should be the estimated fatigue strength reduction factors rather than the theoretical stress concentration factors. Methods are available for estimating these factors for most notch geometries and even for defects such as cracks. When allowable cyclic stresses are based on strain-fatigue data, the same factor should be used for low-cycle as for high-cycle conditions. When calculating the effects of combined mean and alternating stress, the fatigue strength reduction factor should be applied to both the mean and the alternating component, but then account must be taken of the reduction in mean stress which can be produced by yielding.

The complete fatigue evaluation of a pressure vessel, including calculation of discontinuity stresses, strength reduction factors and thermal stresses, and study of cumulative damage from all possible pressure and temperature cycles, can be a major task for the designer. It can be shown, however, that under conditions which include a large number of applications, the complete fatigue evaluation can be omitted provided certain requirements are met regarding design details, inspection, and magnitude of transients.

Although the emphasis in this paper is on pressure vessel design, the same principles could be applied to any structure made of ductile metal and subjected to limited numbers of load cycles.

VI Acknowledgments

The author is indebted to several of his associates for suggestions and contributions to the development of the design methods proposed in this paper. Particular mention should be made of Mr. R. E. Peterson, Westinghouse Research Laboratories, Dr. L. F. Coffin, General Electric Research Laboratories, Dr W. E. Cooper, General Electric Knolls Atomic Power Laboratory, and Mr. J. L. Mershon, Bureau of Ships. The development of eq. (8) resulted from discussions held by a Task Group of the ASME Boiler and Pressure Vessel Committee under the chairmanship of Dr. W. T. Lankford, U. S. Steel Corporation. The statistical study of the data in Fig. 2 was made by Mr. T. Shimamoto, Westinghouse Bettis Atomic Power Laboratory. The assistance of Mr. W. J. O'Donnell, of Bettis, is also appreciated.

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DISCUSSION

Elmer O. Bergman[4]

During the past few years considerable attention has been given to the capability of pressure vessels to withstand being stressed repeatedly by repeated applications of pressure and changes in temperature. Earlier studies of fatigue of structural elements deal with those that were subjected to a great many cycles of loading, while pressure vessels are ordinarily subjected to a limited number of cycles, a few thousand at the most. This paper is a valuable contribution to the growing field of literature on the subject of low-cycle fatigue.

The construction of a fatigue curve such as the one shown in Fig. 2 requires a great deal of test data. The author's equation (8) permits the construction of a fatigue curve when the per cent reduction of area in tensile test and the endurance limit of a material is known. This curve is on the conservative side with the largest deviation in the region of $N = 1000$.

The design of nuclear reactors and primary vessels requires that a fatigue analysis be made. The stresses resulting from pressure and from steady state and transient thermal stresses must be known to make such an analysis, at least for the more severe conditions. Part IV, Determination of Need for Fatigue Evaluation, is in the writer's opinion the most valuable part of the paper. It makes it possible to eliminate a great deal of detailed calculations for vessels that are subjected to pressure and temperature cycles of moderate severity.

The ASME Boiler and Pressure Vessel Committee is working on the preparation of rules for nuclear vessels and other vessels subject to fatigue. The methods outlined in this paper are being considered for these rules. The Committee is grateful for the developments in low-cycle fatigue made available by the author and his co-workers.

S. S. Manson[5]

The practical approach described in this paper will be extremely welcome to designers who wish to base their calculations on a relatively sound basis, but without resort to detailed computations of plastic flow normally required for strict analysis in the low cycle fatigue range.

The author is correct in pointing out that the writer's proposed equation relating total strain range to fatigue life does not take proper cognizance of the possible presence of an endurance limit. The writer is aware of this limitation, but the proposal was made in the same spirit as are many of the procedures discussed in the paper, namely that strictly speaking they cannot be rigorously correct, but practically they conform sufficiently to observed material behavior to be regarded as correct while at the same

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time bypassing complications that add relatively little to the final accuracy. It will be indicated in the following brief discussion that consideration of an endurance limit does not influence significantly the representation of material behavior in the life range below approximately 10^6 cycles, which is the basic range of interest in this report.

One method of approaching the problem is to accept that the basic relation between cyclic life and plastic strain

$$\epsilon_p = MN_f^z \tag{25}$$

and to hypothesize a relation between stress and plastic strain.

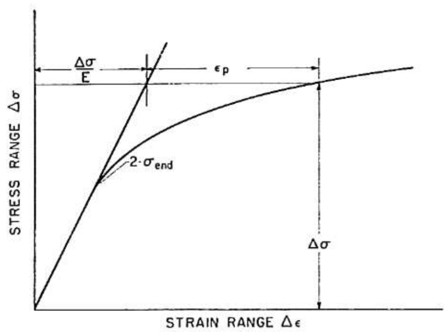

Fig. 14 Schematic representation of asymptotic cyclic stress-strain characteristic

The graphical relation is shown in Fig. 14. No plastic strain is assumed to occur as long as the stress is below the endurance limit (or the stress range below twice the endurance limit). If the stress range is above twice the endurance limit, a cyclic plastic flow is assumed to occur. The magnitude of the plastic strain is then related to the excess of stress range over the endurance limit by the conventionally assumed power law. Thus

$$\epsilon_p = A(\Delta\sigma - 2\sigma_{\text{end}})^d \tag{26}$$

where A and d are material constants, $\Delta\sigma$ is the stress range, and σ_{end} is the endurance limit. Substituting into the relation $\epsilon_p = M\,N_f^z$ we can solve for $\Delta\sigma$

$$\Delta\sigma = 2\sigma_{\text{end}} + FN_f^q \tag{27}$$

where $F = (M/A)^{1/d}$ and $q = z/d$

Thus it can be seen that a log-log plot of $\Delta\sigma$ versus N_f is not a perfect straight line. However, in a limited range of life it is possible that eq. (27) can be adequately linearized to make difficult distinguishing between the curve and the straight line. For example, Fig. 15 shows the experimental relation between cyclic life and stress range for 4130 steel. The heavy solid line is the least-squares straight line through the data represented by the equation

$$\Delta\sigma = 365{,}000\,(N_f)^{-0.085} \tag{28}$$

This equation, in principle, represents SAE 4130 steel as a material having no endurance limit. The dotted line, however, is a curve having the equation

$$\Delta\sigma = 60{,}000 + 330{,}000\,(N_f)^{-0.120} \tag{29}$$

This curve represents the material as having an endurance limit of 30,000 (60,000/2) psi, and is seen to fit the data almost as well in the experimental range. At cyclic lives above 10^7 cycles the discrepancy between the curves represented by eqs. (28) and (29) can become very appreciable, as indicated by Fig. 16 which is drawn to a more condensed time scale in order to include the higher cyclic lives.

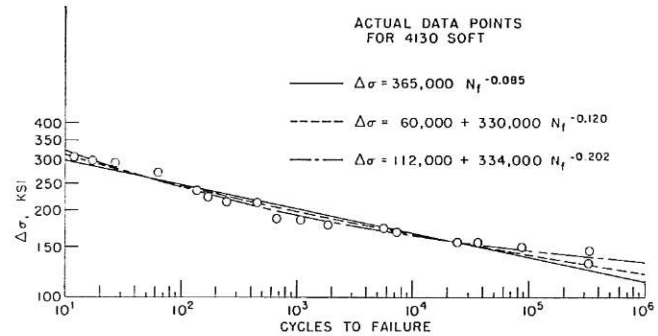

Fig. 15 Representation of experimental data for 4130 steel by several analytical expressions in life range to 10^6 cycles

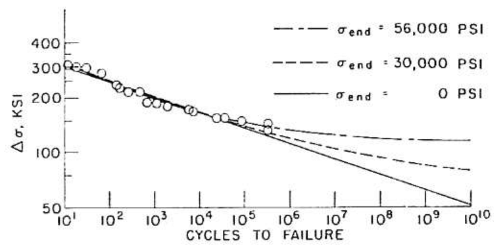

Fig. 16 Representation of experimental data for 4130 steel by several analytical expressions in life range to 10^{10} cycles

Similarly, the dot-dash curves in Figs. 15 and 16 graphically depict the equation

$$\Delta\sigma = 112{,}000 + 334{,}000\,(N_f)^{-0.202} \tag{30}$$

on which the endurance limit is 56,000 psi. Again it can be seen that at the high lives the curve deviates appreciably from the straight line, but in the experimental range little difference can be seen between the degree of fit of the experimental points and the heavy line or the dot-dash line.

In general it can be said, therefore, that the assumption of an equation in the form $\Delta\epsilon_{el} = \Delta\sigma/E = G/EN_f^{\gamma}$ is equivalent to linearizing a curve which, for most materials, has relatively little curvature in the life range up to approximately 10^6 cycles. If the purpose of the representation is to determine the relationship at much higher lives than the experimental range, the linear extrapolation can be expected to result in appreciable error, but in the experimental range the linearization is generally acceptable. In view of the extreme simplicity that results from the linearization, the procedure should not be objectionable from an engineering viewpoint.

D. B. Rossheim,[6] J. J. Murphy,[6] and C. Honigsberg[6]

The author has provided an excellent and timely interim treatment of pressure vessel design to include the influence of repeated loadings, as limited by currently inadequate fundamental materials behavior knowledge.

In the Introduction the author considers fatigue analysis essential for "the utmost in reliability and efficient utilization of material . . . ." It should be appreciated that efficient utilization of material applies to all vessels, not only those in critical services, and includes an economic association of material quality, design refinement, fabrication soundness, and nondestructive examination extent with the level of general and local stress. A weakness of the present approach of all codes is that the refined construction enforced by material sensitivities and low ductility, and strength enhancement, provide no return in the relative level of allowable stresses.

With respect to Coffin's equations (6) and (7) relating fatigue

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cycles to plastic strain range and reduction of area, it would be expected that the elastic portion of the strain would become increasingly a factor with higher yield and lower ductility; for example, would this relationship provide satisfactory correlation for a 200,000 lb/sq in. yield strength 5 per cent *RA* material? It would be desirable for Coffin to make his raw data available for correlation studies to other parameters, in particular the conventional tensile strength, for which test values would be expected to be more consistent. The author has not made clear the reason for employing the total strain (elastic and plastic), other than perhaps convenience in being able to use calculated stresses directly.

The design fatigue curve for austenitic stainless steel of Fig. 2 is stated to be based on a safety factor of either 2 on stress, or 20 on cycles applied to the "best fit" curve, using the lower value. A safety factor of 10 has been widely applied to specimen and prototype data, usually to minimum values. Recent opinion questions whether this is over-conservative, assuming such data duplicates the stress intensification due to internal defects, and for prototypes those due to surface discontinuities, which are considered again in design. Valid factors to be applied to fatigue data must reflect a statistical approach which considers all factors contributing to the scatter.

Calculated concentration factors at defects are of only academic interest, since pressure vessels involve possible defects of extreme acuity whose extent and distribution may involve integrated magnification. Prototype or full scale tests of limited extent, and not fully assessed as to all variables, can prove grossly misleading. The principal factors which enter into the rationalization of local stress with material cyclic loading behavior are:

1 A clearer concept of soundness and structure effects on test specimen *S/N* data, with only the surface presently closely controlled. Such effects probably account for much of the scatter particularly in low cycle data.

2 An appraisal of potential local stress buildup with coincidence of the magnification effect of geometry (general contour, intersections, etc.), surface (fillets, scratches, weld surfaces, and undercuts, etc.), and internal (defects, structure discontinuities or inhomogeneities in materials, and weld deposits).

3 A statistical evaluation to guide logical assumptions as to the coincidence of the preceding effects under pressure, structural (weight, wind, earthquake, etc.), and thermal loadings.

4 Knowledge with respect to ductility exhaustion, i.e., what is the influence of mill, fabrication, or other deformation on fatigue life.

5 Information with respect to the effect of sequence of loadings, in particular the greater effect of a few cycles of wide strain range early as compared with late in cyclic loading history.

The author's proposed comparison of individual mechanical and thermal load-cycle situations with the design fatigue life and subjecting only those in excess to fatigue analysis would seem to ignore total cyclic demands. The statement that individual load cycle situations exceeding the design fatigue basis may nevertheless be acceptable, is not clear.

There is a lack of appreciation that process pressure equipment is subjected to repeated overloads during "normal" operation. Instabilities of operation and control swings involve temperature changes whose fatigue significance reflects not only magnitude but also rate of change. Such temperature swings are specifically provided for in the Power Boiler Code (ASME Code, Section I), and the Refinery Piping Code (ASA B31.3). Pressure fluctuations of insufficient duration to life relief valves fully develop potential stress with potential dynamic or resonant magnification. Cyclic stress is also introduced by structural effects, such as wind, mechanical vibrations, flow vibrations, etc. Less frequent but more extreme overloads are introduced by auxiliary conditions. The Navy AEC Reactor Specification classifies cyclic loading demands into three categories similar to those just mentioned for process pressure equipment; this provides a baseline which can easily be adjusted to suit other load-cycle patterns, where they can be established for design. Where this is not the case the design must be based on assumed fatigue demands, and vessel retirement established by the integrated actual service history.

The author points to "reliability" and "efficient utilization of materials"; these benefits of a fundamentally sound code are not limited to critical vessels. Programmed calculations can be used to provide curves or formulas which minimize approximation errors, and a selection of requirements influenced by evaluation rather than a consensus of opinion alone will allow much needed stability.

The unfortunate emphasis of recent years on higher allowable stresses based on the yield strength (neglecting simultaneously consideration of the U.T.S. which has provided a brake on fatigue capacity) has apparently spread from the Continent to England. While extremely ductile carbon steel of considerable fatigue capacity permits wide latitude with cyclic stresses, this approach is hazardous for high strength and/or limited ductility materials, and also for ductile normal strength materials at higher stress levels. It is all too common to be unaware of or to neglect sizable local stresses due to the weight and thermal reactions of piping and equipment, also stresses due to fabrication deviations from contour, misalignment, and poor fitup (particularly of nozzle pads).

Improved realization of inherent structural capacity must start with materials, i.e., meaningful levels of quality with respect to soundness, chemistry (particularly contaminants), structure, mill practices, etc. Greater usage would provide reduced costs for tests and nondestructive examination, whose sensitivity and extent of application could be proportioned to successive levels of quality. Similarly, design and fabrication including quality control would benefit from an all out emphasis on progress toward better construction, as opposed to acceptance of limitations imposed by current equipment and skills. Progress stems from the pressures of dissatisfaction with inadequacies—the Code Committee should properly lead the way. Such improvement will not necessarily add to costs; it is just as likely that a reduction will be achieved.

Material behavior knowledge is widely recognized as woefully inadequate, with unfortunately little appreciation that the current painfully slow progress results from a totally unorganized approach of limited and unconnected investigations, and lack of career emphasis for fertile minds in this field. For knowledge of materials behavior to progress from present empiricisms and speculations, a broad examination of present concepts of fracture, ductility, ductility exhaustion, intereffects of short and long time deformation, deformation-induced structure changes, etc., is necessary. This involves study and appraisal of the many variables which affect materials structural behavior, as well as test methods and specimens. It is regrettable that some minor part of the sizable government research expenditures in this field is not used to provide general background and guidance for the selection, priority, and planning of individual projects.

Little real progress results from an emphasis on test data and relationships for correlating low cycle fatigue. An immediate broad program aimed at exploring fundamental concepts of fracture and ductility exhaustion, and assigning areas of interest to individuals and laboratories would, in these discussers' opinion, greatly accelerate progress.

It should be noted that the author confines this paper to the elastic range—i.e., below the creep range. Creep and creep-rupture as presently employed generally ignore structure and structure changes with time and temperature, previous deformation, first stage creep, effect of cycles, and variations of load and temperature, etc. Accordingly, high temperature design, particularly at extreme temperatures and with materials of low ductility (especially castings), is highly empirical, and constitutes a bar to the effective utilization of materials. A broad study, similar to that mentioned for the elastic range, is long overdue.

Author's Closure

The author wishes to thank Dr. Bergman for his comments and for pointing out the relationship between this paper and the proposed ASME rules for nuclear vessels. The rules finally proposed to ASME will be based on the principles described here, but in condensed form and making use of more recent studies of fatigue data.

Mr. Manson is quite correct in his demonstration of the point that the absence of an endurance limit in his mathematical formulation is of no importance in the range up to 10^6 cycles.

Messrs. Rossheim, Murphy, and Honigsberg have brought up many interesting points in their discussion. The author is in essential agreement with most of them. One possible disagreement is with their statement, "Little real progress results from an emphasis on test data and relationships for correlating low cycle fatigue data." If we did not have the low cycle fatigue data now scattered through the literature, we would be in a much weaker position than we are. Knowledge of the fundamentals must eventually be obtained, but the author sees no objection to using empirical concepts during the interim.

The following is in reply to some specific questions of Messrs. Rossheim, Murphy, and Honigsberg:

1 The author does not know whether a material with a reduction of area as low as 5 per cent would correlate with the Coffin relationship, but reference [8] shows correlation for materials with RA values ranging from 17.8 to 82 percent.

2 The reason the author used the total strain rather than the plastic strain was entirely for the convenience of the designer in comparing calculated stresses with allowable design values.

3 For the data of Fig. 2 there is not much difference between using a factor of 20 on cycles applied to the "best fit" curve and a factor of 10 applied to the minimum curve. The former has more statistical significance because it is less dependent on an accidental low value. Preliminary results from cyclic fatigue tests now being made on full-size vessels by the Pressure Vessel Research Committee indicate that the factors suggested by the author are adequate but not overly conservative.

4 The cumulative effects of the total cyclic life of the vessel were considered by the author in the last part of section IV of the paper.

Two points which do not appear in the printed discussion have been called to the author's attention. Dr. Bergman has noted that in Fig. 1 the dimension S should be $2S$ and should extend to the bottom of the diagram.

Mr. O'Donnell has noted that the K used in section IV in the discussion of allowable pressure transients does not have the same meaning as the K used in the discussion of allowable thermal transients. For the pressure transients, K includes both secondary discontinuity effects and stress concentrations and thus might be as large as 5 or 6, whereas for the thermal transients it covers only stress concentrations and should not be larger than about 2 in a well-designed and well-fabricated vessel.