

Fig. 15 Theoretical transmission response with roller cage locked

Interpretation of the transfer function, equation (53), for the free roller-cage condition is not as straightforward. Equation (53) in factored form and with the high-frequency break-point terms, referred to in equation (55) as insignificant, omitted is

$$\frac{\Delta\omega_2}{\beta_c}(S) = \frac{343 \left[\frac{S^2}{(44)^2} + \frac{2(0.8)}{44} S + 1 \right]}{\left(\frac{S}{33} + 1 \right) \left(\frac{S}{32} + 1 \right) \left[\frac{S^2}{(114)^2} + \frac{2(0.542)}{114} S + 1 \right] \left[\frac{S^2}{(354)^2} + \frac{2(0.03)}{354} S + 1 \right]} \quad (62)$$

Comparison of equation (62) with its graphical solution in Fig. 16 indicates that all terms are significant. Roller-mechanism response has not been modified since the 33-rps and 354-rps frequency break points are unchanged. However, the 204-rps term caused by transmission creep has been affected due to interaction with the roller-cage dynamics. These new dynamic relationships can be evaluated by using a revised version of equation (53) which is

$$\frac{\Delta\omega_2}{\beta_c}(S) = \frac{\beta}{\beta_c}(S) \left\{ \frac{2\omega_1 F_0 \left[\frac{(I_{RC} + 4I_2)}{K_{RC}} S^2 + \frac{B_{RC}}{K_{RC}} S + 1 \right]}{\left(1 + \frac{4I_2\omega_1}{3P_n f_s C_n(PD)} S \right) \left[\frac{(I_{RC} + 4I_2)}{K_{RC}} S^2 + \frac{B_{RC}}{K_{RC}} S + 1 \right] - \frac{4I_2}{K_{RC}} S^2} \right\}$$

Numeric evaluation of the numerator quadratic gives

$$\frac{(I_{RC} + 4I_2)}{K_{RC}} = \frac{1}{(43.9)^2}$$

and

$$\frac{B_{RC}}{K_{RC}} = \frac{2(0.803)}{(43.9)}$$

Solution of the denominator shows that it has roots corresponding to frequency break points at 32 and 113 rps. This agreement substantiates the assumption that roller-cage velocity in the gyroscopic torque term of equation (39) has a negligible effect.

Conclusion

The primary goal of a theoretical analysis is to be able to correlate system physical parameters with performance. With the theoretical derived relationships, the engineer has the ability to adjust the parameters rationally in designing a particular system to meet a given set of performance specifications; this is the significance of equations (58), (60), and (63). While the derived transfer functions specifically describe one type of commercial

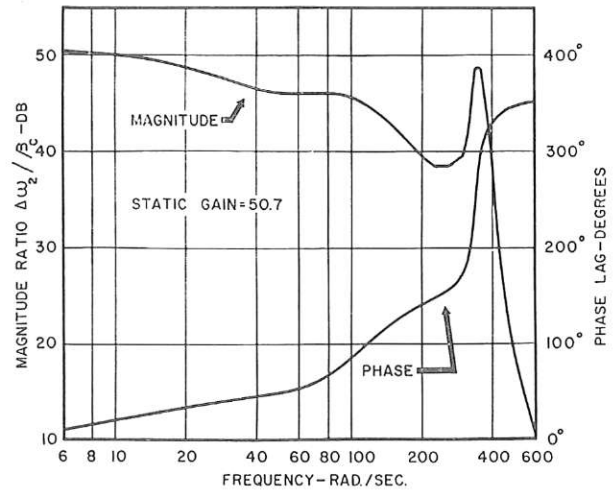


Fig. 16 Theoretical transmission response with roller cage free to rotate

transmission, most of the basic concepts such as creep, roller steering, and friction-contact forces are inherent in any toric variable-speed drive. Therefore these equations will provide a qualitative

parametric interpretation of the basic factors controlling linear toric dynamic performance.

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DISCUSSION

F. R. Erskine Crossley⁷

The authors do not refer to any previous engineering experience with a toric variable-speed drive; I wonder therefore whether they know that an automobile transmission of this type was available as an option with the larger Austin passenger cars in England, in, I believe, the years 1935 and 1936. I had one, or rather it was our family car: The transmission was called the "Hayes Gear," if I remember correctly. It was controlled from a quadrant under the steering wheel; with it, it might have been possible to maintain the engine at either the revolutions for peak power in acceleration, or for maximum fuel economy, at varying road speeds; but unfortunately the Austin people neglected to equip the car with an engine tachometer. A remarkably smooth

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drive was therefore its chief characteristic, and a startling acceleration was obtainable with practice.

Its fault was inherent and inevitable: Every automobile is driven mostly in high gear. The pressure and friction gradually wore a step in this position on the inside surfaces of the torus. After a couple of years there were two steps worn. It required quite a jerk on the lever to get the roller out of the grooves. So we were back again where we started, with several discrete velocity ratios, but with a poor shift-lever. But the novelty had been most enjoyable.

J. S. Rubin⁸

For the past several years, Lycoming has been building toric variable ratio transmissions for use in aircraft constant speed drive systems. The toric geometry is used as the power transmission with an appropriate speed control for maintaining constant output speed through the input speed range. This type of drive was selected because of its "good dynamic response and high accuracy."

In the course of the constant speed drive program, dynamic analyses similar to the one described have been performed. Derivation of the transfer function for the transmission was based on the same criteria used by the authors. However, these criteria were applied in a different manner.

During the preliminary analysis, test results obtained with hardware showed very poor agreement with the equations. Further analysis showed that the cause of this disagreement was a term which was eliminated from the equation as being insignificant. This term was the velocity change in the roll caused by deflection of the roll cage.

When the roll cage is restrained from rotating, it is subjected to reactive loads which are twice the total tractive force at the frictional contacts. For a relatively low powered drive such as was used by the authors, these reactive loads are extremely small. The roll cage, therefore, appears to have infinite stiffness. In the relatively high powered drive built by Lycoming, these loads are considerable. The cage no longer has extreme stiffness, but deflects considerably under these loads. Since these loads are a function of both output load and input speed, the resultant deflections are of significance in the dynamics of the high powered drive. To illustrate this difference in power levels, it will be sufficient to state the power to size ratio of each drive. The drive used by the authors has a ratio of approximately 3.5×10^{-3} hp/in.³ while the Lycoming drive has a ratio of approximately 80×10^{-3} hp/in.³ or about 25 times as much power per unit of volume.

It must be pointed out, however, that the elimination of this velocity term from the equations is considered valid for the low powered drive used by the authors.

With the velocity term reintroduced into the equations, the test results obtained from the hardware were in close agreement with the equations written for the test unit.

The approach taken by the authors and the criteria used in

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deriving the transfer function were much the same as those used in the analysis of the Lycoming drive. A good correlation between test results and equations was obtained in both cases; therefore, it can be assumed that the conclusions reached by the authors are valid.

C. F. Schwan⁹

It is quite probable that a portion of the tractive force is carried through the shear stresses developed in the oil film existing between the driving and driven members, as well as through actual contact of these parts. Has the proportion of the total tractive effort transmitted through shear stresses in the oil film been investigated for the toric drive in establishing values of creep for different loads, or forces between the rollers, and toric wheels, and, if so, what relationships are believed to exist?

What are the maximum Hertzian sub-surface shear stresses in the rollers and wheel?

Authors' Closure

The authors wish to thank Professor Crossley, Mr. Rubin, and Mr. Schwan for their interesting comments. The authors are aware of several applications of the toric transmission including the one referred to in Professor Crossley's remarks. As regards Mr. Rubin's comments, it is really a great pleasure to know that the results arrived at by the Lycoming group are similar to the ones described in the paper.

In reply to C. F. Schwan's question on the portion of the traction force contributed by the shear stresses developed in the oil, it is reasonable to assume that a portion of the tractive force is supported by the reactive forces produced by the oil shearing stress present in the contact area. At high normal loads and low speeds the oil film is squeezed out of the contact area and is only present in the boundary region of the contact area where the normal pressures are low. Experimental measurements have found that the magnitude of the tractive force is the largest at this condition. Keeping the normal load constant and increasing speed creates an increasing hydrostatic film pressure which tends to separate the two metallic bodies, thus reducing the traction force. There is some speed at which the whole contact area will have a hydrostatic film covering; at this speed the traction force is extremely low. Oils of the naphthenic type allow greater traction forces.

The portion of the tractive force due to oil shearing stresses has not yet been determined. A method of determining oil viscosity and oil thickness at the friction contact will have to be devised, since both oil viscosity and film thickness are needed to calculate shearing stresses of the oil.

In reply to Mr. Schwan's question regarding subsurface stresses: The maximum Hertzian stresses at the contact area vary with normal load and roller design but usually are limited to 3-400,000 psi. Exact values depend upon the application of the toric transmission.

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