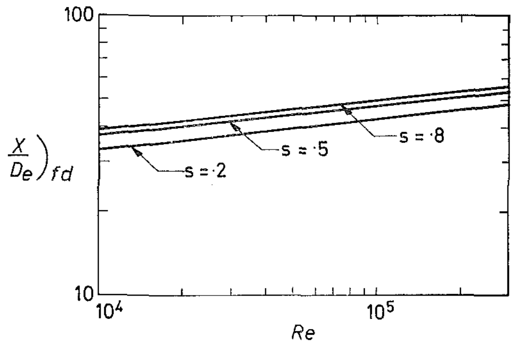

Fig. 9 Variation of predicted thermal development length with Reynolds numbers and radius ratio for air (Pr = 0.7)

perature profiles are evaluated from equation (25) where the dimensionless temperature at the edge of the thermal boundary layer has been calculated using the appropriate value of $\delta_T{}^+$ at the predetermined value of x/D_e. Fig. 6 shows the developing temperature profiles as a function of x/D_e with the fully developed profile compared with the data of Lee referred to earlier.

The temperature profiles may be integrated to obtain the dimensionless bulk temperature (cf. equation (26)) and hence the local Nusselt number variation with x/D_e is determined from equation (29). For the two radius ratios considered, generally excellent agreement is found between prediction and the experimental data gathered by Roberts and Barrow for air, as shown in Figs. 7 and 8. The small discrepancies that do appear at the high values of Reynolds number are, as pointed out by Roberts and Barrow, due to the trip wires being larger than necessary, thus enhancing the heat transfer rates in that region. Nevertheless, it is felt that the agreement achieved by the analysis in the developing thermal boundary layer is attributed to the fact that tripping wires were used in the experiments. This insured that the boundary layers were fully turbulent from the entry point as postulated in the analysis. Unfortunately, they did not present any developing pressure gradient data in their paper which might have afforded a more realistic test of the analysis in the developing hydrodynamic boundary layers.

Finally, the variation of the thermal development length as a function of radius ratio and Reynolds number is shown in Fig. 9. The general trends of these curves are in accordance with those predicted by Lee [20] who analyzed the developing thermal boundary layer in hydrodynamically fully developed flow in an annulus. In the case analyzed here, a slightly greater development length is required.

Conclusions

An analytical investigation, using integral techniques, has been made of the local flow and heat transfer characteristics of a fluid in turbulent motion in the entry region of a concentric annulus which has a uniformly heated core and an insulated outer wall. The important conclusions are:

1 Fully developed hydrodynamic flow is attained in an entrance length of approximately 10 equivalent diameters in the range of Reynolds number studied. The radius ratio range investigated was found to have no influence on x/D_e.

2 Fully developed turbulent heat transfer is obtained within a further 30 equivalent diameters. Increasing the Reynolds number and radius ratio leads to an increase in the required x/D_e.

References

1 Rothfus, R. R., Monrad, C. C., and Senecal, V. E., "Velocity Distribution and Fluid Friction in Smooth Concentric Annuli," *Industrial and Engineering Chemistry*, Vol. 42, No. 2, 1950, p. 2511.

2 Rothfus, R. R., et al., "Isothermal Skin Friction in Flow Through Annular Sections," *Industrial and Engineering Chemistry*, Vol. 47, 1955, p. 913.

3 Barrow, H., "Fluid Flow and Heat Transfer in an Annulus With a Heated Core Tube," *Proceedings, Institution of Mechanical Engineers*, Vol. 196, 1955, p. 1113.

4 Barrow, H., "A Semitheoretical Solution of Asymmetric Heat Transfer in Annular Flow," *Journal of Mechanical Engineering Science*, Vol. 4, 1960, p. 331.

5 Kays, W. M., and Leung, E. Y., "Heat Transfer in Annular Passages—Hydrodynamically Developed Turbulent Flow With Arbitrarily Prescribed Heat Fux," *International Journal of Heat and Mass Transfer*, Vol. 6, 1963, pp. 537–557.

6 Olson, R. M., and Sparrow, E. M., "Measurements of Turbulent Flow Development in Tubes and Annuli With Square or Rounded Entrances," *AIChE Journal*, Vol. 9, 1963, p. 766.

7 Lee, Y., and Barrow, H., "Turbulent Flow and Heat Transfer in Concentric and Eccentric Annuli," Paper 12, Thermodynamics and Fluid Mechanics Convention, Cambridge, 1964.

8 Barrow, H., and Lee, Y., "Heat Transfer With Unsymmetrical Thermal Boundary Conditions," *International Journal of Heat and Mass Transfer*, Vol. 7, 1964, p. 580.

9 Brighton, J. A., and Jones, J. B., "Fully Developed Turbulent Flow in Annuli," *Journal of Basic Engineering*, TRANS. ASME, Series D, Vol. 86, 1964, pp. 835–844.

10 Barrow, H., Lee, Y., and Roberts, A., "The Similarity Hypothesis Applied to Turbulent Flow in an Annulus," *International Journal of Heat and Mass Transfer*, Vol. 8, 1965, p. 1499.

11 Jonsson, V. K., and Sparrow, E. M., "Experiments on Turbulent Flow-Phenomena in Eccentric Annular Ducts," *Journal of Fluid Mechanics*, Vol. 25, 1966, pp. 65–86.

12 Levy, S., "Turbulent Flow in an Annulus," JOURNAL OF HEAT TRANSFER, TRANS. ASME, Series C, Vol. 89, 1967, pp. 25–31.

13 Quarmby, A., "An Experimental Study of Turbulent Flow Through Concentric Annuli," *International Journal of Mechanical Science*, Vol. 9, 1967, pp. 205–221.

14 Wilson, N. W., and Medwell, J. O., "An Analysis of Heat Transfer for Fully Developed Turbulent Flow in Concentric Annuli," JOURNAL OF HEAT TRANSFER, TRANS. ASME, Series C, Vol. 90, 1968, pp. 43–50.

15 Diessler, R. G., and Taylor, M. F., "Analysis of Fully Developed Turbulent Heat Transfer and Flow in an Annulus With Various Eccentricities," NACA-TN-3451, Mar. 1955.

16 Okiishi, T. H., and Serovy, G. K., "An Experimental Study of the Turbulent-Flow Boundary-Layer Development in Smooth Annuli," *Journal of Basic Engineering*, TRANS. ASME, Series D, Vol. 89, 1967, pp 823–836.

17 Roberts, A., and Barrow, H., "Turbulent Heat Transfer to Air in the Vicinity of the Entry of an Internally Heated Annulus," Paper 23, Thermodynamics and Fluid Mechanics Convention, Bristol, 1968.

18 van Driest, E. R., "On Turbulent Flow Near a Wall," Paper 12, Heat Transfer and Fluid Mechanics Institute Symposium, University of California, Los Angeles, Calif., 1955.

19 Jonsson, V. K., "Experimental Studies of Turbulent Flow Phenomena in Eccentric Annuli," PhD thesis, University of Minnesota, 1965.

20 Lee, Y., "Turbulent Heat Transfer From the Core Tube in Thermal Entrance Regions of Concentric Annuli," *International Journal of Heat and Mass Transfer*, Vol. 11, 1968, pp. 509–521.

21 Lee, Y., "Turbulent Flow and Heat Transfer in Concentric and Eccentric Annuli," PhD thesis, University of Liverpool, 1964.

DISCUSSION

E. M. Sparrow[2]

Since the fluid flow and heat transfer results depend on the choice of a turbulent transport model, it is relevant to compare the ϵ_M representation, equation (15), with available data. To this end, the authors may wish to numerically evaluate equation (15) and compare the resulting ϵ_M with experimentally determined ϵ_M profiles measured by Jonsson and Sparrow.[3] The experiments covered the Reynolds number range from approximately 30,000 to 180,000 and encompassed radius ratios from 0.28 to 0.75. These ranges are relevant to those investigated in the present paper.

[2] Department of Mechanical Engineering, University of Minnesota, Minneapolis, Minn. Mem. ASME.

[3] Jonsson, V. K., and Sparrow, E. M., "Turbulent Diffusivity for Momentum Transfer in Concentric Annuli," *Journal of Basic Engineering*, TRANS. ASME, Series D, Vol. 88, 1966, pp. 550–552.

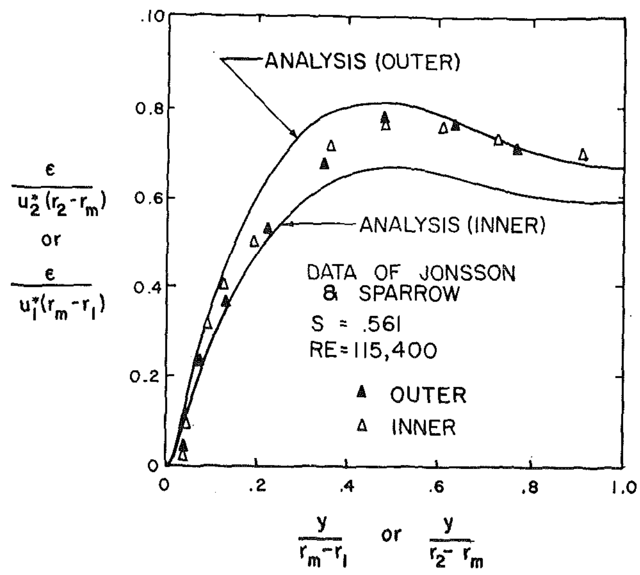

Fig. 10 Comparison of the assumed eddy diffusivity distribution to experimental data

Authors' Closure

We thank Professor Sparrow for his contribution, and we wish to point out that in the preparation of the paper, comparisons were made to available data although this is not specifically mentioned in the paper.

ϵ_M was in fact calculated using equation (17) which was derived from equation (15) as is explained in the text. For fully developed flow, $\delta_1^+ = (r_m - r_1)u_1^*/\nu$, $\delta_2^+ = (r_2 - r_m)u_2^*/\nu$, and for regions away from the wall $[1 - \exp(-y^+/A^+)]$ approaches unity. Thus the statement of equation (17) reduces essentially to the form of equation (15) with the exception that $b = 2$ in the inner region of the annulus and $b = 2.5$ in the outer region. A comparison with the data of Brighton [22] is made in the thesis [23] from which the present paper has arisen and the agreement is good within about 10 percent. For the sake of comparison Fig. 10 shows a similar sort of agreement with Jonsson and Sparrow's results.

Additional References

22 Brighton, J. A., "The Structure of Fully Developed Turbulent Flow in Annuli," PhD thesis, Purdue University, August 1963.

23 Wilson, N. W., "Turbulent Flow and Heat Transfer in Concentric and Eccentric Annuli," PhD thesis, University of Wales, March 1970.