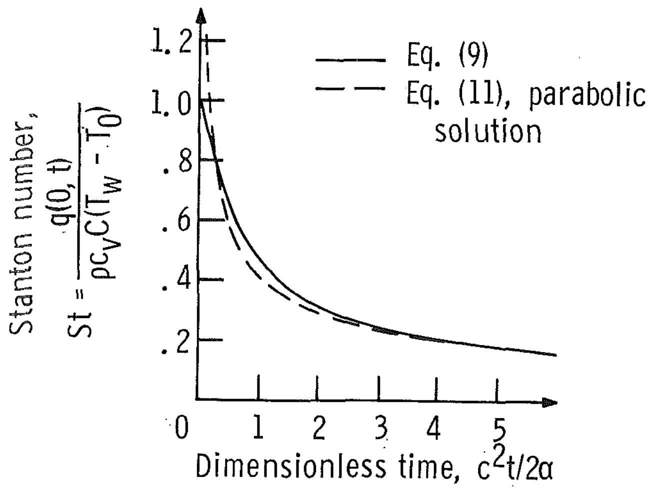

Fig. 1 Heat flux at wall as a function of time

where T is temperature, t is time, q is heat flux, C is speed of heat propagation, α is thermal diffusivity, k is thermal conductivity, and x is position.

We were interested in seeing how the effect of the second derivative of temperature and the thermal inertia term, $\frac{\alpha}{C^2}\frac{\partial q}{\partial t}$, affected the temperature and heat flux distribution. It turns out that equation (6) is an unnecessary constraint, which overspecified the problem, as will now be shown.

The general solution to equation (2) is given in reference [1] as

$$q = -\frac{C^2 k}{\alpha} e^{-C^2 t/\alpha} \int_0^t \frac{\partial T}{\partial x} e^{C^2 \xi/\alpha} d\xi \tag{7}$$

and the solution for the temperature distribution is given in reference [1] as

$$\frac{T(x, t) - T_0}{T_w - T_0} = u(Ct - x) \left\{ \exp\left(\frac{-Cx}{2\alpha}\right) + \int_{x/C}^{t} \frac{Cx}{2\alpha} \exp\left(\frac{-C^2\tau}{2\alpha}\right) \frac{I_1\left[\frac{C^2}{2\alpha}\sqrt{\tau^2 - \left(\frac{x}{C}\right)^2}\right] d\tau}{\sqrt{\tau^2 - \left(\frac{x}{C}\right)^2}} \right\} \tag{8}$$

Differentiating equation (8) and substituting the temperature gradient into equation (7) gives the heat flux as a function of x and t. The differentiation of the unit step function gives a contribution only when $x = Ct$. For the special case of x equal to zero, $q(0, t)$ is

$$\mathrm{St} = \frac{q(0, t)}{\rho C_v C(T_w - T_0)} = e^{-C^2 t/2\alpha} I_0(C^2 t/2\alpha) \tag{9}$$

in terms of a conduction Stanton number.

At t equal to zero the Stanton number equals 1; thus, the above solution predicts a unit jump in the Stanton number associated with a step change in temperature. This differs from the incorrect solution for Stanton number given in reference [1] which satisfied equation (6) and gave a value of zero. Equation (9) is shown in Fig. 1 along with the conventional parabolic solution to the same problem.

For large times, the zeroth-order term of an asymptotic expansion of I_0 is of the form

$$I_0\left(\frac{C^2 t}{2\alpha}\right) \sim \frac{\exp\left(\frac{C^2 t}{2\alpha}\right)}{\sqrt{2\pi\left(\frac{C^2 t}{2\alpha}\right)}} \tag{10}$$

thus for large times

$$\mathrm{St} = \frac{1}{\sqrt{2\pi(C^2 t/2\alpha)}} \tag{11}$$

which is the usual heat parabolic conduction solution.

Free Convection From a Two-Dimensional Finite Horizontal Plate[1]

Zeëv Rotem.[2] The author considers free convective heat transfer from a horizontal, downward-facing plate. Reference is made to previous work by Schmidt [2],[3] interpreted as demonstrating the existence of a type of stagnation-point flow below the plate, and of Suriano and Yang [3] who are reported to have shown the existence of this flow by numerical computation.

The author then applies a similarity transform which in actual fact is a boundary layer transformation.[4] The result is a set of three simultaneous ordinary differential equations. The transformation can, of course, only be used for two-dimensional flows or three-dimensional flows with rotational symmetry where a Stokes stream function may be assumed.

It has recently been pointed out [12, 13] that Schmidt's flow is not of the type stipulated by Chen; also, that Suriano and Yang's [3] solution was erroneous in principle [14], quite apart from the fact that it referred to a plate heated on *both* its sides, for values of the Grashof number for which the transformation of Chen does not apply.

Further, the proof that there is no flow of the type stipulated by Chen for free convection from underneath a horizontal plate heated and facing downward seems to be well established [15, 16]. Therefore correlations will have to be based on careful tests alone and/or on solutions of the complete equations (2) through (5) of Chen, using a numerical technique which is properly conservative [14].

Additional References

12 Rotem, Z., and Claassen, L., "Natural Convection above Unconfined Horizontal Surfaces," *Journal of Fluid Mechanics*, Vol. 38, 1969, pp. 173–192.

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[1] By Ching-Jen Chen, published in the Aug. 1970 issue of the Journal of Heat Transfer, Trans. ASME, Series C, Vol. 92, No. 3, p. 548.

[2] Professor, University of British Columbia, Vancouver, B. C., Canada.

[3] Numbers in brackets designate References continued from original paper.

[4] When λ is finite (!). λ is a measure of plate width.

13 Rotem, Z., and Claassen, L., "Free Convection Boundary-Layer Flow Over Horizontal Plates and Discs," *Canadian Journal of Chemical Engineering*, Vol. 47, 1969, pp. 461–468.

14 Panton, R., "Laminar Free Convection along a Vertical Plate at Extremely Small Grashof Numbers," *International Journal of Heat and Mass Transfer*, Vol. 11, 1968, pp. 615–616.

15 Gill, W. N., Zeh, D. W., and Del-Casal, E., "Free Convection on a Horizontal Plate," *Z.A.M.P.*, Vol. 16, 1965, pp. 539–541.

16 Rotem, Z., "Free Convection from Heated, Horizontal Downward-Facing Plates," *Z.A.M.P.*, Vol. 21, 1970, pp. 472–475.

Author's Closure

To discuss Professor Rotem's comments we recognize the following points:

1 The theoretical solution obtained in my case is for a finite, horizontal plate (i.e., a plate with two edges) heated facing downward or cooled facing upward, while the case solved by Rotem and Claassen [12, 13, 16] or the proof of Gill, Zeh, and Del-Casal [15] are for a semi-infinite plate (i.e., a plate with one edge). We must be very careful in applying the semi-infinite plate solution to a finite, horizontal plate since the existence of continuous flow at $X \rightarrow \infty$ now must be stipulated at the midpoint of the finite plate. For a heated finite plate facing upward the lighter fluid at the midpoint may move upward to give a flow that approximates the condition at $X \rightarrow \infty$. However, for a heated finite plate facing downward, the lighter fluid at the midpoint will be stagnated near the wall and remain there because the heavier fluid is below it. Therefore, the semi-infinite plate solution does not apply to the heated (or cooled) finite plate facing downward (or upward).

2 Experimentally, for a heated, finite plate facing downward to exhibit a stagnation point flow was well demonstrated by Kraus' direct measurement of the flow direction (see Figs. 22 and 23 of [1]). Consider Fig. 12 of Schmidt [2]. If the air near the midpoint were to move down on the underside of the plate the photo would not show smooth isothermal lines near it. Furthermore, a stagnation-point flow for a heated, finite plate facing downward was made visually by Singh and Birkebak.[5] Note that $\lambda \simeq 2.5$ is also applicable in Singh and Birkebak's photo.

3 I agree with Panton's [14] comment on the work of Suriano and Yang [3] that to treat an infinite domain by a finite domain will introduce errors in numerical values because of the uncertainty in the temperature gradient at the wall. In fact, I was unable to plot Suriano and Yang's [3] data in my Fig. 3.

[5] See Fig. 4, *Z. angew. Math. Phys.*, Vol. 20, 1969, p. 454.