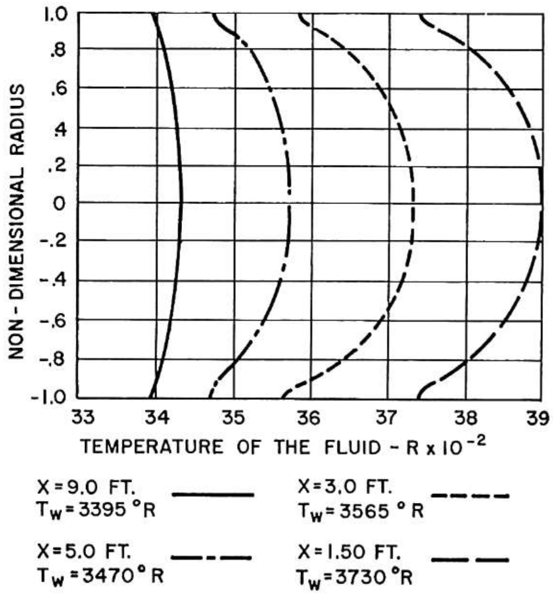

Fig. 5 Radial temperature profile for specific T_w and x

ψ_1 = $10^8\sigma\epsilon$
σ = Stephan Boltzmann constant
ϵ = emmissivity
A_{I} = cylindrical area

and

A_{II} = surrounding room area. Since the ratio A_{I} to A_{II} is small the term $\frac{A_{\text{I}}}{A_{\text{II}}} - \left(\frac{1}{\psi_{\text{II}}} - \frac{1}{C_b}\right)$ is neglected.

The results obtained for the case where $\alpha = 0.7$ are presented in Fig. 4 as plots of axial temperature distribution along the pipe. Radial temperature profiles located at certain axial locations are presented in Fig. 5.

Discussion of Results

The heat losses from a poorly insulated pipe were determined for a fully developed turbulent flow. Along the inner wall, neither the heat flux nor the temperature variation is known in advance, since they are determined by the losses from the outer wall. Consequently, at each axial position, it was necessary to match the heat flux with the corresponding wall temperature variation. In addition, the radial temperature distribution at the entrance to the pipe was assumed to be uniform. The accuracy in approximating this inlet condition was dependent upon the number of eigenfunctions used. With only four eigenfunctions the accuracy was poor near the wall but excellent near the center line. However, the accuracy of the radial distribution at the wall improves considerably at subsequent axial positions.

For the numerical example chosen, the fully developed thermal field is attained when the gas has passed through 136 tube diameters. Also, the center line temperature dropped rapidly from its maximum value in 96 tube diameters, then asymptotically approached a minimum value, while the corresponding radial distribution became uniform.

Acknowledgments

The author expresses his appreciation to Mr. H. E. Fettis from the Aerospace Research Laboratories for many helpful suggestions in formulating the analytical expressions and adapting these to numerical computations. Also appreciated are the efforts of Mr. J. C. Caslin of the same laboratories for carrying out the computations. I also express my indebtedness to the Department's Clerk-Stenographer, Miss Geraldine Adams for her generous assistance in preparing the manuscript.

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DISCUSSION

John C. Chato[2]

The method presented may be applicable to moderate temperature changes, but the assumption of constant properties is very questionable for the large temperature ranges given in the example and Fig. 4. An explanation of how the numerical values of the properties were determined and how their variation affects the calculations would be quite helpful. Fig. 4 and the discussion also seems to indicate that the temperatures of the fluid and the external wall reach constant temperature levels of about 3380 deg R and 705 deg R, respectively. With such temperature differences, the heat losses should be still appreciable and, therefore, the temperatures should continue to drop. As a matter of fact, all temperatures should approach the ambient value of 540 deg R at least at large enough axial distances. For the entrance portion of the tube shown in Fig. 4, the center line and internal wall temperatures indicate increasing heat loss from the fluid. The differences between the inner and outer temperatures of the wall, however, become less in this regime and seem to indicate a decrease of the amount of heat transferred. Some explanation of these results would also be quite necessary.

H. C. Perkins,[3] J. W. Mitchell,[4] and G. E. Myers[5]

If one studies the author's example as shown in Figs. 4 and 5 several questions arise concerning the results. These questions are based only on the physical results, but they appear to cast some doubt on the mathematical analysis.

If the author's results of Fig. 4 are replotted as shown in Fig. 6, the results appear very unrealistic physically. The fluid temperatures are seen to decay asymptotically to approximately 3400 R, and the external wall temperature to approximately 700 R. This asymptotic condition is reached in spite of the fact that the temperature difference between the gas and the ambient at 14 ft (approximately 2960 R) is nearly as large as the temperature difference at zero ft (approximately 3460 R). Could the author explain why, with this still large temperature difference, the center line and inside wall temperatures reach an asymptote and become equal, rather than decreasing to 540 R? This equality of fluid temperature implies that there is no heat transfer from the gas, which appears unlikely in view of the large temperature difference between the gas and the ambient that still exists.

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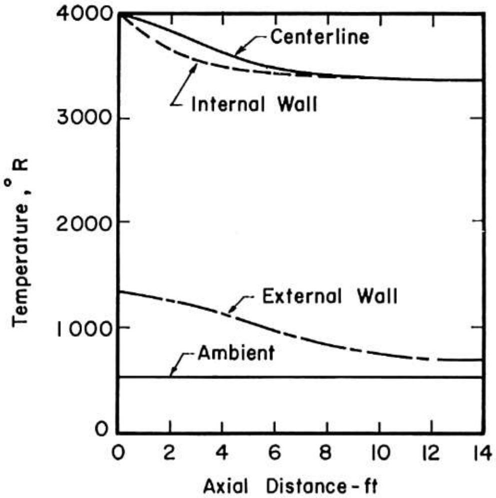

Fig. 6 Temperature distribution versus axial distance

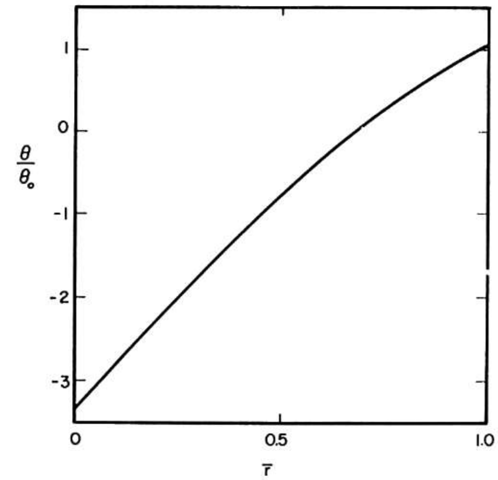

Fig. 7 Initial temperature distribution

In addition, it is seen that the temperature difference across the wall is approximately constant (2700 R), while that from the wall to the ambient decreases from 760 R at zero ft to 160 R at 14 ft. In view of the assumption made by the author that the overall thermal conductance is constant, this would imply that the heat flow through the wall is a constant, while the heat flow from the wall to the ambient decreases with distance. Could the author resolve this apparent anomaly?

If the solution, author's equation (56), is evaluated at $x = 0$, the resulting temperature profile does not correspond to the assumed uniform entering temperature. The apparent initial condition is plotted in Fig. 7. In addition, the large negative values are unrealistic.

In Fig. 5 the gas temperature profiles appear to give an almost zero temperature gradient at the wall. Physically one knows that the eddy diffusivity approaches zero at the wall so that a large temperature gradient is required to sustain the heat flux indicated by the profiles on the figure. Could the author comment on this discrepancy?

Lastly, could the author state his criterion for a "fully developed thermal field?" The 136 diameters mentioned in the paper appears very long for a combined thermal and hydrodynamic entry length, as is the case physically occurring in the example.

D. M. McEligot[6]

While others during the oral discussion have wondered about the validity of the general solution in this paper, I would first like to ask for clarification of some points regarding the numerical example. Without an indication of tube diameter, one cannot relate the dimensional distances to the nondimensional conclusions, e.g., 136 tube diameters for development of the thermal field. It also appears that the initial temperature distribution in the insulation will have a significant effect on the heat transfer in the central tube; this distribution is not specified.

The respective temperature differences in the example show the external resistance to be about ten times the internal convective resistance. Thus the two-dimensional variations within the tube are of second order of importance. I suspect that the experimental evidence shown in the presentation would agree as well with a simplified one-dimensional, or "thin rod," analysis based on a constant, approximate value of the internal heat transfer coefficient. Alternatively, if the Biot number based on the outside heat transfer coefficient is too high, then the appropriate system of elliptic partial differential equations must be solved for the insulation. In either case, the example appears to be predominantly a conduction problem.

Referring to the method of solution for a case with dominant internal resistance, I feel a more general result could be evolved by the approach of Sleicher and Tribus with appropriate modifications. For steady flow with constant fluid properties, negligible axial diffusion, a constant external heat transfer coefficient, and a fully established velocity profile, the energy equation can be written as

$$\frac{1}{r}\frac{\partial}{\partial r}\left[(\alpha + \epsilon)\frac{\partial T}{\partial r}\right] = u\frac{\partial T}{\partial x}$$

for both the fluid and its surroundings. Since the solution for the insulation and environment is known under the given conditions, they can be represented by a "frozen" region of the fluid extending from R to a fictitious radius, r_{eff}. The mathematical statement is then the turbulent Graetz problem with a velocity profile:

$$u = u(r, \text{Re}) \quad \text{for} \quad 0 \leq r \leq R$$

$$u = 0 \quad \text{for} \quad R \leq r \leq r_{\text{eff}}.$$

By studying the rate equations for the surroundings, we see that r_{eff} may be given by

$$\frac{2\pi k_{\text{fluid}}}{\ln\left(\dfrac{r_{\text{eff}}}{R}\right)} = \frac{1}{\sum\limits_j \dfrac{\ln(r_{0j}/r_{ij})}{2\pi k_j} + \dfrac{1}{2\pi r_1 \bar{h}}}$$

Useful entering conditions for $r < R$ are either (a) $T = T_0$, (b) a fully established cooling temperature profile for the entering Reynolds number, or (c) a fully established heating temperature profile at the start of the cooling. The second case might permit an approximate means to allow for fluid and insulation property variation; as suggested by Professor Sparrow, axial zones along the tube could be used for a "piecewise constant" treatment.

Since the case of dominant internal resistance is more likely to occur with gas as the fluid than with liquids, a number of operationally useful solutions can be obtained by the approach of Sleicher and Tribus or by implicit finite difference techniques (the explicit approach would probably be oscillatory in the "frozen" layer). For entry condition (a), by holding the Prandtl number at 0.7, the significant nondimensional parameters would be only the Reynolds number and the radius ratio, r_{eff}/R. It is hoped that this discussion will stimulate such work.

One additional caution comes to mind regarding the author's paper and the approach suggested by this discusser. While neglecting the velocity development still allows reasonable downstream predictions for described wall heat flux boundary

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conditions, it is not clear that the same philosophy can be adopted for a cooling situation with prescribed "surface" temperatures. In the latter case, errors introduced at the inlet will be propagated downstream through the analysis, since the overall energy balance will remain in error. Thus an additional safety factor should be used in design calculations for cooling unless there is an adiabatic entry of sufficient length.

Author's Closure

Comments to a Discussion Submitted by Dr. John C. Chato

A simple way to estimate the influence of property variation across the flow of a gas in a circular tube is to examine the local wall to bulk temperature difference. The bulk temperatures were obtained by numerically integrating the radial temperature profiles at selected axial locations and the corresponding temperature differences are tabulated in the following:

x	T_{wi}	T_b	$T_b - T_{wi}$	T_c
0.5	(3890)[(a)]	3957	67[(a)]	3975
1.0	3810	3870	60	3925
2.0	3660	3702	42	3800
5.0	3460	3470	19	3550
8.0	3395	3407	12	3440
11.0	3370	3379	7	3390

[(a)] Value obtained by extrapolating the temperature profile to the wall.

Since the temperature differences $(T_b - T_w)$ are not large and do not change drastically, one assumes that the flow is not greatly affected by the degree of property variation present. The fluid properties were evaluated at the center-line temperatures.

Replying to the question about temperature drop between the inner and outer surfaces of the pipe, since the problem considered was for steady flow and heat transfer in an insulated pipe, then at a given axial position, the gas temperature could never approach the ambient temperature.

Referring to the discusser's comment concerning the entrance portion of the tube, the paper stated that by choosing only four eigenfunctions the approximation of the inlet profile was poor, which is usually the case with all eigenvalue problems. This error occurs near the pipe wall, but decreases at succeeding stations in the axial direction. Thus better wall temperature gradients can be obtained by extrapolating the temperature profiles to the wall. Also the heat lost from the gas continuously decreases from $x = 0$ as is demonstrated by examining succeeding axial $T_b - T_{wi}$. The bulk temperature is more representative of the heat loss, since the outer portion of the fluid loses most of the heat near the inlet. This behavior is demonstrated by the temperature profiles which first change from a constant to a parabolic shape and these at large axial distances again approach a uniform constant profile.

Comments to a Discussion Submitted by Dr. John W. Mitchell

The numerical example in the paper 65—WA/HT-16, with the results as shown in Figs. 4 and 5 was chosen to provide a comparison of the analysis with existing experimental data. This particular case was that of a pipe with considerable insulation around the thermal section. Thus, for the purpose of clarity, let us first consider the pipe as being perfectly insulated some distance away from the thermal inlet. At this position the temperature profile would approach a uniform profile due to the radial thermal conduction and turbulent mixing in the gas. Thus the center line and inner wall temperature approach each other. Now, when the wall is not perfectly insulated, the heat flux at the wall must be matched with the temperature of the gas and the heat flux through the wall (for constant wall conductance) will decay asymptotically in the axial direction to some value that is determined by the insulation, the ambient temperature, the gas temperature, and the heat transfer processes at the outer wall. Also, the temperature profile will first deviate from the uniform entering profile due to the losses at the wall and then again approach a uniform profile, but at a greater distance than if the wall were perfectly insulated, due to the radial conduction and turbulent mixing in the gas. In addition, even though the wall conductance is constant in the axial direction, the heat flow and temperature difference through the wall is not constant but decreases due to the temperature drop in the gas, and this heat flow through the wall is influenced by the changing free convection and radiation at the outer wall.

Since the problem considered was for steady flow and heat transfer, then at a given axial position the gas temperature could never approach the ambient temperature. In the case of a perfectly insulated pipe, larger differences between the gas and ambient temperatures exist.

In reference to the discusser's comment about evaluating equation (56) at $x = 0$, it was stated in the paper that by choosing only a few eigenfunctions the approximation of the assumed inlet profile was poor, which is usually the case in eigenvalue problems. The same type of numerical error occurs near the pipe wall, but the error decreases at succeeding stations in the axial direction. Thus better wall temperature gradients can be obtained by extrapolating the temperature profiles to the wall.

"Fully developed thermal field" in this paper is defined as occurring at the approximate axial position where the temperature profile is about uniform and the change in temperature along the pipe becomes small as compared to the inlet temperature. The velocity profiles were assumed to be fully developed prior to entering the cooling region and to be unaffected by the subsequent heat loss. Thus 136 diameters refers only to the development of the temperature field.

Comments to a Discussion Submitted By Dr. D. M. McEligot

Omitting the tube diameter when relating the dimensional distances to nondimensional conclusions was an oversight on my part. The tube diameter is 0.75 in. and the entire pipe diameter including the concentric layers of insulation is 16 in.

Referring to the comment on the initial temperature distribution in the insulation, in the problem it was assumed that $T_\infty < T_{\text{wall}} < T_{\text{hot}}$ gas. The temperatures and temperature gradients were matched at each surface and the flow and heat transfer were steady; therefore, the initial temperature distribution in the insulation did not enter the problem.

I agree with the comments given in the second paragraph of the discussion. The general problem was to examine the temperature behavior in the hot gas due to a poorly insulated pipe. However, the particular example was chosen for comparison with existing experimental results.

On page 8, the 6th paragraph, the comments are very interesting. However, I cannot make any comment on the comparison of methods until the details of the suggested method are made available to me. I would appreciate the discusser forwarding some references concerning the suggested method if any are available.