journal of heat transfer

# discussion

## Heat Transfer to Horizontal Gas-Solid Suspension Flows[1]

**W. J. Danziger.**[2] The values of Nu in Figs. 5 and 7 show considerable difference from published data for vertical flow of air-solids mixtures. In [1][3] it was shown that up to $W_s/W_a$ of about 1.0 there was little effect of the solids (although there was a tendency toward a slight increase in Nu), and that over $W_s/W_a$ of 2.0, Nu varied as $(W_s/W_a)^{0.45}$. In [7] it was reported that Nu varied as $Re^{0.66}$.

Fig. 7, however, shows zero effect of $W_s/W_a$, and indicates values of Nu for air-solids mixtures only 80–90 percent of those for solids-free-air instead of approximately 230 percent that would be predicted for the maximum solids concentration used. Fig. 5 shows some points that indicate no dependence on either $W_s/W_a$ *or* Re, and others that indicate no dependence on Re but show a greater effect of $W_s/W_a$ than would be predicted. Both [1] and [7] were based on cracking catalyst of about 50$\mu$ average particle diameter, but extrapolation to either 30$\mu$ or 200$\mu$ average diameter glass particles could not be expected to introduce errors of the magnitude of the discrepancies indicated, and change from vertical to horizontal flow would not change the relationships noted as long as no solids accumulated on the bottom of the tube.

At least some of the discrepancy results from use of $(T_w - T_{mm})$ in calculation of the heat transfer coefficient. This gives a false value of $h$, since it is $(T_w - T_a)$ that represents the true driving force for transfer of heat from the hot surface. Though $T_a$ wasn't measured, $T_{mm}$ wasn't measured either, and only incorrect values of $h$ can be calculated with it. It is suggested that useful information might be derived from the data by use of the equation given in [7] for the MTD between air and a solid particle, together with a heat balance, to estimate $T_a$. The possible error in the assumed value of $h$ between gas and particle will result in some possible error in the calculated value of $T_a$, and therefore of $h$ between fluid and wall, but $h$ so calculated will still be more correct than the value used by the authors.

It it is assumed that a higher temperature at the top of the tube would stem from stratification, the reduced heat capacity of the resulting solids-poor mixture at the top of the constant-flux tube would require that the air temperature be higher there than that in the lower portion, thus intensifying the error in $h$ calculated on the basis of $T_{mm}$. However, it is inconceivable that gas flowing at a given mass rate does not prevent stratification of small beads yet does prevent stratification of beads of perhaps 300 times the mass of the small beads. A more reasonable explanation of the temperature discrepancy, top versus bottom, affecting the small bead but not the large bead might be that an electrostatic charge on the beads caused fines to plate out on the upper tube surface. At the bottom of the tube there would be enough scouring action to prevent such plating, whereas at the top of the tube the scouring would be less intense, with gravity acting to reduce particle impact, thus allowing an insulating coating to remain. Even the smallest diameter sphere in the size range of the large bead, however, is many times larger than any particle that would be expected to adhere to the wall because of its charge.

Fig. 8 shows pressure drop at Re of 30,000 remaining unchanged, whether air-only or the same quantity of air with up to about 4 lb of 30$\mu$ beads per pound of air was flowing. Such constancy would seem to be more indicative of manometer leads plugged with beads than of a novel flow effect.

[1] By C. A. Depew and E. R. Cramer, published in the February, 1970, issue of the JOURNAL OF HEAT TRANSFER, TRANS. ASME, Series C, Vol. 92, No. 1, pp. 77–82.

[2] The M. W. Kellogg Company, New York, N. Y.

[3] Numbers in brackets designate References at end of article under discussion.

### Authors' Closure

The authors wish to thank Mr. Danziger for his suggestion. The influence of the estimated mean air temperature on the evaluation of Nu is being studied, and the results will be presented along with additional data in the near future. More recent pressure-drop data have confirmed the previous findings, and plugged pressure taps can be ruled out as a possibility.

## Heat Flux Through a Strip-Heated Flat Plate[1]

**Frederick A. Costello.**[2] An exact solution to the subject problem that is more convenient than that presented by Schmitz [8][3] can be obtained by use of the Schwarz-Christoffel transformation [9]. A typical section of the strip-heated plate is shown in Fig. 1. The transformation is given by $(d > 1)$

$$\frac{dz}{d\tau} = K \frac{1}{\sqrt{\tau(\tau - 1)(\tau - d)}} \tag{1}$$

An elliptic integral of the first kind, defined by

$$F(k, \sin^{-1} R) \equiv \int_0^R \frac{dr}{\sqrt{(1 - r^2)(1 - k^2 r^2)}} \tag{2}$$

results, with the one complication that the second argument is complex.

[1] By R. A. Schmitz, published in the February, 1970, issue of the JOURNAL OF HEAT TRANSFER, TRANS. ASME, Series C, Vol. 92, No. 1, pp. 201–202.

[2] Associate Professor, University of Delaware, Newark, Del.

[3] Numbers in brackets designate Additional References at end of Discussion.

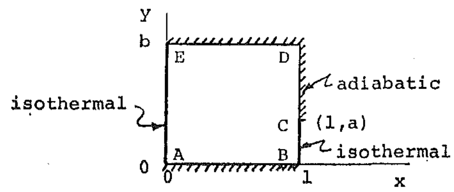


**Fig. 1 Typical section**

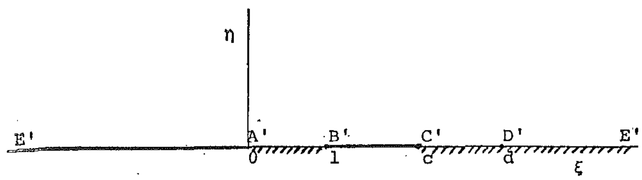


**Fig. 2 Mapping onto upper half-plane**

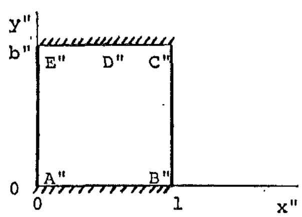


**Fig. 3 Inverse mapping**

The integral of equation (1) can be expressed in terms of elliptic integrals if the following transformation is used:

$$\tau = r^2$$

Then, for $\tau < 1$ and for $\tau = 0$ corresponding to $z = 0$,

$$z = \frac{2K}{\sqrt{d}} F\left[\frac{1}{\sqrt{d}}, \sin^{-1}(\sqrt{\tau})\right]$$

For $\tau > 1$, a second transformation is required:

$$d - r^2 = (d - 1)w^2$$

so that

$$z = \frac{2K}{\sqrt{d}} F\left(\frac{1}{\sqrt{d}}, \frac{\pi}{2}\right) - \frac{2K}{\sqrt{d}} i \left[F\left(\sqrt{\frac{d-1}{d}}, \sin^{-1}\left(\sqrt{\frac{d-\tau}{d-1}}\right)\right) - F\left(\sqrt{\frac{d-1}{d}}, \frac{\pi}{2}\right)\right]$$

For $z = 1$ to map onto $\tau = 1$ and for $z = 1 + ib$ to map onto $\tau = d$,

$$\frac{\sqrt{d}}{2K} = F\left(\frac{1}{\sqrt{d}}, \frac{\pi}{2}\right) \quad (3)$$

$$b = \frac{2K}{\sqrt{d}} F\left(\sqrt{\frac{d-1}{d}}, \frac{\pi}{2}\right) = F\left(\sqrt{\frac{d-1}{d}}, \frac{\pi}{2}\right) \Big/ F\left(\frac{1}{\sqrt{d}}, \frac{\pi}{2}\right) \quad (4)$$

The location of $C'$ can be determined from the known value of $z(= 1 + ia)$ at $C$:

$$a = \left[F\left(\sqrt{\frac{d-1}{d}}, \frac{\pi}{2}\right) - F\left(\sqrt{\frac{d-1}{d}}, \sin^{-1}\sqrt{\frac{d-c}{d-1}}\right)\right] \Big/ F\left(\frac{1}{\sqrt{d}}, \frac{\pi}{2}\right) \quad (5)$$

The conductance of the strip heater as shown in Fig. 1 can be determined from the above transformation as follows:

1 The value of $d$ is determined from equation (4) for the known value of $b$.
2 The value of $c$ is determined from equation (5) for the known values of $a$ and $d$.
3 The mapping in the $(\xi - \eta)$ plane, Fig. 2, is transformed via the same equations to the $(x'' - y'')$ plane shown in Fig. 3 to a problem with known conductance. The location of $b''$ is found from equation (4)

$$b'' = F\left(\sqrt{\frac{c-1}{c}}, \frac{\pi}{2}\right) \Big/ F\left(\frac{1}{\sqrt{c}}, \frac{\pi}{2}\right)$$

4 The conductance of Fig. 1 is calculated from [2]

$$\frac{Q}{k(\Delta T)} \equiv S = b''$$

It appears that the above solution is the only closed-form solution to the subject problem expressible in terms of tabulated functions.

## Additional References

8 Schmitz, R. A., "Heat Flux Through a Strip-Heated Flat Plate," Journal of Heat Transfer, Trans. ASME, Series C, Vol. 92, No. 1, Feb. 1970, pp. 201–202.
9 Carslaw, H. S., and Jaeger, J. C., "Conduction of Heat in Solids," Oxford University Press, Oxford, England, 1959.

# Taylor-Goertler Vortices and Their Effect on Heat Transfer[1]

**Arun S. Mujumdar.**[2] Although boundary layer development over curved surfaces is of great practical importance, to my knowledge there are no accurate prediction methods available which incorporate transverse and/or streamwise curvature effects. The effect of surface curvature is, of course, a special case of the more general effect of streamline curvature [12].[3] The curvature effects could be significant even on plane walls near separation. The problem dealt with by Professor McCormack and his coworkers is, therefore, a significant contribution to the existing literature, and will provide guidelines for future work in this area.

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[1] By P. D. McCormack, H. Welker, and M. Kelleher, published in the February, 1970, issue of the Journal of Heat Transfer, Trans. ASME, Series C, Vol. 92, No. 1, pp. 101–112.
[2] Mechanical Engineer, Research Division, Carrier Corp., Syracuse, N. Y.
[3] Numbers in brackets designate Additional References at end of Discussion.