At low mass velocities, the change in flow level is small for a wide range of quality; however, at large mass velocities, there is found to be considerable increase in the flow level as the quality decreases. These results stand in contradiction to the results found earlier by Chaddock and later by Chato who have based their analyses on a different flow model which predicts that the condenser tube would flow less full at the outlet end.

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DISCUSSION

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There are at least two conditions which may exist near the end of a horizontal condenser tube pass. The first is a condition where the liquid completely fills the tube. It forms the starting point for the analysis by the authors of this paper. The second is a condition where the condensate flows in the bottom portion of the tube, with an essentially, stagnant blanket of vapor over it, and never completely fills the tube. The negligible vapor velocity leads to a smooth vapor-liquid interface, i.e., stratified flow.

That the second condition can be physically realized (contrary to the statements of the authors of this paper) there is no doubt. It was observed by Chaddock [9] and observed, measured, and photographed by Chato [10] in an experimental condenser tube. Since the shear force at the interface is negligibly small, the energy required to overcome friction, and to discharge the condensate at the tube end, can come only from a decreasing hydrostatic head of liquid. In a 28.25 in. long, 0.572 in. ID tube with Refrigerant-113 condensing, the *decrease* in the condensate angle φ was found to be between 5 and 20 degrees for condensing conditions which produced our flow model [10].

The beginning point of our analyses was also the writing of the basic continuity, momentum, and energy equations. The momentum equation considered the important forces acting on both phases of the fluid, but the vapor and liquid were treated separately. Upstream from the discharge end Chato's analysis also included the interface shear.

Under the physical conditions for which our model prevails, the outlet geometry *does influence the upstream flow*, insofar as it controls the mode of liquid discharge. According to well-developed theories in hydraulics this control exists if the flow is "subcritical," i.e., it is slow and deep as it approaches the discharge. Consequently, if the tube is horizontal, the outlet can indeed control the flow upstream. With a slight downward slope, however, the flow becomes "supercritical." In this case the outlet no longer influences the depth of flow, and the establishment of a control point becomes extremely difficult, as was explained by Chato.

The authors attempt to establish a different control point in their analysis by assuming the boundary condition that the condensate depth is zero at the tube inlet. They state that our assumption of an initial condensate height is physically unrealistic, whereas, it seems to us exactly the opposite is true. In early experiments simulating condensation with negligible vapor velocity, the formation of a liquid level at the tube inlet was readily achieved [9].[9] Further, visual observations by one of us [10] at the very inlet of a condenser tube showed the liquid depth was always finite under conditions which produced stratified flow. When the vapor flow was extremely high, this depth became very shallow, but in this case the flow was more annular than stratified. The almost infinite slope of the liquid level at the tube inlet, as shown in Figs. 5, 6, and 7, under the assumed conditions of stratified flow with negligible vapor shear on the condensate, must surely be physically unrealizable. The authors' zero depth assumption may still be usable, however, since the calculated depths downstream may not be too sensitive to the actual depth assumed at the inlet. This may be compared with a similar tendency in laminar boundary layers near a leading edge.

Consider now the first condition stated in the foregoing, where the tube completely fills with condensate. Practical condenser applications assure this condition must certainly prevail, but the conditions under which it does, require careful consideration.

First it should be ascertained whether the tube is capable of discharging the total amount of liquid condensed without filling

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[9] Also, J. R. Dixon, "Condensate Flow in Horizontal Tubes," MS thesis, M.I.T., May, 1953.

the tube. A method is presented in reference [10]. If the condensate flow is too high or the end geometry does not allow a free discharge, then the tube will indeed fill up completely. The solution presented by this paper approaches the fill-up point with a positive slope and actually reaches it with an infinite slope. The maintenance of a positive slope on the liquid surface requires either an adequate negative pressure drop in the vapor phase or an appreciable interface shear or both. In the authors' model, the shear, however, was assumed to be negligible. The negative pressure drop then could come only from a rapid enough increase in the liquid depth to cause adequate acceleration of the vapor. Whether the vapor shear remains negligible under such circumstances should be demonstrated. From physical considerations the question arises whether the flow really remains stratified near the fill-up point or will it become, for example, slug flow?

It is regrettable that the authors chose not to present a brief description of their experiments and their experimental results with condensation occurring in a tube. It is our understanding, however, that some of these data will be included in the final form of the paper and that these agree reasonably well with the analytical calculations. We presume that some of their assumptions are based on their own experimental results.

In regard to the hold-up data it should be noted that the experimental points shown in Fig. 3 from [22][10] must represent mean values of the depths since the original data show definite variations in depth along the length of the tube. The authors' assumed condition that if the condensation rate is zero the flow level remains constant [equation (37)] is satisfied only if all shear, including wall shear acting on the liquid, is negligible.

In connection with the effect of tube inclination a result of reference [10] was that as the downward slope increased, the heat transfer rate went through a maximum, usually in the range of 10 to 20 deg downward angle, beyond which it decreased until it reached the rate corresponding to the vertical tube arrangement. This last value was in general below that obtained in the horizontal configuration.

It would be useful if the authors would give some details on the values for the α's and β.

The work in reference [10] was stated to be usable below an entrance vapor Reynolds number of 35,000. Beyond this value the flow was essentially annular rather than stratified in a considerable portion of the condenser tube. The regime in which the present analysis is valid should be ascertained more precisely.

Authors' Closure

The authors thank Professors Chato and Chaddock for offering a stimulating discussion and for raising the points that they do. It is our aim in this closure to bring these points into proper perspective. To begin with, it should be kept in mind that we are dealing here with a condenser tube which may be part of a vapor compression refrigeration system or any other two-phase stratified flow system in which condensation of the vapor phase takes place at the inside of the tube and under an *imposed pressure gradient*. It is precisely this pressure gradient which makes the difference in the flow models. The discussers in their remarks refer to the observation of a different condition at the outlet of the tube, which is that of a partially filled cross section, in contrast to the outlet condition employed in the present treatment. This possibility was certainly not overlooked by the writers who are attempting to define more precisely a model agreeing with the physical behavior of a condenser tube. Their concept of the outlet was indeed avoided because it is not considered to be realistically related to the physical problem being treated in this paper. They have treated a condensation process taking place inside a tube in which no imposed pressure gradient acts. With this in mind let us now look at the conditions under which Chato and Chaddock have observed, measured, and correlated the depth of the condensate flowing at the bottom of a horizontal tube and discuss them in light of the conditions prevailing in an operating condenser tube.

Chaddock checked his analysis using the flow experiments reported by Dixon. In Dixon's experiments, measured quantities of water were uniformly supplied along the length of a tube which was open at the top (an open channel flow) and the variation in flow depth was measured at different positions. It is clear that these experiments were not actual two-phase flow experiments but they do represent an attempt at simulating condensation and condensate flow which they may do under conditions of zero imposed pressure gradient.

Chato, in his experiments, employed a modified form of the experiment of Dixon. In the modified arrangement, which he referred to as a fluid-mechanics analogy, the tube was not open at the top but means were provided at 12 points along its 54-in. length for admitting water on both sides and for venting air at the top. Although means were provided for supplying air at the inlet of Chato's tube and also for bleeding it off at the 12 points along the tube, as mentioned, this crucial feature of the setup unfortunately *was not utilized*. If this feature had been used, a closer simulation of the behavior of a condenser tube probably would have been achieved. Since it was not, flow-level measurements made using this simulated experiment are not representative of two-phase flow conditions but rather of channel-type flow such as that found in a sewer pipe. In view of these considerations, it is not surprising at all that the measurements taken under the experimental conditions of both Dixon and Chato are reported to show a decreasing liquid level with increasing tube length. Furthermore, one does not doubt that the flow level in the type of experiments they discuss and use is found to depend on the conditions at the outlet of the tube, just as is the case for flow in an open channel.

Chato certainly recognized at one time that his flow-analogy experiment was lacking in certain basic respects in its correspondence to actual flow of condensate inside a horizontal condenser tube because in his dissertation (reference [10], p. 89) he states: "It (the analogy) could not fulfill all the requirements of a really close analogy of the condensation process, but it provided very useful quantitative and qualitative results that could not be obtained from a condensation experiment." It seems that a properly conceived condensation experiment would provide the most useful results because it would deal with the physical phenomenon at first hand. We are in absolute agreement with Chato's observation that his fluid-analogy experiment did not simulate the process of condensation and condensate flow and this is exactly why we have made an attempt in the present work to build a flow model which describes the real physical situation more accurately.

In the condensation experiments used to supply a check for the theory presented here, a complete refrigeration system with a precondenser and test and after condensers, evaporator and a variable speed compressor and an oil separator was used. The test section consisted of a water-jacketed copper pipe, $^1/_2$-in. in dia and 24-in. long, having glass sections at both inlet and outlet. Sixteen thermocouples soldered to the tube surface allowed the determination of the average tube wall temperature. With the precondenser, any condition ranging from a quality of unity to zero could be established at the inlet of the test section. Also measurements of the flow-level angle could be made at both glass sections. Measurement of the flow-level angle was accomplished using the following arrangement: The glass tube was provided with 0-ring seals and so arranged that it could be rotated. A line parallel to the axis of the tube was etched on the inside tube surface. An accurate protractor rigidly fixed to the downstream coupling of the tube was used in conjunction with a pointer fixed to the body of the tube. The flow-level angle is measured by lining up the line inside the glass tube with the edges formed by the liquid-vapor interface and the glass wall and reading off the two angles indicated by the pointer on the protractor. This method of measuring the flow-level angle is not affected by optical

[10] See also: O. P. Bergelin and C. Gazley, Jr., "Co-Current Gas-Liquid Flow, I. Flow in Horizontal Tubes," Heat Transfer and Fluid Mechanics Inst., 1949, pp. 5–18.

distortion caused by the extra heavy curved glass surface. Data points taken in this experimental apparatus using R-22 as refrigerant are shown in Fig. 2 for the special case of zero rate of condensation in the test section itself. Additional data (measured at the outlet of the experimental condenser) are shown in Fig. 9 for a temperature difference of 12 deg in the test section.

Since the discussers analyzed the condenser-tube problem from the viewpoint of an open-channel type of flow while we have analyzed the same problem on the basis of two-phase flow principles, one ought not expect to find much in common between the two sets of results except perhaps when the pressure gradient is very small so that the gravity forces predominate. This applies to the effect of outlet geometry on the upstream flow level as well as to the maximum height of flow level at the inlet.

Let us first look at the conditions at the *inlet* of the tube. The discussers dispute our statement that their assumption of maximum condensate depth at the inlet is physically unrealistic, and in turn, they claim that our assumption of zero depth is unrealistic. Fig. 10 is a schematic drawing of the situation being discussed. If one considers a horizontal tube, part of which is insulated as indicated in Fig. 10 by the cross-hatched portion while the other part is held at a temperature below the saturation temperature the film condensation on the wall of the tube can take place. The solid line inside the tube corresponds to the condensate profile as apparently proposed by Chaddock and Chato (see also Fig. 3 of reference [9]) while the dashed line corresponds to flow levels calculated by the present theory. It is indeed very difficult if not impossible to visualize the existence of a vertical wall of liquid rising approximately one half the tube diameter at the beginning point of condensation as the discussers suggest. One is forced to ask the question: What keeps this vertical wall of liquid from collapsing or from being blown away? Such an abrupt rise of flow level has never been observed and we know of no other observation to substantiate this view and furthermore surface tension cannot account for this strange behavior if one thinks of it as a possibility. The description of the flow level given by Chato in the discussion of this paper is perplexing because his description seems to correspond more to what has been observed in this study and to what the present analysis predicts rather than to the flow model depicted in Fig. 3 of reference [9] and reproduced in Fig. 10 by the solid drawn line.

A typical flow level obtained from the present analysis is indicated by the dashed line in Fig. 10 and it is seen that at $z = 0$ (the beginning point of condensation) the slope is infinite, however, at the position dz the slope is finite and the function describing the slope is continuous. Furthermore, we would like to recall that the flow-level equation (29) has a singularity at the point $z = 0$ and, as has been described in the text, this point was excluded. The procedure proposed for evaluating the flow-level equation at this point has also been described in the text.

It can be easily shown that the flow-level profile indicated by the dashed line in Fig. 10 is indeed a typical one by making a careful study of Fig. 5. Consider for example, the curve for 50 lb_m/hr of R-22 flowing inside the 0.625 in. ID tube with a $(t_{sat} - t_w) = 12$ deg F. At the position $z = 1$ ft from the inlet, the flow-level angle is found to be approximately 90 deg corresponding to a condensate height of 0.092 in. At a position $z = 2$ ft, the flow-level angle is 105 deg corresponding to a condensate flow depth of 0.125 in. While the gradient is indeed steep near the inlet and decreasing with increasing tube length except near the position where the quality is very small and the tube runs practically full, the condensate flow depths involved are quite shallow and certainly physically realizable. One suspects that the discussers may have failed to convert the flow-level angles to flow heights in order to correctly visualize the flow situation. It should also be stressed that while the effect of the vapor shear acting on the thin condensate film is assumed negligible, the same vapor shear acting on a deep flow of a condensate is not assumed negligible and it appears that the discussers failed to grasp this point because the interaction between the two phases is accounted for by the use of the two-phase pressure drop correlation. Nevertheless, when it was convenient to assume vapor shear to be negligible (after the use of the two-phase pressure drop correlation) in order to simplify parts of the analysis without causing much loss in accuracy, this has been done, as shown by equation (37). Furthermore, near the position where the tube runs full, the flow changes from stratified to plug flow and the model assumed ceases to be valid and therefore this end point (see equation (34)) has been excluded, as already stated.

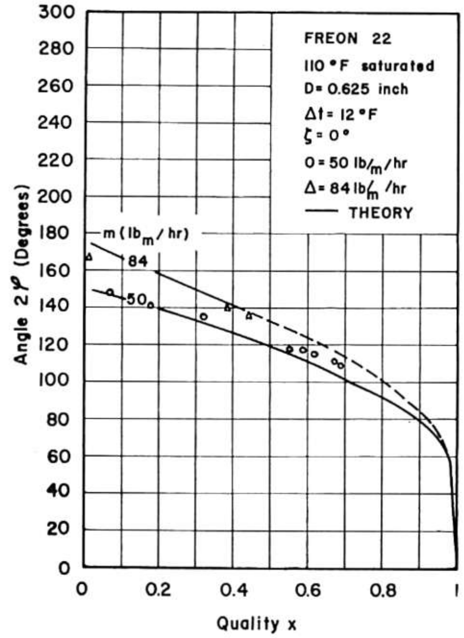

Fig. 9 Vapor quality versus flow-level angle during condensation of refrigerant 22. Comparison of theory with experimental data.

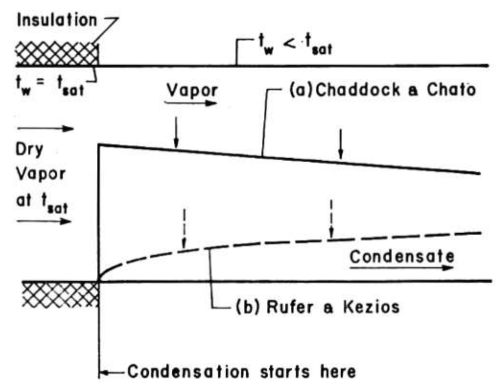

Fig. 10 Schematic diagram of condensate profile at inlet to condensate tube. (a) Chaddock & Chato (reference [9] Fig. 3) (b) Rufer and Kezios.

We are in perfect agreement with Professors Chato and Chaddock as well as with a number of other investigators before them regarding the effect that the outlet geometry has on an open-channel flow, but we cannot agree that the identical effect is present in two-phase flow. It is clear, however, that at extremely low mass velocities, much smaller than those usually employed in commercial refrigeration condensers, the two-phase flow may indeed approach that of an open channel and when this is the case, Chato's flow model may be employed as was already stated in the paper. In summarizing this point, for mass velocities where the shear effect is no longer of negligible magnitude, the pressure forces become much larger than the hydrostatic head forces and the flow cannot be regarded as an open-channel flow anymore.

Now let us focus our attention on the *outlet* conditions by considering a long horizontal tube of length $z = L$ followed by a *free* discharge as described by Chato. Assume that at some position z_1 upstream of $z = L$ all vapor has been condensed and further assume that the portion of the tube between z_1 and L is insulated so that no subcooling of the liquid can take place. If at the position $z = z_1$ all vapor is condensed then according to the present flow model, the portion of the tube between z_1 and L must run full of condensate. According to Chato, the tube is not supposed to run full, in fact the condensate has reached a height close to its minimum height. One is forced to raise the question: Since all vapor has been condensed into liquid, what occupies the space above the liquid? Once the tube is filled with liquid, it is rather obvious that the outlet geometry has no effect on the flow upstream. However, the flow downstream may be affected because due to pressure drop (liquid friction) some liquid could flash back into vapor unless at the same time a corresponding amount of heat is removed from the liquid. It can thus be seen that the fill up process may be quite complicated if the liquid is not allowed to become subcooled a few degrees, and a rather unstable condition may set in.

The discussers are quite correct in pointing out that Gazley's original data showed variations in depth along the tube; as an example, the case of turbulent-liquid, turbulent-gas flow, the flow depth decreased with increasing tube length. The data shown in Fig. 3 are the values measured at a location 10 ft downstream from Gazley's mixing zone. This was the *first* measuring station, the last one being 18 ft from the mixing zone. If the depths measured at the 18 ft location had been used instead, the discrepancy shown in Fig. 3 between the data and equation (39) would have been *reduced by nearly one half*! We chose not to do this because the original aim was to compare this theory with data in relatively short tubes such as are usually found in refrigeration condensers. It is therefore seen that in spite of the assumption made in equation (37) which lead to the relatively simple equation (39), excellent agreement with experimental data is obtained even if relatively short tubes are considered. This is quite remarkable because usually a considerable tube length is required to establish a stable flow pattern. It should also be noticed that in Fig. 3, the flow-level angle increases with a decreasing quality, a characteristic which is in agreement with the present theory, but which stands in disagreement with the findings of both Chato and Chaddock.

The optimum angle of inclination may be found by maximizing equation (32). With increasing angles of inclinations there is a decrease in the condensing heat transfer coefficient h_m (approximately 1.5 percent at $\zeta = 20$ deg) and an increase in the film angle Ω. If, as an approximation the relatively simple equation (38) is used instead of the more general equation (29) for the determination of the local values of $\Omega = \pi - \varphi$, the optimum inclination yielding the shortest tube length may easily be found by trial and error. For the cases considered here, the optimum inclination is from 10 to 15 degrees resulting in an approximately 25 percent reduction in the required tube length. However, a five degree inclination already achieves over 90 percent of the maximum possible reduction in tube length.

In response to the discussers remarks concerning the regime of validity of the present analysis and the suggestion that it be more precisely ascertained using as a criterion the entrance vapor Reynolds number (as was done by Chato in his work), it must be recognized that the Reynolds number is neither an adequate nor suitable criterion for determining the flow regime and therefore the range of validity of the present work. It is pointed out at several places in the paper (see also footnote 2) that the flow pattern correlation of Baker reference [14] may be used as a basis to establish the limits of applicability of the model employed to develop the present theory.