crude approximations may be tolerated. Nevertheless, h can vary an order of magnitude; therefore, one should not use this estimation premise as a license for laxity.

Suh and Tsai's equation (6) has been taken from a paper by Nydick. This author partially based his work upon mine. Mutations or rearrangements which occurred during the two reworkings will be discussed below.

Reference [9] of their paper is an analysis of the *steady-state* problem of a thermocouple in a *nonablating* medium. The thermocouple is *normal* to the heated surface. The equation given for the heat transfer coefficient is

$$h = \frac{2k/R}{3 \ln(2l/R)} \tag{17}$$

where l is *not* the *length* of the wire but a characteristic length depending upon the distance that the wire temperature, compared with the ambient, is depressed. The ambient temperature was considered to decrease linearly with y.

In one of Nydick's cases, the relation

$$h = \frac{2k/R}{3 \ln(2l_2/R)} + k\beta \tag{18}$$

was given for a wire of length L *parallel* to an ablating surface and in a region where the temperature gradient is proportional to β. The first term on the right-hand side is similar to equation (17) and the second is added to account for "the gradient at that point in an undisturbed material." No derivation of equation (18) was given, although Nydick refers to the heat flux

$$\dot{q} = k \left.\frac{dT}{dr}\right|_R \tag{19}$$

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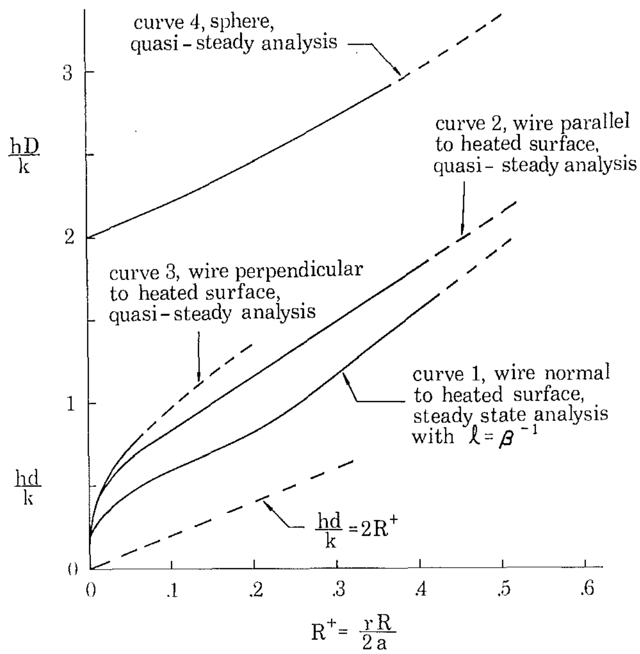

Fig. 12 Heat transfer coefficients in solids for certain cases

which for his orientation varies between $\pm\beta(T_s - T_0)k$ but has the average value of zero. Nydick also modified the $k\beta$ term to a form similar to the last term of equation (7). Note that as α goes to zero this term becomes

$$\frac{\beta k}{2}(1 + e^{-\beta L}) \tag{20}$$

Clearly this value is not zero, yet it should be in order to reduce to equation (17). Nydick accepted equation (17) for the case of a wire normal to the heated surface. I conclude that the second term in equations (7) and (18) should not be present.

Nydick did not give equation (7) exactly as Suh and Tsai wrote it. For example, instead of ln $(2L/R)$, he gave ln $(2l_2/R)$; however, he did not explicitly state what l_2 is. For equation (18) Nydick considered the geometry of the wire to be the shape of an inverted L with the shorter leg parallel to the heated surface. For this case the length of the short leg might possibly be identified with l_2. Nydick also considered the case of a wire at angle α (as shown by Fig. 1) for length L' with the wire then bent normal to the heated surface. Although not specified, Nydick may have intended α to be near 90 deg and L' to be small, in which case l_2 in equation (18) might possibly be represented by L' (but the βk term should be dropped). Suh and Tsai considered an unbent wire but retained the Nydick analysis and let L' become large and equal to L, the length of the thermocouple wire embedded in the insulator. They then set l_2 equal to L. After these modifications, equation (7) becomes incorrect for the use intended.

Rather than this expression, several alternatives are suggested. Instead of $l = L$, the wire length in equation (17) would be better expressed as $l = \beta^{-1} = r/a$ where $a = k/\rho c$. Then equation (17) can be written

$$\frac{hd}{k} = \frac{4}{3 \ln R^+} \tag{21}$$

where

$$d = 2R \qquad R^+ = \frac{rR}{2a} \tag{22}$$

Implicit in the derivation of equation (21) is that R^+ must be small. Equation (21) is shown as curve 1 of Fig. 12.

The above analysis is for steady state. One can obtain quasi-steady h values for a constant ablation velocity. Two cases may be obtained fairly directly from Carslaw and Jaeger [15] using line and point sources in a medium moving at velocity r. An infinite line source perpendicular to the direction of movement (analogous to a wire parallel to the heated surface) produces

$$\frac{hd}{k} = \frac{2}{K_0(R^+)} \tag{23}$$

where $K_0(x)$ is the zero-order modified Bessel function of the second kind. The result is shown as curve 2 in Fig. 12. Again R^+ is assumed to be small.

In the case of an infinite wire normal to the heated surface, the describing equation for transient radial heat transfer outside an infinite cylinder is

$$\frac{k}{r'}\frac{\partial}{\partial r'}\left(r'\frac{\partial T}{\partial r}\right) = \rho c_p \frac{\partial T}{\partial t} \tag{24}$$

The equation for steady-state heat transfer from outside an infinite cylinder to a fluid flowing parallel to its axis is

$$\frac{k}{r'}\frac{\partial}{\partial r'}\left(r'\frac{\partial T}{\partial r}\right) = \rho c_p r \frac{\partial T}{\partial z} \tag{25}$$

where r' is the radial coordinate and r is the axial velocity. Notice that any solution of (24) is also a solution of (25) with t replaced by z/r. For the boundary conditions for $T(r',z)$

$$\begin{aligned} T(R,z) &= T_0 \\ T(\infty,z) &= T_\infty \\ T(r',0) &= T_\infty \end{aligned} \tag{26}$$

one can derive for small R^+

$$\frac{hd}{k} = 2\left\{\frac{-1}{\ln R^+ + \gamma} - \frac{\gamma}{2[\ln R^+ + \gamma]^2} + \ldots\right\} \tag{27}$$

utilizing Carslaw and Jaeger's equation [16] and employing $z = a/r$ as the characteristic length. This result is shown as curve 3 of Fig. 12. In equation (27), $\gamma = 0.57722\ldots$ is Euler's constant.

A heat transfer coefficient for the spherical junction can be obtained by using the moving point source solution [17]. For small R^+ one can derive

$$\frac{hD}{k} = 2e^{R+} \tag{28}$$

for steady state. This result is the upper curve (curve 4) of Fig. 12. Unlike that of the wire, the sphere value of h has a non-zero value as R^+ approaches zero.

It is not apparent how these results may be compared with those of Suh and Tsai. If there is a sufficiently large value for L/R in equation (7), it reduces to

$$\frac{hD}{k} = 2R^+ \tag{29}$$

which is the lower straight line in Fig. 12. Compared to curves 2 and 3, this can be 100 percent or more in error.

Several other aspects of the paper are worthy of comment, for example, the choice of finite-difference scheme. Other approximations, such as the Crank-Nicolson, are more accurate and stable for any time step. Also, a generalized transient analysis could be developed, together with some correction kernels, for use in other integrals [18]. Further comments will be deferred.

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Authors' Closure

It is indeed gratifying that our paper aroused sufficient interest on the part of Dr. Pfahl and Professor Beck to write such extensive comments. We would like to thank them for the thorough study of the paper. It is rather amusing, in a sense, that what we considered to be the most trivial part of the paper generated more interest than any other aspect of the paper. Since it is clear that the major purpose of the paper has not been conveyed to the readers, the basic purpose of the work will be stated here and our approach to this type of problem will be discussed.

The major purpose of the paper is to convey a *concept* which enables the "measurement" of the surface temperature of a rapidly deflagrating solid. The purpose of the paper *is not* to evaluate various heat transfer coefficients in solids. The method used by others in the past in "measuring" the surface temperature was to use a thermocouple and then through analyses determine the probable error involved in the measurement. Such a method is not reliable since the validity of the analyses cannot be checked. Therefore, we decided to *predict* the surface temperature by measuring its value using several sizes of thermocouples and by extrapolating the results to zero wire diameter based on the most probable temperature profile in the solid.

In order to accomplish the stated purpose, reasonable values for the heat transfer coefficients were going to be assumed in order to by-pass the difficulty of knowing the exact extent of the region disturbed by the presence of the thermocouple. We felt that as long as the temperatures measured by various sized thermocouples could be predicted by a model to the desired accuracy, the exact form of the heat transfer coefficients would not be important for our purpose. Then, we found that the Nydick's expressions were available. We used Nydick's expressions as the first approximation, since the whole question of the heat transfer coefficients in solids has not been verified experimentally to a satisfactory degree, as Dr. Pfahl pointed out in his comments. Our method might not have been a good one if it depended sensitively on the heat transfer coefficient. However, it turned out that the solution does not depend on the coefficient sensitively, as shown in the paper. For our purpose any expression or values for the heat transfer coefficient that correctly predicted the surface temperatures measured by a large number of different sized thermocouple wires would have been acceptable. For example, we could have assumed the heat transfer coefficient to be a polynomial function of pertinent parameters with undetermined coefficients and then varied the coefficients until it fit all the experimental results.

We would like to thank both Dr. Pfahl and Professor Beck for commenting on the numerical scheme. We are certain that the comments will be useful to those who want to refine the concept presented in our paper. However, we think that our solutions are sufficiently accurate. Furthermore, one should realize that the assumed model is an approximation of a complex, real phenomenon. Reasonable solutions, i.e., reasonable both in time, cost, and accuracy, have to be provided to this type of real problem without losing sight of one's objectives. The accuracy of a numerical result cannot, after all, exceed the accuracy within which a model can approximate the actual phenomenon.

Finally, a few comments will be made on the optimum ratio of the bead diameter to the wire diameter. The exact value for the optimum ratio will be subject to the assumptions made in the paper such as the dependence of the heat transfer coefficient on the radius of the thermocouple bead. Therefore, if the expression given by Dr. Pfahl, i.e., equation (15), is used, the optimum ratio will assume a different value. It will be interesting to use the equation suggested by Dr. Pfahl and to determine if it yields a better correlation with the experimental results. When the total heat transfer rate from the surrounding material to the bead does not depend on the size of the thermocouple, obviously the smaller the bead the more accurate will be the result. The purpose of discussing the question of the optimum bead size was to point out that making the bead diameter as small as possible at a great expense does not necessarily guarantee accurate measurements.