is not as good as one might prefer, the variation being 6.0 and 11.5 percent, respectively. Furthermore the theory leads to a lower value than actually occurs, implying that the square section is not quite as rigid as the theoretical assumptions presume. On the other hand, the theoretical prediction for the transient velocity in the rectangular section pipe is 20 percent above the observed result. A slightly greater difference applies if equation (10) from Jenkner is used. It should be noted however that one of the difficulties encountered by Hill [6] in his experimental work was that there was some variation in the wall thickness of the pipes that he used. Since the wave speed is a function of the ratio (a/e) cubed, small variations in the wall thickness can have a marked effect on the surge velocity.

Table 3 Observed and theoretical surge velocities for three examples of pipeline

Pipeline				Surge velocity (m/s)	
Wall material	Dimensions (mm) a	b	$\frac{a}{e}$	Observed	From equation (27)
Steel	60.3	60.3	18.73	651	611
Aluminum	49.174	49.174	30.73	208	184
Steel	47.6	22.2	14.99	476	572

As might be expected, the pressure variations associated with a "rapid" valve motion i.e. one within one pipe period, conform to the standard Joukowski relationship (assuming $c >>> v$):

$$\Delta p = -\rho \cdot c \cdot \Delta V \tag{29}$$

Table 4 provides a comparison of the observed and theoretical results for the square section aluminum pipe. The observed surge velocity was used in the calculation for $-\rho c \Delta V$

Table 4 Comparison of the observed pressure change Δp in the aluminum pipe with the Joukowski value of $(-\rho c \Delta V)$ for various changes in flow rate

ΔV (m/s)	Observed Δp (kPa)	Calculated $-\rho c \Delta V$ (kPa)
−0.339	69	70.5
−0.195	42	40.56
−0.307	68	63.9

The square section aluminum pipe was also used for a series of experiments to determine the deflection profile of the side walls for various internal pressures above atmospheric pressure. It was found that the actual pipeline profile agreed very closely with the theoretical one from equation (18). Such slight differences that did occur were mainly in the vicinity of the corners.

Conclusions

1 A theoretical equation to predict the velocity of surge propagation in thick-walled conduits of rectangular cross section has been developed. It takes account of pipe wall deflections due to bending moments, shear and transverse tensile forces in the pipe wall.

2 It is shown that the effects of shear forces and tensile stretching of the side walls can have a significant effect on the predicted surge velocity in a liquid contained in thick-walled pipes.

3 Theoretical predictions are compared with experimental data, showing moderately good agreement.

4 The deflection profile of the side wall of a square section aluminum pipe measured at three different pressures, has been found to agree closely with the profile predicted by simple beam theory.

5 Pressure changes associated with the surge have been measured and found to compare favorably with the Joukowski formula.

References

1 Streeter, V. L., and Wylie, E. B., *Hydraulic Transients*, McGraw Hill, 1967.

2 Parmakian, J., *Water Hammer Analysis*, Dover, 1963.

3 Massey, B. S., *Mechanics of Fluids*, 2nd ed., D. van Nostrand Reinhold Co., 1970.

4 Thorley, A. R. D., "Pressure Transients in Liquids in Slightly Elastic Pipes of Rectangular Cross-Section," Research Memorandum No. ML65, Aug. 1973, The City University, London.

5 Hutarew, G., "Einfuhrung in die Technische Hydraulik," *Introduction to Technical Fluid Mechanics*, Springer Verlag, 1965.

6 Hill, D. J., "Fluid Transients in Pipes of Non-Circular Cross-Section," Final Year Project Report No. 385, July 1970, The City University, London.

7 Timoshenko, S., *Strength of Materials*, Vol. 1, 3rd ed., D. van Nostrand Inc., 1963.

8 Roark, R. J., *Formulas for Stress and Strain*, McGraw Hill, 1954.

9 Jenkner, W. R., "Über die Druckstossgeschwindigkeit in Rohrleitungen mit quadratischen und rechteckigen Querschnitten," *Schweizerische Bauzeitung*, Vol. 89, No. 5, Feb. 1971, pp. 99–103.

10 Skalak, R., "Water Hammer," *Water Power*, Dec. 1955, Vol. 7, No. 12, pp. 458; and Jan. 1956, Vol. 8, No. 1, pp. 17.

11 Thorley, A. R. D., "Pressure Transients in Hydraulic Pipelines," *Journal of Basic Engineering*, TRANS. ASME, Vol. 91, No. 3, Sept. 1969, pp. 453–461.

12 Guymer, C., "Pressure Transients in Pipes of Square Cross-Section," Final Year Project Report No. 598, June 1974, The City University, London.

DISCUSSION

Don J. Wood[2]

The authors have presented a clearly developed relationship for surge velocities in rectangular sections. The agreement with experiments is really quite good considering the significant effect of small deviations in the conduit wall thickness on the surge velocities. The authors also presented a small amount of data in Table 4 comparing observed and calculated values for pressure surge magnitudes and although these are in quite good agreement the observed values tend to be slightly higher than the calculated values (an average of 2.6 percent for this data). It turns out that this is the case for most published data comparing observed and calculated pressure surge magnitudes. Sometime ago this writer showed that the higher values for the observed pressure surge magnitudes could be partially attributed to the additional kinetic energy due to velocity distribution within the conduit which is not accounted for in equation (29).[3] For higher Reynolds numbers this effect is practically negligible but for low Reynolds number turbulent flow observed pressures 10 to 20 percent higher than predicted by equation (29) have been reported. In noncircular conduits even greater deviations would be expected and should be considered when estimating surge magnitudes.

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[3] Wood, D. J., and Stelson, T. E., "Energy Analysis of Pressure Surges in Closed Conduits," *Developments in Theoretical and Applied Mechanics*, Vol. 1, Plenum Press, 1963

J. S. Walker[4] and J. W. Phillips[4]

This paper represents a significant contribution to the fluid transient literature. The authors begin with the usual one-dimensional waterhammer equations (1), (2), which are based on several approximations, and under certain, perhaps unusual, circumstances, the error in these approximations could be larger then the effects, such as wall shear, which are included here. The results of Lin and Morgan [15][5] indicate that the celerity given by equation (7) here is, in fact, the phase velocity of periodic waves when the speed of sound in the pipe material is much greater than that in the fluid and when the wave length is much greater than the pipe diameter. The authors do point out later that the inertia of the pipe wall is neglected, which is equivalent to the assumption that the speed of sound in the pipe is infinite, citing Skalak [10] as justification of this assumption. However, Professor Thorely has himself observed the effect of wall inertia in the occurrence of a precursor traveling at the speed of sound in the pipe material [11]. The restriction that equations (2), (7) here only hold for long wave-length periodic waves becomes a restriction on the closure time for the waterhammer problem, namely that this time must be much longer than D/c for circular pipes and than a/c for rectangular conduits [16]. For the three conduits listed in Table 3, closure times much greater than 9.26×10^{-5} sec, 2.36×10^{-4} sec, and 1.00×10^{-4} sec, respectively, are required; these conditions are certainly met. These comments are not intended as criticism of this outstanding paper, since the lack of justification of the basic waterhammer equations (1), (2), (7) in terms of formal asymptotic expansions is virtually universal in the fluid transient literature.

Additional References

15 Lin, T. C., and Morgan, G. W., "Wave Propagation Through Fluid Contained in a Cylindrical, Elastic Shell," *Journal of the Acoustical Society of America*, Vol. 28, No. 6, 1956, pp. 1165-1173.

16 Walker, J. S., and Phillips, J. W., "Pulse Propagation in Fluid-Filled Tubes," submitted for publication.

[4]Associate Professors, Department of Theoretical and Applied Mechanics, University of Illinois, Urbana, Ill.

[5]Numbers 15-16 in brackets designate Additional References at end of discussion.

Authors' Closure

The authors welcome the discussion from Professor Wood and from Professors Walker and Phillips and offer the following comments.

Professor Wood's remarks concerning the influence of the additional kinetic energy effects on the magnitude of the surge pressures are worthy of further experimental study, particularly in the laminar flow regime. However, in the case of turbulent flows, which cover the majority of real flow situations such influence could well be masked by likely variations due to the sensitivity of the wave speed to the ratio of wall width to wall thickness. Further work is also required to assess the extent to which the wave speed, and hence pressure, is a function of the deformation of the pipe cross-section and the distance travelled. In some recent work[6] in ducts of hexagonal cross-section where large pressure changes occurred (to the extent that plastic deformation of the duct took place) it was found that the wave speed in the zone of elastic deformation could increase by more than a factor of two. This was due to the duct, when deforming, tending to change from a hexagonal cross-section into a circular cross-section. In the former case the initial increase in cross-sectional area is due to the bending deflection of the side walls. For a given incremental change of pressure this is much greater than the subsequent changes at the higher pressures when area change is due to the hoop straining of the duct wall of circular cross-section.

For small changes in pressure in the hexagonal section ducts the wave speed could be estimated from[6]

$$C = \sqrt{\frac{1}{\rho\left[\frac{1}{k} + \frac{1}{15\sqrt{3}'E}\cdot\left(\frac{l}{e}\right)^{3}\right]}}$$

The additional restriction pointed out by Professors Walker and Phillips on the application of equations (2) and (7) are well taken. However, it is felt that for most mechanical and civil engineering situations they are of academic rather than practical significance, though this is probably less true of aeronautical and aerospace hydraulic systems.

The authors wish to conclude by thanking the discussers for their contributions and for their kind remarks about the paper in general.

[6]Thorley, A.R.D., and Twyman, J.W.R., "Propagation of Transient Pressure Waves in a Sodium-Cooled Fast Reactor," Submitted for presentation at the Second International Conference on Pressure Surges, organized by B.H.R.A. Fluids Engineering, Cranfield, Beds., to be held at The City University, London, Sept., 1976.