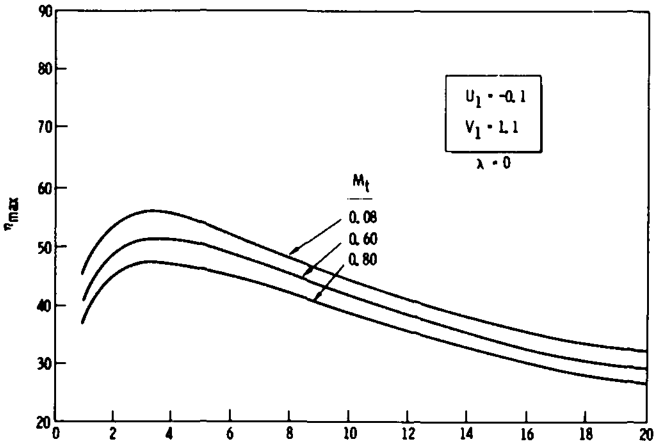

Fig. 10 Maximum turbine efficiency versus Reynolds number with turbine tip Mach number as a parameter

efficiency. Maximum efficiency occurs for incompressible flow and decreases with increasing Mach number. It should also be noted that at the higher Mach numbers only a limited region of the outer disk can be used efficiently.

Turbine efficiency increases with decreasing throughput as was shown in the incompressible analysis [10]. Momentum is transferred to the disk by the shear force generated by the relative tangential velocity between the gas and the disk. Viscous forces associated with the radial velocity component only serve to reduce the available enthalpy of the gas stream. Mach number effects are less pronounced for low throughputs as shown by comparing the results given in Fig. 8 with those given in Fig. 9. It is clear that low throughputs will be required if disk turbines are to be operated efficiently in the compressible flow regime.

An optimum Reynolds number of 3 to 4 exists for the range of inlet parameters investigated (Fig. 10). A large number of runs are required, however, to generate performance plots for the compressible regime similar to those presented in reference [10] for incompressible flow. The analysis presented herein provides a method by which such plots may be generated efficiently. Once generated, these plots can provide a ready assessment of the degradation in turbine performance to be expected as a result of compressibility effects in the flow medium. Experimental verification of these results will be required before disk turbine designs can be undertaken with confidence.

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DISCUSSION

W. Rice[3]

The authors are commended for the formulation of the problem together with an efficient solution. The information presented has been needed to further consideration of applications of multiple-disk turbomachinery. The paper covers the same topic, but with different emphasis, as a recent contribution by Bassett.[4] All of the conclusions of the two papers are in agreement. The essential correctness of the results is further confirmed by work completed but not yet published by this discusser and coworkers, in which flow of a compressible fluid laden with particles is calculated; the results of the present paper are obtained closely when the particle field density is made negligible.

It is interesting and important that the Reynolds number at which maximum efficiency occurs, and the value of maximum efficiency, decrease because of compressibility. While Mach number is a parameter of the flow one would expect little effect due to Mach number change below sonic velocities because the main cause of density change is pressure change rather than kinetic energy change.

This discusser agrees that the radius at which inflection of the radial component velocity profile occurs is of little practical interest. The question of criteria for occurrence of laminar flow is far from settled; several investigators are known to be pursuing this question at this time much more comprehensively than has been done before.

It has been the experience of this discusser that computer execution time increases drastically for cases with small Reynolds number, especially if the flow rate parameter is also small; this was also experienced by Bassett. Did the present investigators also find this and if so is the reason for it clear? Furthermore, when uniform velocity profiles are assumed at the outer periphery, a large number of terms of the various series are required to approximate the uniform profiles satisfactorily. In earlier work, it was found that a separate starting solution method had to be used when uniform inlet profiles were assumed, because of this. Did the authors experience this also, and if so what was done to adapt the solution for use with uniform inlet profiles?

Lastly (and trivially), should not the word "dimensionless" in Note 1 of the Nomenclature be "dimensional" and sum signs included in equations (18), (19), (20)?

C. E. Bassett, Jr.[5]

This investigation, in large part, duplicates an analytical study presented at the Tenth Intersociety Energy Conversion Engineering Conference in August, 1975.[6] The present authors have used temperature-dependent viscosity, parabolic inlet profiles, 6th-order polynomials for velocity components, and Runge-Kutta

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[4] C. E. Bassett, "An Integral Solution for Compressible Flow Through Disc Turbines," Proceedings Tenth Intersociety Energy Conversion and Engineering Conference, Newark, Delaware, Aug. 1975, pp. 1098–1106.

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integration; whereas, constant viscosity, uniform inlet profiles, 8th-order polynomials for velocity components, and predictor-corrector integration were used in the other investigation. In cases where the runs from the two independent studies were duplicated, good agreement of results is obtained.

An interesting observation by the authors, depicted in Fig. 6, showing the location of maximum efficiency to be near the point where the maximum local tangential velocity component departs from the local disk velocity, is generally confirmed in our work.

The inclusion of curves showing pressure recovery at the exit from the disks seems of questionable value. The flow exits the disks in a plane that is normal to its eventual exit path from the turbine (axial). The recovery of pressure from this velocity field seems difficult at best.

The generally displayed cases of high throughput (excluding Fig. 9) may lead to some inaccurate general impressions by the reader. Typical designs would, in most cases, utilize lower values of U_0 (such as that in Fig. 9) so that a more reasonable efficiency might be obtained. In these cases, choking is *not* a factor to be concerned about at any subsonic value of entrance Mach number. Also, maximum efficiency is typically obtained at $0.25 \leq r \leq 0.35$ for all subsonic M_t so that one is not limited to using the outer regions of the disk at high M_t. Finally, the effect of Re on η_{max} shown in Fig. 10 is not so extreme at the higher values of Re for $U_0 \sim 0.02$. For example, for $U_0 = 0.015$, $V_0 = 1.1$, $M = 0.5$, η_{max} drops from about 86 percent at $Re = 4.0$ to about 80 percent at $Re = 15.0$ in our study.

An extensive continuing program is being conducted at this time at the Naval Undersea Center examining the problems peculiar to disk turbine design. Two areas of concern are the maintenance of disk spacing using thin disks and high entrance losses at the stator surface prior to disk admission. An experimental turbine is under construction and results should be obtained by mid-1976.

Authors' Closure

The authors are pleased to note the agreement with the work of Bassett and of Rice. Uniform velocity profiles were not used in our study because of the difficulty cited by Professor Rice, whom we thank for the errata he was courteous enough to put as questions.

Dr. Bassett feels that our use of high throughput may mislead the reader, and that in "more reasonable" cases choking need not be reckoned with. High throughput must often be accepted in cases where weight or size of the turbine must be limited. Approximate sizing calculations have been made by us using what seem to be very optimistic assumptions for blade thickness for turbine output of from 100 HP for automotive applications to 100 MW for central power generation. These calculations, although approximate and preliminary, indicate that to use a disk turbine one must often either accept lower efficiency than is available with a bladed turbine, or have a much bigger and heavier machine. The latter option is not always acceptable.

We also feel that choking, as a manifestation of compressibility, is of interest in its own right.