$$\frac{(p_1 - p_2)}{\rho} = \frac{1}{2g}({}^*w_2^2 - w_1^2) \quad (39)$$

the velocity equation becomes

$$\tfrac{1}{2}({}^*w_2^2 - w_1^2)\sin\beta = w_1 \sin(\beta - \alpha)[{}^*w_2\cos\theta - w_1\cos\alpha] \quad (40)$$

or

$$2\cos\theta = \left[\frac{1}{{}^*R} + {}^*R\right]\cos\alpha + \left[\frac{1}{{}^*R} - {}^*R\right]\cot(\beta - \alpha)\sin\alpha \quad (41)$$

APPENDIX B

Brumfield's Criteria

The cavitation number (k) is related to the cavitation parameter (τ) by the following equation

$$\tau = \frac{\text{NPSH}}{u^2/2g} = k(1 + \phi^2) + \phi^2 \quad (42)$$

where equation (42) is the energy equation without prewhirl. Substituting into the suction specific speed (N_{ss}) equation

$$\frac{N_{ss}}{\sqrt{1 - \xi_h^2}} = 8150\,\frac{\sqrt{\phi}}{\tau^{3/4}} \quad (43)$$

differentiating with respect to ϕ (holding k constant) and setting the result to zero, the value of k for which N_{ss} is a maximum is expressed as

$$k = \frac{2\phi^2_{\text{opt}}}{1 - 2\phi^2_{\text{opt}}} \quad (44)$$

therefore, equation (42) becomes

$$\tau = \frac{3\phi^2_{\text{opt}}}{1 - 2\phi^2_{\text{opt}}} \quad (45)$$

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DISCUSSION

Alan H. Stenning[9]

It is shown in Fig. 7 that small variations in ω produce large changes in suction specific speed. No explanation is offered for the apparent variation of ω with ϕ. Reductions in stream tube area can be produced by boundary layers or by radial displacements, and the neglect of radial equilibrium effects in this analysis may be responsible for part or all of the observed behavior. Even though the observations were made at the breakdown point, the whirl velocities leaving the inducer could still be quite substantial since the efficiency at and near this point is close to zero.

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Ward W. Wilcox[10]

Mr. Stripling is to be congratulated on a fine piece of work which adds much to the open literature. Although the cylindrical helical inducer has been known and used for many years, no such correlation of data and theory for a comprehensive family of inducers has appeared. One may hope that the same methods may be applied to the variable lead inducers in common use and extended to include cambered and twisted inducers.

At the Lewis Research Center, detailed radial surveys before and after helical inducers have shown quite drastic radial shifts in streamline location as flow coefficient is varied (including flow reversals at the hub at outlet and at the tip at inlet). This situation, of course, further complicates the application of two-dimensional theories to the three-dimensional machines. On the other hand, it has been shown that, when loss coefficients are known, the outlet axial velocity profile may be computed quite accurately by the simplified radial equilibrium equation. Thus the use of the mixing loss coefficient derived herein should allow better estimation of the outlet vector diagrams which are necessary for proper matching with the main stage impeller.

Glenn M. Wood[11]

The authors are to be congratulated on this very informative series of papers on the subject of cavitation in axial pump inducers. The theoretical models presented in Part 1 are a valuable addition to the present state of the art of cavitation research in rotating machinery.

In discussing Part 2 represented by this paper, I feel that the key figures are numbers 3, 4, 6, 7, and 16 and I shall limit my discussion to these. With regard to Figs. 3 and 4, is there a logical explanation as to why the boundary curves for the 6.5 in. and 5.5-in. OD inducers should be different? This is particularly true of the noncavitating zero head region. It is also apparent in these two figures that the ideal two-dimensional theory deviates substantially from the experimental head breakdown curves. I believe it is very significant that the variation of the slopes of the curves for various tip blade angles as predicted by the ideal theory are opposite to the trend of the data. This could possibly indicate the exclusion of a major variable in the formulation of the theory. It is very interesting to note that the experimental data plots linearly with incidence over such a wide range of design parameters.

The results depicted in Fig. 6 are also significant. This shows that the ideal theory predicts that the tip region is the most susceptible to cavitation, which is consistent with experimental results. However, as I interpret the curve, the introduction of the correction factor ω^*, reflected in eq. (22), indicates a rapidly increasing value of NPSH near the hub. Does this mean that the modified theory would suggest that cavitation would occur at the hub before the tip?

A major experimental contribution of this paper is presented in Fig. 7. This plot consolidates more specific data on the leading edge profiling effect than is currently available in the literature and fills an important gap.

The correlation shown in Fig. 16 is a very interesting comparison between the two-dimensional modified theory and the experimental results for inducer A. This would indicate that the loss mechanism postulated is fairly accurate in predicting the shape of the head drop off curve in cavitation if the head breakdown point can be brought into agreement with the theory. Has this approach been attempted for any impeller other than the constant helix axial inducer?

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Finally, has any work been done to correlate data based on the partial cavitation model approaching incipient cavitation conditions? For long life pumping applications, a definite need exists to predict this incipient point to avoid the possibility of cavitation damage.

Authors' Closure

The authors wish to express their gratitude to Messrs. A. Stenning, W. Wilcox, and G. Wood for their discussions to these papers. As to Mr. Wilcox's comment pertaining to variable lead inducers, the author has attempted limited data correlations with tapered hub, tapered tip, variable lead inducers. The correlations were made with the blade angle and incidence angle, as indicated in this paper, measured from a line constructed tangent to the pressure side at the blade's leading edge. The results were very encouraging but inconclusive at this time.

Mr. Wood's comments are well presented but require further elaboration. If the noncavitating, zero-head boundary curve for the 6.5-in. OD inducers (Fig. 3) were superimposed onto Fig. 4, one would find that a majority of the 5.5-in. OD inducers require a lower incidence angle to produce the zero head region. This effect is due to the smaller blade thickness of the 5.5-in. OD inducers as indicated in Table 1, the exception being inducer N which has an rms blade thickness of 0.146 and, therefore, requires a larger incidence angle to produce zero head. It should also be noted that inducer M (which requires the smallest incidence) has a tip solidity of 1.07 as compared to tip solidities greater than 1.6 for the remaining 5.5-in. OD inducers. Since the cavitation performance (values of τ_t^*) of the 5.5-in. OD inducers are considerably lower for higher blade angles, the curvature of the zero head boundary curve is much more pronounced. The influence of blade thickness on this boundary may be reduced by correcting to a constant blade thickness. It was felt, however, that the data should be presented of the actual test hardware.

Figs. 3 and 4 indicate that in all cases (with the exception of inducer G at high incidence) the values of τ_t^* as predicted by the ideal theory are considerably less than the experimental results. In addition, the experimental curves have a greater negative slope than indicated by the ideal curves. It was felt that this increase in the experimental curves was due to the blockage effects excluded from the ideal theory. It was for this reason that the ω^* term was introduced into the theory. For a given inducer (i.e., blade angle) equations (21) and (22) indicate the same results as shown in Figs. 3 and 4. In addition, if $\phi = \tan(\beta - \alpha)$ is expressed as $\phi = (\beta - \alpha)$ for small fluid angles and substituted into equations (20), (21), and (22), the suction performance of inducers may be shown to vary linearly with the incidence angle.

It is true that Fig. 6 does indicate that cavitation will occur near the hub before the tip when estimating NPSH with the modified equation (22). On several occasions it has been observed by the author that a cavity has occurred near the hub without occurring at the tip. These conditions, however, are the exception and not the rule. The main purpose of Fig. 6 is to demonstrate the radial influence on NPSH when choosing a radial station in estimating the suction performance of an inducer from the theoretical equations. Fig. 6 indicates that for all practical purposes any radial station for constant lead inducers may be chosen to estimate the ideal suction performance, whereas care must be exercised in choosing the radial station when applying equation (22). For small values of Z, equation (22) reduces to $\tau = 3Z$ and the influence of the radial station may be ignored, i.e., equation (22) behaves in the same manner as the ideal equation (21).

The author has mainly confined his investigations to conditions of cavitation where the values of NPSH are below the incipient point. Mr. Wood's comment does rise the question, however, that if the blade thickness were to completely fill the cavity for a given value of NPSH, then would not the incipient point correspond to that value estimated by equations (23), (24), (25), and (26) for a given blade profile as illustrated in Part 1?

The radial equilibrium effects which are discussed by Mr. Stenning may be responsible for the apparent variation in ω with ϕ as shown in Fig. 7. However, the author feels that the radial equilibrium effects are not the sole cause for the ω variations. This conclusion by the author is based on recent experimental tests which have shown that the NPSH value at the 5 to 10 per cent head drop-off point (where considerable visual backflow occurs) agrees with that value which occurs at the complete breakdown condition. For complete breakdown conditions the visual backflow was observed to decrease substantially in magnitude and in many cases completely disappear. In addition, recent experimental results obtained from inducers with blade profiles as shown in Fig. 13, Part 1 (Supercavitation), are in accord with the modified theory for values of the correlation parameter Z in the range 0.002 to 0.006 (Fig. 10).

The author has attempted to explain the variations in ω with ϕ by introducing a form drag coefficient into Eq. (41). Limited calculations of the coefficient resulted in values (inducer C) which were independent of the incidence angle except at low incidence where the values of the form drag coefficient increased with decreasing incidence.