Second-order contributions in **e** and **e** are neglected, and u_t is assumed to be constant in θ. Then the integral can be reduced to

$$\int_{CS} \mathbf{u}\rho\mathbf{u}d\mathbf{A} = \rho \bullet \int_0^{2\pi} h \frac{n}{2\pi} \int_0^{\delta} u_{r2}^2 \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} + u_{t2}u_{r2} \begin{pmatrix} -\sin\theta \\ \cos\theta \end{pmatrix} \frac{d\xi d\theta}{\cos\lambda} \tag{A3}$$

At the outlet, the flow will be a fully developed film for the flow rates considered here, and turbulent, even for extremely small flow rates, so we use the velocity distribution (9) to obtain

$$\int_0^{\delta} u_{r2}^2 \frac{d\xi}{\cos\lambda} = \int_0^{\delta} u_{r2\max}^2 \left(\frac{\xi}{\delta}\right)^{2/7} \frac{d\xi}{\cos\lambda} = \frac{7}{9} u_{r2\max}^2 \bullet \frac{\delta}{\cos\lambda}$$

$$= \frac{\delta}{\cos\lambda} = \frac{7}{9} \bullet \left(\frac{8}{7}\right)^2 u_{r2}^2 \bullet \frac{\delta}{\cos\lambda} \simeq u_{r2}^2 \bullet \frac{\delta}{\cos\lambda}$$

$$= u_{r2} \bullet \frac{dV}{d\theta} \bullet \frac{2\pi}{\mathrm{nh}}$$

Looking at the turbulent film as calculated by equation (14), it can be shown that the mean radial velocity is proportional to $V5/12$.

Consequently, using equation (1), u_{r2} can be written

$$u_{r2} = u_{r20} \left(1 - \frac{e_x}{r_d} \cos(\theta - \phi_2) - \frac{e_y}{r_d} \sin(\theta - \phi)\right)^{5/12}$$

and

$$u_{r2} \simeq u_{r20} - u_{r20} \bullet \frac{5}{12} \left(\frac{e_x}{r_d} \cos(\theta - \phi_2) + \frac{e_y}{r_d} \sin(\theta - \phi_2)\right)$$

Using these results, and the same procedure as with the volume integral, the result becomes

$$\int_{CS} \mathbf{u}\rho\mathbf{u}d\mathbf{A} = -\frac{17}{24} \bullet \frac{\rho \bullet V}{r_d} \bullet \mathbf{u}_{r20} \begin{pmatrix} e_x \cos\phi_2 - e_y \sin\phi_2 \\ e_y \cos\phi_2 + e_x \sin\phi_2 \end{pmatrix} - \frac{1}{2} \bullet \frac{\rho \bullet V \bullet u_{t2}}{r_d} \begin{pmatrix} -e_x \sin\phi_2 - e_y \cos\phi_2 \\ -e_y \sin\phi_2 + e_x \cos\phi_2 \end{pmatrix} \tag{A4}$$

In order to calculate the phase function ϕ, it is necessary to estimate the speed V_{dr} with which disturbances propagate in the radial direction.

Following Fulford (1964), the velocity of small disturbances in the flow in a laminar film will propagate along the film surface with a velocity of $3 \bullet u$, where u is the mean velocity in the film.

However, most of the experimental work reviewed by Fulford (1964) indicates a somewhat lower velocity, dependent on the Reynolds number, both for laminar and turbulent films. In this study, a value of $2 \bullet u$ will be used in the fully developed film.

In a free stream with a free surface, it is reasonable to assume that the disturbances will propagate with the free-stream velocity.

Therefore, in the inlet, a mean disturbance velocity of $V_{dr} = u_r (1 + \epsilon/\rho)$ is assumed.

Calculations of the destabilizing force have been carried out with an assumed disturbance velocity in the fully developed film from one to three times the mean film velocity.

The influence of this parameter is very small for high flow rates of water, but considerable for low flow rates or with high-viscosity fluids.

DISCUSSION

William E. Thompson[1]

The author's paper, including some rare experimental results, is a welcome addition to the literature on working fluid-induced effects that may lead to rotordynamic instability. It possesses two attributes that I would like to emphasize.

(*a*) Unsteady flow in a centrifugal impeller alone (without volute or vaned or vaneless diffuser) *can* induce both stiffness and damping effects, which may have a destabilizing influence on the impeller/rotor/bearing system. Perhaps this idea was first introduced by Thompson (1978). Investigation on the topic has been widened by Ohashi et al. (Shoji and Ohashi, 1987, and previous work) by including both vaned and vaneless diffusers. Bolleter et al. (1987) give major results from the EPRI-Sulzer project, in which measurements from a pump with a vaned diffuser were made. The author cites the work on volute pumps (Chamieh et al., 1982; Adkins, 1986). As distinct from the results of Shoji and Ohashi (1987) and Bolleter et al. (1987) as well as those of Chamieh et al. (1982) and Adkins (1986), however, the present paper strongly reinforces Thompson's contention that a destabilizing influence can arise from impeller passage flow. This is suggested to occur at design point operating conditions in the absence of volute or diffuser but when slight circumferential asymmetries are induced from impeller passage to impeller passage.

(*b*) The rotary atomizer is a simple apparatus and gives experimentalists an alternative test bed for observing the influence of asymmetric impeller passage flow. The control over the flow through the distributor yields significant control over the asymmetric passage flow, which is not as directly available to the experimenter using a pump or compressor.

Notwithstanding these features, as well as the interesting computational model for simulating the fluid force on the atomizer wheel, the paper has many frustrations for the reader. I suggest the following topics where further explanation and clarity are desirable.

(*c*) The author emphasizes much too strongly the occurrence of whirl (or lack of it) due to the destabilizing or stabilizing effect of the asymmetric passage flow. Since the atomizer rotor was supported in anti-friction bearings, which may provide little damping to the system, significant whirl may readily follow from small excitation effects. However, consider excitation of rotor vibration due to asymmetric impeller flow, as well as some bearing and seal influences to follow from energy input to the rotor. Further consider attenuation of rotor vibration to result from energy dissipation from the rotor largely in the bearings. Then whirl, as shown by the orbital displacements, is a function of the energy balance between excitation and dissipation. When associating the results of the paper with the more common pump and compressor rotor systems, ascribing whirl solely to the impeller passage flow effects may be misleading.

(*d*) The most important clarification needed is in distinguishing the stationary distributor from the rotating atomizer wheel in Fig. 1. Until that is done, it is not possible to understand satisfactorily the author's physical situation where

[1]Consultant, Rotating Fluid Machinery, 306 W. Victoria Lane, Arlington Heights, IL 60005.

a circumferentially uniform flow leaves the narrow circular ring gap of constant width but an unevenly distributed flow enters the various passages of the wheel. By what means is the flow partially reduced into some of the wheel channels and increased into others?

The uneven flow provides a *forced* asymmetry to the wheel passage flow conditions and is under control of the experimenter. Thus, the results in the paper are all reported as a function of the volumetric distributor flow. In drawing a comparison to a pump or compressor impeller, however, it must be remembered that such impellers are supplied with fluid through an upstream pipe or duct roughly the size of the impeller eye. Whirl of the impeller will not noticeably throttle the inlet cross-sectional area. Rather, asymmetric passage flow may follow from effects *induced* at the impeller blade leading edge. One such mechanism is postulated by Thompson (1978).

(*e*) In comparison to the detail of the first five steps in the Numerical Procedure, the description for calculating K_{xy} and other rotor-dynamic coefficients is inadequate. While the explicit formulas are given, it is the description of the disturbance phenomena, meant to be formulated by the phase function φ, which is so incomplete. Clearly φ is a function of ω, the rotor whirl frequency (and the angular frequency at which the fluctuations occur in the flow entering the atomizer passages). However, it is also a function of r. Hence, its influence is to take into account the differing flow rates entering a channel as the rotor orbit is traversed, as well as the passage flow rate at a radius r, which was initiated at an earlier ϑ_w. Thus, a model for the unsteady passage flow is presumably constructed as a function of rotor orbital position and velocity. The author needs to amplify his conception to a considerable extent in order to be understood on this important point. It does not help that r^* is undefined in the paper and that V_{dr} is given incorrectly in step 5 of the Numerical Procedure.

(*f*) The reader will be aware that the curves of K_{xy} in Fig. 4 are at constant atomizer wheel speed (1900 rad/s = 18,143 rpm). At least an indication of *all* the rotor-dynamic coefficients throughout the speed range would expand the reader's perception of the phenomena.

(*g*) The legend in Fig. 6 is inadequate. In the figure, two curves are drawn through experimental data for each wheel, but there is no clear description of the difference between the curves in each pair either on the figure or in the text. The figure should be completely self-explanatory.

In this paper we have a "rough-cut gem." It is to be regretted that the reviewers did not more effectively assist in polishing the gem for the greater value of the paper and the understanding and appreciation of the reader.

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Author's Closure

The author is grateful to Dr. Thompson for pointing out topics in the paper where more clarity is needed. The reviewers should not be blamed, however, for the lack of clarity, since they have already pointed out several weak points in the explanations. It is the author's responsibility if he has not been successful in trying to incorporate their advice in the final text. It is hoped that the following answers to Dr. Thompson's comments and questions will help in reducing the frustrations of the readers:

(*a*) The author is aware of Dr. Thompson's paper (1978). However, since the most basic physical and computational features of the model presented there could not be disclosed, as they were part of a proprietary development, it was found very difficult to benefit from the analysis.

(*b*) The liquid distributor does *not* control the asymmetric flow. It controls the total flow $\dot{V}$ to the rotating wheel, like a valve in a feedpipe to a pump. Also, it spreads the liquid out into a cone-shaped film, which is symmetric with respect to the distributor axis, but obviously not symmetric with respect to the eccentric rotor axis.

(*c*) The paper only claims to explain the main destabilizing mechanism in a rotary atomizer, not in any type of fluid-handling machinery. However, an adaptation of the analysis to a pump impeller has been submitted as a paper to the next ASME Vibrations conference to be held in Sept. 1989. It is hoped that the relative importance of the mechanism analyzed can be discussed on the basis of that future paper.

(*d*) Figure 1(A) attempts to show the distributor-rotor configuration. The stationary distributor is marked "distributor," and this also includes the inner part, forming the inner boundary for the liquid, which is pumped through in the direction of the arrow, leaving through the ring gap just below the transparent wheel top plate. The rotating parts are the shaft, the wheel bottom with the channels, and the transparent top plate.

The flow is reduced in the channels that move away from the distributor, and increased in the channels that approach the distributor, as the wheel becomes eccentric with respect to the distributor axis. It is postulated that the thin film sticks to the wheel without slip in the tangential direction in the inner wheel and the channel inlet. Extensive slip and backflow will destroy this mechanism, and this is in fact utilized when curing the instability, redesigning the channel inlet to create precisely this type of secondary flow. The uneven flow in the wheel is *not* a forced asymmetry controlled by the experimenter, but is caused by the eccentricity of the rotor, when the rotor whirls freely. This is precisely what creates the self-exciting nature of the resulting vibrations.

(*e*) The symbol r^* was introduced to avoid the appearance of r, both in the boundaries and under the integral sign in the expression for ϕ in Appendix A. V_{dr} is not incorrectly given in step 5 in the numerical procedure. What is given is the disturbance velocity V_d in the direction of the channel. This should have been explained in the list of symbols.

The phase function ϕ describes the asymmetric flow through the wheel in terms of the inlet distribution, given by equation (1), and the radial velocity V_{dr}, of a disturbance that travels as a surface wave on the film in the channel. In an incompressible fluid, completely filling the channel, ϕ would be equal to the angle traversed by the curved channel from the inlet to the outlet. However, free surfaces and compressibility effects will cause the disturbances to move with a finite velocity, introducing a further phase difference between the inlet asymmetry and the asymmetry felt by the channels at some given radius. For the atomizer, this is given as an explicit formula in the paper; for other machines, without the film-type flow in the channels, it would obviously be different.

(*f*) The influence of rotor speed relative to whirl speed has not been investigated, since the experimental program did not allow for this. It has been shown in the paper that for the atomizers, which run at ten times the first critical, the dominating term is the cross-coupling stiffness. Consequently there is no experimental verification of the damping terms in the model, and since these will look quite different in other machines, we propose postponing discussion of the damping terms until after the publication of the mentioned analysis of a pump impeller.

(*g*) In Fig. 6, curves with positive ordinates are the amplitudes of the forward whirl component, and curves with negative ordinates are the backward component amplitudes, as indicated on the ordinate axis.

Clearly, the above only partially answers the comments and questions raised. Since most of the unclarity is obviously connected to the fact that a rotary atomizer is not a very well-known device outside the community of its designers and users in spray-drying, dry scrubbers, and mineral concentration, a further discussion of the relevance of the analysis for turbomachines should perhaps await the publication of the earlier mentioned adaptation of the model to a centrifugal pump impeller.

It is hoped that the constructive criticism of the reviewers and Dr. Thompson has also added to the author's limited experience in technical presentation, so that this next paper will be less frustrating to the readers.