If the wall reflects specularly, radiation is reflected only at an angle $\sigma$ with respect to the normal, and the reflected flux at any position in the absorbing layer is given by $rE_0 \exp[-\alpha(\delta_0 + y) \times \sec\sigma]$. The rate at which energy is absorbed from the reflected beam per unit volume is $\alpha rE_0 \exp[-\alpha(\delta_0 + y)\sec\sigma]$. The total rate of energy absorption per unit volume for a reflecting wall is the sum of the rates of absorption from incident and reflected beams, so for a specularly reflecting wall the absorption term is

$$q_2''' = \begin{cases} \alpha E_0\{\exp[-\alpha(\delta_0 - y)\sec\sigma] \\ + r\exp[-\alpha(\delta_0 + y)\sec\sigma]\}, & y < \delta_0 \\ 0 & y > \delta_0 \end{cases} \quad (60)$$

The intensity of the radiation reflected in any direction at any point on a diffuse wall, is

$$I_w = \frac{rE_0}{\pi}\cos\sigma\exp[-\alpha\delta_0\sec\sigma] \quad (61)$$

since the wall is uniformly irradiated. So each unit area of a plane at $y$ is irradiated by diffusely reflected energy at a rate given approximately by $\int_{\Omega=2\pi} I_w \cos\Theta \exp[-\alpha y\sec\Theta]d\Omega$. (This is a very good approximation for thin absorbing layers.) The integrand is independent of azimuthal angle, so this integral can be put in the form $2\pi I_w \int_0^{\pi/2} \exp[-\alpha y\sec\Theta]\cos\Theta\sin\Theta\, d\Theta$. It follows that the rate at which diffusely reflected energy is absorbed per unit volume is $2\pi I_w \alpha \int_0^{\pi/2} \exp[-\alpha y \sec\Theta]\sin\Theta\, d\Theta$. The absorption term in the case of a diffusely reflecting wall is therefore

$$q_3''' = \begin{cases} \alpha E_0 \Big\{\exp[-\alpha(\delta_0 - y)\sec\sigma] \\ \quad + 2r\cos\sigma\exp[-\alpha\delta_0\sec\sigma] \\ \times \int_0^{\pi/2}\exp[-\alpha y\sec\Theta]\sin\Theta\, d\Theta\Big\}, & y \leq \delta_0 \\ 0 & y > \delta_0 \end{cases} \quad (62)$$

where equation (61) has been substituted for $I_w$. This expression can be simplified by defining $\zeta$ as equal to $\cos\Theta$, making a substitution of variables

$$Z = \frac{\alpha y}{\zeta} \quad (63)$$

and integrating by parts to get

$$\int_0^{\pi/2}\exp[-\alpha y\sec\Theta]\sin\Theta\, d\Theta = e^{-\alpha y} + \alpha y Ei(-\alpha y) \quad (64)$$

where the exponential integral $Ei(-\alpha y)$ is defined by

$$-\int_{\alpha y}^{\infty}\frac{e^{-z}}{z}dz.$$

The form of the absorption term for a diffusely reflecting wall becomes

$$q_3''' = \begin{cases} \alpha E_0\{\exp[-\alpha(\delta_0 - y)\sec\sigma] \\ + 2r\cos\sigma[e^{-\alpha y} + \alpha y Ei(-\alpha y)] \\ \times\exp[-\alpha\delta_0\sec\sigma]\}, & y \leq \delta_0 \\ 0 & y > \delta_0 \end{cases} \quad (65)$$

# DISCUSSION

## D. R. Jeng[4]

The authors are to be complimented on adding another piece of work to the literature regarding stagnation point heat transfer. The validity of the analysis presented in this paper is limited to the case where the properties of the injected fluid are identical to or not much different from those of the free-stream fluid. However, I believe that, even if the injected fluid is the same as the free-stream fluid except for a large number of absorbing particles dispersed in the former, the properties may differ greatly.

In connection with the evaluation of the integrals which appear in equation (43), it may be worth noting that they most likely can be integrated by using the method of steepest descent.[5]

As an alternate method of evaluating the double integral which appears in the numerator of equation (43), we first deduce the power series solution of $f(\eta)$ in the form

$$f(\eta) = \sum_{n=0}^{\infty}\frac{a_n\eta^n}{n!} \quad (66)$$

By successive differentiations of equation (14), using the boundary conditions (15) and (16), we find

$$a_0 = f_w = f(0), \quad a_1 = 0, \quad a_2 = f''(0), \quad a^3 = -a_0a_2 - 1, \quad a_4 = -a_0a_3,$$

etc. The integrand involved in the outer integral may alternatively be written as

$$\exp\left\{-N_{\rm Pr}\left[\int_0^\eta f(\gamma)d\gamma - a_0\eta\right]\right\}\cdot\exp[-N_{\rm Pr}a_0\eta] \quad (67)$$

We observe that

$$\int_0^\eta f(\gamma)d\gamma - a_0\eta = \eta^3\sum_{m=0}^{\infty}B_m\eta^m$$

where

$$B_m = \frac{a_{m+2}}{(m+3)!}$$

Following Meksyn, we put

$$\eta^3\sum_{m=0}^{\infty}B_m\eta^m \equiv \xi$$

and obtain, from the theory of series inversion,

$$\eta = \sum_{m=0}^{\infty}\frac{A_m}{m+1}\xi^{\frac{m+1}{3}} \quad (68)$$

in which the $A_m$'s are the coefficients of $\eta^m$ in the expansion

$$\left(\sum_{m=0}^{\infty}B_m\eta^m\right)^{-\frac{m+1}{3}}$$

Thus,

$$A_0 = B_0^{-1/3}, \quad A_1 = -2/3B_0^{-5/3}B_1, \quad A_2 = B_0^{-1}\left[\left(\frac{B_1}{B_0}\right)^2 - \frac{B_2}{B_0}\right], \text{ etc.}$$

Next, we can reduce the integrand involved in the inner integral to a power series in $\beta$, integrate, change the variable from $\eta$ to $\xi$ by use of equation (68), expand the exponential $\exp[-N_{\rm Pr}a_0\eta]$ in equation (67) in a power series in $\xi$, combine with the inner in-

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[5] D. Meksyn, *New Method in Laminar Boundary Layer Theory*, Pergamon Press, 1961.

tegral, and follow by evaluating the outer integral in terms of the gamma function. To evaluate the sum of this series solution, use may be made of Euler's procedure[5] whenever it is necessary. The integral appearing in the denominator of equation (43) may be integrated in a similar manner as described for the numerator.

## Authors' Closure

The effect of particles on the properties of the mixture in the absorbing layer depends, of course, upon the number density of the particles. The experimental results of Lanzo and Ragsdale [3], as well as those of Marteney[6] for the opacity of suspensions of small carbon particles can be used to show that particle densities on the order of $10^{-4}$ slugs per cu ft of mixture and upstream velocities on the order of 10 fps will produce a value of $\tau_m$ on the order of unity. If the fluid is air at standard conditions, its density is on the order of $10^{-3}$ slugs per cu ft. Hence the ratio of particle density to fluid density is on the order of $10^{-1}$, which is relatively small. The specific heat of carbon in the neighborhood of room temperature is of the same order as that of air, so the specific heat of such a suspension would be approximately the same as that of air. The thermal conductivity would probably be affected little by the presence of such particles, because of the very small volume they occupy. The viscosity of the mixture is not a well-defined quantity. However, Marble[7] indicates that the effect of the particles on the motion of the fluid is very small if the ratio of particle density to a characteristic density of the suspension is much less than unity. This criterion appears to be satisfied by the above ratio, $10^{-1}$.

The alternate methods suggested by Professor Jeng for evaluating the integrals in equation (43) may well be suitable. However, it is felt that the method outlined in equations (44) through (52) has several features which recommend it. It is simple and straightforward. It provides results for temperature distribution as well as the derivative of the temperature at the wall. Finally, it utilizes a method of integration for which the error can be estimated.

[6] P. J. Marteney, "Experimental Investigation of the Opacity of Small Particles," NASA Contractor Report CR-211, April, 1965.

[7] Frank E. Marble, "Dynamics of a Gas Containing Small Particles," in *High Temperature Phenomena,* Fifth AGARD Colloquium on Combustion and Propulsion, Macmillan Co., New York, N. Y., 1963.