mensional theories) and the measured lift coefficient in a propeller pump tested by Oshima, et al. [10] and propeller tested by Himmelkemp [11]. Both of them report large increase in lift coefficient at the root and appreciable decrease at the tip from the corresponding two-dimensional values. Even though some of the discrepancies is due to viscous effect, the three-dimensional inviscid effects, as explained in this paper, alone may account for some of the discrepancies observed by Oshima, et al., and Himmelkemp.

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DISCUSSION

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The aerodynamic designer of turbomachinery has always a keen interest in calculating the three dimensional flow as it passes through the blade row. Along with this line, Mr. Tamura and Prof. Lakshminarayana try to evaluate the influence of the three dimensional flow induced by finite number of blades replaced by vortex and source lines spanning the annulus. The authors assume that the perturbed potential can be applied to the downstream flow field, thus even the axial velocity vanish at far downstream. They mention then that the theory is valid for a free vortex type of flow with any arbitrary blade thickness.

However the writer believes that the assumption made for the perturbed axial velocity should be avoided in any turbomachinery flow problem, at least, in order to account for the three dimensionality. It is then understood that the induced velocity of both tangential and radial direction may vanish at far downstream within the frame work of linear treatment, but the axial velocity induced does not disappear and tends to a finite value with tangential and probably radial dependency towards the far downstream.

Correspondingly the writer wonders what kind of physical importance the present theory has as a result of the free vortex type assumed for the induced velocity components. It should be noted that the type of free vortex is the matter of concern in the two dimensional or quasi three dimensional flow, positively not in the purely three dimensional flow about which they discuss. What shape of airfoils have the authors imagined during the course of their work? It is of the writer's opinion that the authors should point out the limitation and restriction of their theory and its application to the practical problem, not to confuse the turbomachinery designer.

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Authors' Closure

Most of the comments and questions raised are due to misunderstanding of the analysis as well as the paper objectives by the discussor. One has to make a clear distinction between the primary flow and the perturbed flow in such a type of analysis. In this analysis, the primary flow is assumed to be irrotational. From physical as well as mathematical considerations, the perturbations in axial velocity due to vortex lines should vanish at far upstream and downstream. The discussor is probably confused the three-dimensionality induced by blade row with those induced by flared annuli, compressibility effects, etc.

The objective of this paper is to assess the extent of three dimensional effects rather than provide a solution for any particular blade row. What we sought was an analysis (which lies between a two dimensional cascade solution and a fully three dimensional solution in accuracy and computational speed) to provide an indication to the designer as to the extent of three dimensionality. The limitations of the analysis have clearly been stated in the paper. Application of this analysis to one type of blade row and experimental verification will be dealt with in a paper under preparation.