Concerning the total pressure loss in the vaneless diffuser, the following conclusions are obtained:

1 The major part of the total pressure loss is the wall friction loss, but the total pressure loss does not change in proportion to the wall friction coefficient, because the circumferential velocity decreases as the wall friction coefficient increases.

2 Unless the asymmetry of flow pattern is very severe, the total pressure loss in the vaneless diffuser is approximately equal to the sum of the friction loss of the axisymmetric flow and the mixing loss of nonuniform flow at the diffuser inlet.

3 The difference of the time average total pressure and the mass average total pressure increases with flow distortion. Therefore, if the total pressure at the exit of impeller is measured with ordinary instrumentation, the impeller efficiency is overestimated while the diffuser efficiency is underestimated.

4 The flow distortion quickly decays near the inlet of the vaneless diffuser, and as a result the time average total pressure decreases with radius. The radial distribution of the time average total pressure gives the impression that the total pressure loss is very large near the entrance region of the vaneless diffuser.

Acknowledgments

The authors are grateful to Dr. R. C. Dean, Jr., who presented stimulating discussions on their last paper. The wall roughness test of the present paper evolved from the discussion. They also thank Mr. M. Yamaguchi for the design and construction of the experimental apparatus as well as the instrumentation, and Mr. M. Ono for securing the experimental data.

References

1 Brown, W. B., "Friction Coefficients in a Vaneless Diffuser," NACA TN 1311, 1947.

2 Dean, R. C., Jr., and Senoo, Y., "Rotating Wakes in Vaneless Diffusers," TRANS. ASME, Series D, Vol. 82, 1960, pp. 563–574.

3 Johnston, J. P., and Dean, R. C., Jr., "Losses in Vaneless Diffusers of Centrifugal Compressors and Pumps," TRANS. ASME, Series D, Vol. 88, 1966, pp. 49–60.

4 Dean, R. C., Jr., "On the Necessity of Unsteady Flow in Fluid Machines," TRANS. ASME, Series D, Vol. 81, 1959, pp. 24–28.

5 Senoo, Y., and Ishida, M., "Asymmetric Flow in the Vaneless Diffuser of a Centrifugal Blower," *Proceedings of the 2nd International JSME Symposium, Fluid Machinery and Fluidics*, Vol. 2, Tokyo, 1972, pp. 61–69.

DISCUSSION

R. C. Dean, Jr.[2]

I wish to compliment the authors for this most important and excellent contribution to the physics of distorted flow leaving centrifugal impellers. The losses which can be attributed to this discharge process are significant. What are commonly called "impeller discharge mixing losses" often amount to 2 points of stage efficiency and, in extreme cases, have been calculated to be 10 points. Because the flow from a high performance centrifugal compressor apparently is always severely distorted, as a result of intense relative-flow diffusion within the impeller and the three-dimensional nature of the impeller, an accurate means for calculating discharge losses is essential to optimization of stage design.

There has been an on-going scientific debate between me and Professor Senoo for over a decade concerning means for *practically calculating* these discharge losses.

Further, in order to adequately predict fluid-dynamic interaction between an impeller and a vaned diffuser, the noise generation of centrifugal compressors and vibratory stresses in diffuser and impeller vanes, a competent theory for the *decay* of the impeller-discharge asymmetry is essential. Professor Senoo and I published a theory for the discharge mixing process (authors' reference [2]) in 1960. This theory included wall friction, friction between the jet and wake, and reversible work exchange between jet and wake. It led to both a prediction of the net loss and the history of the flow passing outward through a vaneless diffuser. A few years later, Johnston and I published a simple means, called the Sudden Expansion Analysis (SEA), for calculating discharge mixing losses, following a suggestion of Mellor (authors' reference [3]). The SEA has been widely used by centrifugal compressor designers/developers in their flow models.

The central question raised in this paper by the authors is whether the SEA model is adequate.

By the employment of refined, unsteady flow measurements, the authors have produced, for the first time, competent data which can reveal the true nature of the process. Further, they have carefully compared the results of the Dean and Senoo theory with the Johnston and Dean theory and against data for two cases: one purposely distorted in high degree to emphasize differences and the other typical of practical compressors.

Professor Senoo's principal complaint about the SEA model is that it does not incorporate reversible work exchange between the jet and wake in the vaneless space. That work was shown by the Dean and Senoo theory to be significant. In contrast, the SEA theory is nothing more than sudden-expansion mixing plus uniform flow wall friction, which uses the angular and radial momentum equations in order to predict the ultimate axisymmetric state. While Senoo's criticism is justified, the fact remains that Johnston and Dean showed a correspondence within 2 percent of the overall loss predicted by the Dean and Senoo theory and by SEA over all of the practical operating range of centrifugal compressors. This correspondence is perplexing because the physical models are radically different. Unfortunately, I feel that the present paper contributes little toward explaining why this fortuitous and amazingly precise agreement exists.

The concurrence is especially puzzling since the authors have proven, beyond doubt, that reversible work exchange of significance is occurring in the real situation. They have demonstrated at least a 15 percent increase in the absolute stagnation pressure of the wake outside the impeller. This increase is impossible by any other mechanism than the work process embodied in the authors' equation (1). The authors' Fig. 14 is fairly convincing and explains the measured absolute stagnation-pressure variations along relative streamlines, except of course along the jet-wake boundaries, where shear mixing is important and obscures the reversible-work.

I would appreciate the authors explaining how they determined the trajectory of the relative streamlines in their Fig. 12, although obviously the details of their method are not important to concluding qualitatively that a reversible work exchange has occurred. Nevertheless, the method for determining relative streamline location is important to the quantitative check in equation (1) attempted in Fig. 14.

So there is no question, after the authors' demonstration, that the unsteady static-pressure field in absolute coordinates has produced a finite energy exchange between jet and wake. No doubt this phenomenon does affect the rate of distortion decay. But the question remains whether it affects the overall loss associated with the discharge process. This brings us to my second point.

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[2] President, Creare Inc., Hanover, N.H.

Table 3 of the authors' paper shows that about the same net loss is calculated from the Dean and Senoo and the SEA for both smooth and rough walls. The calculated difference between the theories does increase with distortion intensity. However, even for the extremely distorted case employed by the authors to accentuate differences, the total losses calculated by the two theories differed at most by 25 percent. And we must remember that this difference applies normally to a loss of between 2 and 4 points of stage efficiency. Hence, the maximum inaccuracy apparently predicted by the authors in using the simple Johnston and Dean theory would be one point in stage efficiency for the extremely distorted case.

In the case of an ordinary distortion (bottom row in Table 3), the loss difference is only about 5 percent, which is only 0.2 points of stage efficiency and is insignificant compared to other uncertainties in both theories (e.g., wall friction coefficient, distortion magnitude for a real compressor, etc.).

When we compare the two theories, we realize the Dean and Senoo theory has a higher wall loss and a much lower mixing-shear loss between the jet and the wake. (It is not plain what shear coefficient the authors have employed between the jet and the wake for their calculations with the Dean and Senoo theory; elucidation would be appreciated.)

So the question remains as to why these very different physical models give quite similar results in practical cases. Even when the authors roughened the diffuser wall in order to enhance the prime loss mechanism of the Dean and Senoo theory, the two theories still agreed fairly well. Perhaps the following hypothesis may provide a hint as to the real explanation.

In reality, there is coupling between wall-friction effects and the internal-shear/jet-wake mixing because wall shear stretches and the interface between jet and wake.

Eckardt's paper plainly demonstrates this distortion.[3] As the wall shear coefficient is increased by roughening the walls, the thickness of the wall layer δ increases. The stretching of the boundary between the jet and the wake should enhance interface-shear mixing. So maybe in this *three-dimensional* mechanism, there is a physical means to explain the strange coincidence between the Dean and Senoo and Johnston and Dean *two-dimensional* theories. If this ultimately proves to be the true explanation that will show again the pragmatic power of simple mixing models.

The last matter I would like to discuss is the rate of mixing between jet and wake on the jet-leading and the jet-trailing faces as clearly revealed by the authors' data. I have explained before that, *within the impeller,* Coriolis effects tend to suppress mixing on the leading face of the jet. This has been amply confirmed by experimental data. The situation proves to be fortuitous because it allows simple impeller-flow models, which are extensively utilized.

Outside of the impeller, one would expect the same phenomenon to suppress mixing between jet and wake on the *leading* face of the jet and to enhance mixing on the *trailing* face of the jet.

[3] Eckardt, D. "Instantaneous Measurement in the Jet-Wake Discharge Flow of a Centrifugal Compressor Impeller," ASME Paper No. 74-GT-90.

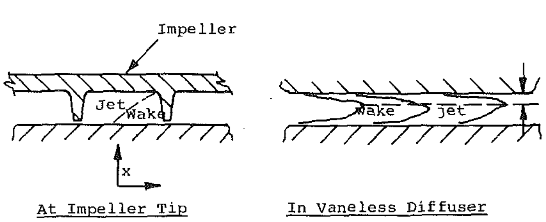

Fig. 21 Modulation of jet-wake geometry by wall shear

However, if we examine the authors' Figs. 11 and 12, we are immediately struck by the slow rate of decay of the strong stagnation-pressure gradient at the rear of the jet (center of Fig. 12) and the rather rapid decay, in relative coordinates, of the absolute stagnation-pressure gradient at the leading face of the jet (see especially Fig. 11). This behavior is opposite to my expectations for the influence of Coriolis forces. At first, I thought that the authors had misinterpreted the direction of time on their hot wire traces. But, the authors checked this point about a year ago and produced undeniable confirmation of their data.

It appears as if the Coriolis effect *suppressed* mixing at the trailing face of the jet where I would expect it to *enhance strongly* the mixing process.

Note also in both Figs. 11 and 12 that the mixing situation seems to change radically at $R \simeq 1.15$. Beyond that, the trailing face of the jet mixes very rapidly. Some new phenomenon apparently causes a wild burst of turbulent mixing.

While I do not have a physical explanation for the authors' data, which I believe is valid, I do observe that the preservation of the jet-wake pattern out to $R \simeq 1.15$ does allow time for reversible work exchange to ameliorate the differencies between jet and wake. If discharge mixing near $R = 1.0$ were strongly enhanced by Coriolis forces, then perhaps the reversible work exchange would be thwarted. So, for a completely valid theory, this strange mixing situation must be explained. I encourage the authors to delve into this matter, for I believe it is important to the ultimate theory.

In closing, I again want to compliment the authors for their excellent data, their persistence and their unique attention to this phenomenon and to express the appreciation of the centrifugal compressor fraternity for their important work.

P. G. Hill[4]

A remarkable feature of the experimental results is the extraordinary lack of mixing at the interface between jet and wake regions in the neighborhood of streamline 19 (Fig. 12). Fig. 7 shows the very steep velocity gradient at the interface persisting to at least $R = 1.10$; after $R = 1.5$, turbulent decay is fairly rapid. Near the other interface between jet and wake, intensive mixing is apparent throughout the vaneless space.

Bradshaw[5] shows that the Brunt-Vaisala frequency for flow over the curved blades of a radial turbomachine is given by

$$\omega_{BV}^2 = \frac{2W}{r^2}\frac{\partial}{\partial r}(Wr) - 2\Omega\left(\frac{\partial W}{\partial r} - 2\Omega\right)$$

in which r is the radius of curvature of the relative streamline. The flow is stable if the right-hand side is greater than zero, or if

$$\left(\frac{W}{U} - \frac{r}{r_0}\right)\frac{r}{U}\frac{\partial W}{\partial r} + \left(\frac{W}{U}\right)^2 + 2\left(\frac{r}{r_0}\right)^2 > 0$$

in which W and U are the relative fluid velocity and tip speed and r_0 is the tip radius.

From Fig. 9 (or Fig. 12 with its stretched vertical scale) it may be deduced that the radius of curvature of the mean streamline in the shear zone is in the range

$$0.3 < \frac{r}{r_0} < 0.4 \quad \text{for} \quad 1.04 < R < 1.10$$

[4] Visiting Commonwealth Professor, S. R. C. Turbomachinery Laboratory, University Engineering Department, Cambridge, England.

[5] Bradshaw, P., "Effects of Streamline Curvature on Turbulent Flow," AGARDograph No. 169 Aug. 1973.

Also

$$\frac{W}{U} \sim 0.4 \text{ and } \frac{r}{U}\frac{\partial W}{\partial r} \sim 5$$

Thus the magnitudes are approximately

$$(\frac{W}{U} - \frac{r}{r_0})\frac{r}{U}\frac{\partial W}{\partial r} + \underbrace{(\frac{W}{U})^2 + 2(\frac{r}{r_0})^2}$$

$$(0.4 - 0.4)5 \quad + \quad 0.5 \quad > 0$$

For this condition the interface is stable, but only marginally so. Changing the radius of curvature from 0.4 r_0 to 0.6 r_0 makes the flow unstable so one can see why rapid turbulent mixing takes place past $R = 1.15$ where the streamline radius of curvature is in fact larger.

The impeller blades have very much larger radius of curvature ($>2r_0$) so that the flow near the pressure surface will no doubt have been unstable.

In the left-hand zone of the jet (Fig. 8) instability would require $r/r_0 < W/U$ since $\partial W/\partial r$ is negative. It is not easy to estimate the streamline curvature for this zone. However, it may be noted from Fig. 10 that for streamline 15, $\partial P/\partial \theta$ is mainly positive for $1.04 < R < 1.30$. This would indicate that curvature acceleration is larger than coriolis acceleration in this zone so that, with $\partial W/\partial r < 0$, the flow is unstable.

Rationalization after the fact is all very well of course; what is valuable is the contribution of this excellent paper in showing so clearly the unusual behavior of these impeller exit shear layers.

A. Mobarak[6]

The authors are to be congratulated for their experimental work to assess the theory of energy exchange at impeller exit. I do agree with the authors, that the energy exchange in the vicinity of impeller and at diffuser inlet is mostly governed by the energy transfer process due to the unsteadiness of the flow. If the sudden expansion approach leads to satisfactory results, when calculating the pressure and pressure losses, it should be accepted only as a theoretical flow model, but it is far from being the real mechanism of energy exchange process, which occurs at impeller exit. In fact, the sudden expansion mixing is an unsteady-state process, but one considers only the steady-end conditions of flow and not the actual mechanism of mixing. However, I have some comments concerning the theory applied, because I believe that the theory used is inherently not correct enough to render this astonishing agreement with the measurements. The theory assumes that the relative flow is steady, here I am going to show, that to render the asymmetric flow uniform with increasing radius, the flow must be unsteady not only in an absolute system of coordinates but also in the relative one. We have proved[7] that for a rotating frame of coordinates and considering the viscous forces, the energy equation can be written in the form:

$$\frac{D(\bar{H} - q)}{Dt} = \frac{1}{\rho}(\frac{\partial p}{\partial t})_{\text{Rel}}, \tag{5}$$

where

$$\bar{H} = h_{\text{tot}} - UV_u = h + \frac{W^2}{2} - \frac{U^2}{2} \tag{6}$$

is the total transformed enthalpy, h_{tot} the total enthalpy, h static enthalpy, V_u the circumferential component of the absolute velocity and q is the heat transfered to the fluid element.

If the flow is considered adiabatic, we get

$$\frac{D\bar{H}}{Dt} = \frac{D(h_{\text{tot}} - UV_u)}{Dt} = \frac{1}{\rho}(\frac{\partial p}{\partial t})_{\text{Rel}} \tag{7}$$

For a fixed frame of coordinates ($U = 0$), equation (7) reduces to the well-known equation

$$\frac{Dh_{\text{tot}}}{Dt} = \frac{1}{\rho}\frac{\partial p}{\partial t} \tag{8}$$

If the relative flow is steady $(\partial p/\partial t)_{\text{Rel}} = 0$, the transformed total enthalpy $\bar{H}$ is constant and we must write:

$$\frac{D\,h_{\text{tot}}}{Dt} = \frac{DUV_u}{Dt} = \frac{1}{\rho}\frac{\partial p}{\partial t} \tag{9}$$

For this case, we can also write (as given in this paper)

$$\frac{1}{\rho}\frac{\partial p}{\partial t} = -\frac{\Omega}{\rho}\frac{\partial p}{\partial \theta} \tag{10}$$

But if the relative flow is unsteady, the following relation is valid:

$$\frac{1}{\rho}\frac{\partial p}{\partial t} = \frac{1}{\rho}(\frac{\partial p}{\partial t})_{\text{Rel}} - \frac{\Omega}{\rho}\frac{\partial p}{\partial \theta} \tag{11}$$

Combining equations (11), (8), and (7), we get

$$\frac{DUV_u}{Dt} = -\frac{\Omega}{\rho}\frac{\partial p}{\partial \theta} \tag{12}$$

Equation (7) can also be written in the form

$$W\frac{\partial h_{\text{tot}}}{\partial s} + (\frac{\partial h_{\text{tot}}}{\partial t} - \frac{1}{\rho}\frac{\partial p}{\partial t})_{\text{Rel}} = \frac{DUV_u}{Dt} \tag{13}$$

Substituting equation (12) in (13) we get

$$W\frac{\partial h_{\text{tot}}}{\partial s} + (\frac{\partial h_{\text{tot}}}{\partial t} - \frac{1}{\rho}\frac{\partial p}{\partial t})_{\text{Rel}} = -\frac{\Omega}{\rho}\frac{\partial p}{\partial \theta} \tag{14}$$

Equation (14) is a general form of equation (6) in the paper, so that if we divide equation (14) by $\Omega\, U^2/_2$ we get:

$$A + \psi = B$$

where ψ is

$$2\,(\frac{\partial h_{\text{tot}}}{\partial t} - \frac{1}{\rho}\frac{\partial p}{\partial t})_{\text{Rel}}/\Omega U^2 \tag{15}$$

It is now clear that the deviation of the values A and B from the theoretical values which is given by the line of 45 deg slope angle Fig. 14 is not due to inaccuracies in measurements or the assumption of frictionless flow, but is mainly due to the neglection of the term which accounts for the unsteadiness of the relative flow and which is given by ψ in equation (15). This is also evident from Table 1, where the discrepancies between A and B values often exceed 100 percent, which cannot be attributed to simplified equations or in accurate measurements.

We come now to the important conclusion: that if it is correct to assume that the severely asymmetric relative flow at impeller exit is steady and that the flow has different total transformed enthalpies due to the presence of punched plates at inlet, then the assumption could not any more be valid, when the severely asymmetric flow proceeds in the diffuser and turns to a symmetric one. This is because—according to equation (7)—it is only possible if the relative flow is unsteady. In other words, if the total enthalpy of an adiabatic flow can only be changed through pressure fluctuation in a fixed frame of coordinates, the total transformed enthalpy as well can only be changed if the pressure fluctuates in the rotating frame of coordinates. The output signals of Fig. 7 show clearly too, that if the flow at the station $R = 1.014$ is assumed steady in the relative flow system, it grows unsteady (less periodicity) as the asymmetric flow decays at larger radii. It

[6] Asst. Professor, Mechanical Department, Cairo University, Cairo, Egypt. Presently at Institut für Strömung Maschinen, Hannover, Germany.

[7] Bammert, K., Mobarak, A., and Rautenberg, M., "Energy Transfer in Centrifugal Compressors," von Karman Institute for fluid Dynamics, Lecture Series (Advanced Centrifugal Compressors) Mar. 1974.

may be worthy mentioning, that since equations (7) and (8) are also valid for viscous flow, and since the entropy production is great in such diffusers with shallow depths, I think, the statements "isentropic energy exchange" and "reversible work exchange" should be replaced merely by energy exchange.

Returning to the outstanding agreement between theory and measurements, I believe, that this is because the authors have varied the flow and wake parameters so long (within range) until the agreement is achieved. For example, if we compare the values of dimensionless jet and wake velocities given in Table 2 with the measured values of Figs. 8 and 17, we may conclude, that this choice is to certain extent arbitrary. The same could be said for the ratio of wake width and pitch of value 0.75 for the severely asymmetric pattern and of 0.5 for the ordinary flow. Also the measured relative angle β is far from being 54 deg.

I would like to mention too, that if we accept that the relative flow must be unsteady, then the concept of stream lines is not any more valid. At last I think that Fig. 15 is for a rough wall, while Fig. 16 is for the smooth one.

Author's Closure

The authors are grateful to Dr. Dean, and Professors Hill and Mobarak for their informative comments and discussions.

The static pressure rise from the impeller exit to a radius where the flow is axisymmetric depends on three factors, i.e., the change of radial momentum, the radial component of wall friction force, and the centrifugal force of fluid in the annular control volume. If the condition of flow at the inlet and exit of the control volume is fixed, the change of radial momentum through the control volume is constant regardless of the flow condition inside the control volume or of the mechanism of decay of distortion. It is obvious that the radial component of wall friction force is not much different for the two theoretical flow models. Therefore, the major cause of the difference of the static pressure rise for the two flow models is the centrifugal force of fluid, which depends upon the flow pattern in the control volume. It is clear that the centrifugal force of the distorted flow is larger than that of the uniform flow for a fixed rate of entry of angular momentum. According to reference [2], the distortion decays rather quickly, therefore the excess centrifugal force due to distortion in the control volume is not very large anyway. It is noticed further, that the wall friction force is roughly proportional to the centrifugal force, therefore the larger the centrifugal force the larger the friction force which decreases the swirl and consequently the excess pressure rise due to distortion is reduced.

If there is no friction force on the wall and no shear force between the wake and the jet, the pressure rise based on SEA model is less than the pressure rise based on the isentropic work exchange model by the mixing loss, which must be equal to the difference of the centrifugal forces for the two cases. Therefore, in the case of isentropic vaneless diffuser the distortion persists long enough to create the large excessive centrifugal force, which balances to the extra pressure rise compared to the SEA model. The rate of decay of distortion depends upon the ratio of wake and jet flow rates, the wall friction coefficient and other factors, and the analysis of reference [2] predicts how far does the distortion persist in the vaneless space. The prediction is important for design of vaned diffusers.

Concerning the stable interface between the trailing face of the jet and the wake up to a radius ratio of 1.10, a quantitative explanation is presented by Professor Hill in his discussion to this paper. The stability criterion for Coriolis acceleration is

$$\frac{d}{dr}(2\Omega W)_0 \geq \frac{d}{dr}(2\Omega W) \qquad \text{(A-1)}$$

where r is the radius of curvature. The left-hand side of this equation represents the case of invisid flow without distortion, e.g., a potential flow through a rotating impeller, where the following relation exists

$$\frac{W^2}{2} + \frac{p}{\rho} - \frac{1}{2}(Rr_0\Omega)^2 = \text{const} \qquad \text{(A-2)}$$

Differentiating equation (A-2)

$$\frac{d}{dr}\left(\frac{p}{\rho}\right) = -W\frac{dW}{dr} + \frac{1}{2}\frac{d}{dr}(Rr_0\Omega)^2$$

Due to Coriolis acceleration there is

$$\frac{d}{dr}\left(\frac{p}{\rho}\right) = -2W\Omega + \frac{1}{2}\frac{d}{dr}(Rr_0\Omega)^2$$

Comparing these two equations

$$\frac{dW}{dr} = 2\Omega \qquad \text{(A-3)}$$

By substituting this equation to the left-hand side of equation (A-1) the stabilizing effect of Coriolis acceleration is measured by

$$-\frac{d}{dr}(2\Omega W) + 4\Omega^2 \equiv S_{\text{cori}} \qquad \text{(A-4)}$$

If this is positive the flow is stable.

The stability criterion for centrifugal acceleration of a curved stream is

$$\frac{d}{dr}\left(\frac{W^2}{r}\right) \geq \frac{d}{dr}\left(\frac{W^2}{r}\right)_0$$

or

$$2\frac{W}{r}\frac{dW}{dr} \geq 2\frac{W}{r}\left(\frac{dW}{dr}\right)_0 \qquad \text{(A-5)}$$

The right-hand side of these equations represents the case of inviscid flow, where the flow satisfies the free vortex condition $Wr = K$ (constant) and

$$2\frac{W}{r}\left(\frac{dW}{dr}\right)_0 = -2\frac{W}{r}\frac{K}{r^2} = -2\left(\frac{W}{r}\right)^2 \qquad \text{(A-6)}$$

By substituting equation (A-6) to the right-hand side of equation (A-5) the stabilizing effect of centrifugal acceleration is measured by

$$2\frac{W}{r}\left(\frac{dW}{dr} + \frac{W}{r}\right) \equiv S_{\text{cent}} \qquad \text{(A-7)}$$

The sum of equations (A-4) and (A-7)

$$\left(\frac{W}{r} - \Omega\right)\frac{dW}{dr} + \left(\frac{W}{r}\right)^2 + 2\Omega^2 \equiv S$$

is the stabilization criterion of the flow with the two kinds of acceleration, and the flow pattern is stable if this value is positive. This equation is identical to the second equation of Professor Hill's discussion.

The authors drew relative streamlines for different kinds of impellers and noticed that the radius of curvature was small immediately after they left the impellers. It is especially so if the backward leaning angle of the impeller blades is small. At the interface between the trailing face of the jet and the wake dW/dr is a large positive value and S_{cori} is negative or the flow is unstable due to Coriolis acceleration, but it is expected that the instability due to Coriolis acceleration is overcome by the stabilizing effect of centrifugal acceleration or the positive value of S_{cent} if the radius of curvature is very small. One of the authors talked to Professor Hill about the stabilizing effect of centrifugal acceleration relative to Fig. 7. The authors are grateful to him for his quantitative investigation on the matter.

The trajectory of the relative streamlines in Fig. 12 were determined by the following way. The streamline No. 0 or No. 20 is determined as the trace of the middle point of the steep velocity gradient zone at different radii in Fig. 9. A portion of the next streamline No. 1 is assumed between R_0 and R_1 and then the relative flow direction is tentatively determined. As the absolute velocity is known in Fig. 9, the velocity vector triangle is drawn, and the streamline is corrected so that it satisfies the continuity condition. Identical process is repeated to all streamlines between R_0 and R_1 one after another, and if the last streamline No. 20 does not agree with the No. 0 streamline a little adjustment is required. The work is repeated between R_1 and R_2, R_2 and R_3, and so forth.

For calculation of Figs. 15 and 16, the shear coefficient ζ = 0.094 was used.

Professor Mobarak mentions that to render the asymmetric flow uniform the flow must be unsteady in the relative system, because $\bar{H}$ of his equation (2) remains constant for a steady relative flow. If the wall friction force and the shear force between wake and jet is disregarded and the flow is steady in the relative system, the difference of $\bar{H}$ between the wake and the jet remains constant. The difference is mostly due to the difference of the square of relative velocity W. At the exit of impeller where W is small the difference of $\bar{H}$ may correspond to significant difference of W between the wake and the jet. At larger radii the relative velocity W becomes considerably larger and the difference of W becomes much smaller, and at a certain distance from the impeller the flow becomes almost axisymmetric. Although his equation (3) indicates no change of $\bar{H}$ for steady relative flow, the total enthalpy h_{tot} varies together with UV_u and the total enthalpy is the one which people are concerned about.

According to Professor Mobarak's hypothesis, if $\bar{H}$ of the wake and $\bar{H}$ of the jet become equal downstream $(\partial p/\partial t)_{rel}$ must be positive for the wake and $(\partial p/\partial t)_{rel}$ must be negative for the jet. As the time average value of $\partial p/\partial t$ should be zero, the time average value of $\bar{H}$ does not vary and the time average value of $\bar{H}$ of the wake never becomes equal to that of the jet at a downstream station. In other words although $(\partial p/\partial t)_{rel}$ may not be zero momentarily, the time average value is zero and the time variation is not important at all for the present problem.

Professor Mobarak mentions that his equations (3) and (4) are valid for viscous flow. A steady shear force in the flow direction increases or decreases the total enthalpy of fluid. Therefore, these equations must be modified accordingly before they are applied to viscous flow.

For theoretical treatment the observed complicated distorted flow must be expressed as a simple wake and jet model. The mass flow rate and the flow rate of angular momentum of the flow model must be made identical to those of the real flow. It is not easy to replace a real flow with a uniform flow keeping the flow rate and the momentum. In the case of simple one-dimensional boundary layer flow, the momentum thickness is different from the displacement thickness, in other words, it is impossible to replace a real flow with a distorted flow with a uniform flow keeping the flow rate and the momentum correct. In the present case the difficulty was overcome by adjustment of two zones which must be parallel. Therefore, the pattern is somewhat different from the intuitive flow model which satisfies the continuity condition but does not satisfy the angular momentum condition.