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DISCUSSION

W. E. Anderson[6]

We would like to thank the authors for reminding us of the dangers involved when attempting to generalize from limited amounts of data. We think that most experimenters really recognize this point while at the same time they are dangling their legs over the pit, so to speak. Speculative theories about "laws" of behavior are frequently published in several American technologies; and this may be more useful than damaging.

In this light we wish to draw attention to the data in Figs. 4 and 6. Another interpretation of these results is that the slope trend is $^1/_5$, composed of two segments joined by a transition slope of about $^1/_2$. The midtransition occurs at growth rates of about 10^{-5} in. per cycle but the 2024-T3 transition occurs at somewhat lower rates than the transition in 7075-T6.

We shall now speculate that there is a general cracking mechanism, causing the 1:5 slope, supervened, in one region, by another mechanism having superior capacity for cyclic energy dissipation, causing the transition.

It is actually observed that the intermetallic constituent optically visible in these alloys trend downward in size from dimensions on the order of 5×10^{-5} in.; the sizes in alloy 2024-T3 consistently smaller than in alloy 7075-T6.

We can superimpose the two jogged curves we have drawn in Figs. 4 and 6 and note that the 2024 curve is regularly parallel to but shifted to lower rates than the 7075 curve. We recollect that (*a*) the solution treatment temperatures for 2024 are consistently higher than for 7075; and (*b*) there has occassionally appeared, in the literature, suggestions that something equivalent to local annealing occurs at the tip of fatigue cracks in aluminum alloys. The discusser is hesitant, but cannot resist the temptation to continue. The summary thesis being made (with considerable reluctance) that:

1 Somehow, one or another, or perhaps all the intermetallic constituents in commercial alloys 2024 and 7075 act as to inhibit crack growth over regions where the growth rate per cycle is of order of the intermetallic particle sizes.

2 The general trend (1:5 slope) of crack growth due to fluctuating loads in these alloys is perhaps due to the overbearing characteristics of mechanical stress (or strain) state experienced at crack tips and as measured by the intensity factor, but the growth rate differences are directly related to the annealing resistance of the alloys; the more the annealing resistance the slower the growth rate.

The need for much additional and relevant data is at once clear. It must be wide ranging and from tests where loadings are well controlled, as the authors have so clearly demonstrated.

We shall conclude, however, by suggesting that the authors pursue their quest for power laws no further. The course of events has proceeded from Head's linear power law to, now, a fifth power law. Further efforts will surely lead only to flatter slopes; and this will mean no relation at all!

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A. J. McEvily, Jr.[7]

It may be of interest to point out that a recent evaluation [15][8] of fatigue crack propagation theories has led to conclusions which are quite similar to those which have been reached by Paris and Erdogan. Power laws of the type

$$\frac{da}{dN} = Ba^n \tag{41}$$

were assessed in the following manner.

Consider two specimens in which fatigue cracks are growing at equal rates; then

$$\frac{da_1}{dN} = \frac{da_2}{dN} \tag{42}$$

and

$$B_1 a_1{}^n = B_2 a_2{}^n, \tag{43}$$

where the subscripts refer to the two specimens. It has been established [16] that the rate is uniquely defined by the parameter $K_N S_{\text{NET}}$, and it has been further shown [17] that even for specimens of finite width

$$K_N S_{\text{NET}} \approx \sigma_g \sqrt{a}. \tag{44}$$

Here σ_g refers to the average stress based on the gross section area. Since the rates in the two specimens under consideration are equal, it follows that

$$\sigma_{g1}\sqrt{a_1} = \sigma_{g2}\sqrt{a_2}. \tag{45}$$

Equation (44) can be written as

$$A\sigma_{g1}{}^{2n} a_1{}^n = A\sigma_{g2}{}^{2n} a_2{}^n \tag{46}$$

where A is a material constant. Equating the coefficients of the a^n terms in (43) and (46) we find that

$$B = A\sigma_g{}^{2n}, \tag{47}$$

which leads to

$$\frac{da}{dN} = A\sigma_g{}^{2n} a^n \tag{48}$$

which can be rewritten as

$$\log \frac{da}{dN} = \log A + 2n \log \sigma_g \sqrt{a}. \tag{49}$$

Equation (49) was compared with experimental results, and it was found that a value for n of 2 best suited the results for high strength aluminum alloys, in agreement with the authors' finding. On the other hand, the best value for n for a series of copper alloys was closer to 3. However, the net section stresses for some of these alloys were in the plastic range, which, as will now be shown, results in values for n greater than 2.

When one reflects upon the factors responsible for the rate of crack growth in a given material it appears two factors are of primary importance. One of these is the peak stress at the tip of the crack, and the other is the condition of the material immediately ahead of the advancing crack. The way in which both of these factors are coupled is through the strain energy stored in the portion of the plastic zone at the crack tip through which the crack must grow. In a strain-hardening material, we can approximate the energy per unit volume as being proportional to the square of the peak stress. The volume of affected material can

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[8] Numbers in brackets designate Additional References at end of this discussion.

be related to the radius of the plastic zone at the crack tip. However, the entire zone need not be considered, but only the rectangular slice immediately ahead of the advancing crack. This region is given by eR, where e is a constant and R is the radius of the zone, which according to Irwin [18] and McClintock [19] can be expressed as

$$R \approx \sigma_g^2 a. \qquad (50)$$

Upon making the reasonable assumption that the rate of crack growth is proportional to the energy stored in the rectangular slice at the crack tip, one obtains

$$\frac{da}{dN} \approx (\sigma_g^2 a)(e\sigma_g^2 a), \qquad (51)$$

which is the relationship that has been found to be in agreement with the data for the high strength aluminum alloys. However, as the applied stress approaches the yield strength the size of the plastic zone increases more rapidly than predicted by equation (50) [19], so that an even stronger dependence of the rate on the stress and crack length results. This last situation applies to the copper alloys investigated [15] for which a value for n of 3 was found to be appropriate.

With respect to the authors' paper, it is not clear how their treatment deals with sheets of finite width and with the effect of a variation in mean stress. Finally, it is pointed out that at low values of the applied stress the simple form of any power law would have to be modified since a minimum stress is required before a crack will propagate.

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Authors' Closure

The points made by Mr. Anderson and Dr. McEvily in the discussion are most appropriate. They contribute in a constructive way toward emphasizing the relative importance of the views taken in the paper, as well as adding some new ideas.

Mr. Anderson's observation and plausible explanation of the "Jogg" in the data in both Figs. 4 and 6 provides some motivation for further study of the Micro-Mechanisms involved in crack growth. Though such studies were not necessary to arrive at the conclusions drawn in the paper, they would add immeasurably to a better understanding of many detailed phenomena of crack growth. Nevertheless, empirically, the broad trends shown in the Figs. 4 and 6 remain valid.

Dr. McEvily's alternate means of arrival at an acceptable power law is most enlightening. Moreover, it has come to the authors' attention that laws similar to equations (36) and (48) have been expounded by McClintock [20] and Schijve [21] as the results of still different approaches. In all of these, the continuum parameter, k or $K_n\ \sigma_{net}$, was found to be appropriate to employ.

In this connection one may also mention the conclusion reached by the staff of Battelle Memorial Institute in their analysis of the crack propagation data from various sources [22]. They observed that, under constant load level, the crack growth rate was approximately proportional to the crack length.

Further, McEvily's observation that the exponent of the power law apparently changes when the net section stress exceeds the yield point is a result which is expected. Since the parameter k or $K_n\ \sigma_{net}$, is based on elastic stress analysis, it is evident that its role must change when the behavior of a specimen changes from predominantly elastic to predominantly plastic. Thus, as McEvily notes, an assumption of the power law expressed in the paper is that nominal stresses shall remain below the yield point. The full assessment of this restriction and the influence of the mean stress requires further studies.

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