the evaluation of coefficients λ and A. It is known, for example, that a large recess volume is probably the main cause of instability. This volume can always be reduced when the recess is replaced by a groove along its circumference, while the central area remains at the same elevation as the bearing surface. The static pressure fields of such grooved and recessed pads are identical, but the degree of validity of the assumption that the pressure profile changes quasi-statically will depend on the geometry of the pad, which is known, and the motion of the journal, which is unknown. Experimental verification of an analysis based on a distributed parameter approach [4], is being presently conducted by the author. It is hoped that this work will point to a method of "improving" the coefficients of the characteristic equations for cases where the present assumptions may be deficient.

The use of perturbation techniques may be justified if the nonlinearities involved are of the "slowly varying" kind [5]. The method is still relatively simple in comparison with computer solutions of Reynolds equation in which the term $(\partial/\partial t)(PH)$ is retained and boundary conditions are time dependent.

While it may appear that the sole purpose of the preliminary work would be directed toward the evaluation of coefficients required in stability analyses, very little (and often enough nothing) is known about the static characteristics of externally pressurized journal bearings. The "preliminary" work, therefore, while furnishing the means of performing simplified stability analyses, represents a task worth while undertaking in itself, be it merely by virtue of the fact that determination of such static characteristics as load-carrying capacity, local stiffness, air, and power consumption cannot be accomplished by assuming linear pressure-gradient approximations.

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DISCUSSION

D. F. Hays[9]

In this paper the author has investigated the stability of externally pressurized symmetric multipad journal bearings and has shown that it is possible to factor the characteristic equation, and, hence, to reduce the degree of the equation whose roots must be investigated. The question which arises is whether or not the assumption which resulted in this centro-symmetric matrix and the factorable form of the characteristic equation are realistic, and, therefore, whether the stability criterion is meaningful in the range of operating parameters considered. In raising this question the author has indicated two possible techniques which could corroborate his stability analysis. In the first of these, the Reynolds equation would be solved for a series of geometries. In so doing, the boundary conditions of Equation (24) of the paper must be altered such that the recess pressure P_r extends only along the recess periphery and not entirely across the bearing width; thus

$$P(x_a;\ z) = P(x_b;\ z) = P\left(x;\ \pm\ \frac{l}{2}\right) = P_r$$

$$-\ \frac{l}{2} \leq z \leq +\ \frac{l}{2}, \quad a \leq x \leq b$$

Since the stability coefficients are obtained from numerical differentiation of the computed pressure and mass flow profiles, an extremely small grid size would be indicated along the recess boundary if much accuracy is desired. There are several techniques which will assist in stabilizing the process of numerical differentiation. Such a numerical investigation would certainly be of interest and of value.

The experimental procedure for pressure and flow measurement would be an absolute check on the validity of the assumptions used in this lumped parameter stability analysis and in the solution of Reynolds equation. There are questions, though, as to the accuracy of results obtained from the apparatus shown in Fig. 6 if conical pivots are used. Upon loosening the pivots for rotation of the journal, the subsequent tightening of them may not return the apparatus to the fixed eccentricity due to metallic contact and small clearances. The use of a liquid externally pressurized bearing would allow both greater freedom of movement and accuracy. A similar comment applies to Fig. 7 where a head and tailstock are used to hold the fixture. Such methods would surely create deflections of the journal which would invalidate the data obtained. If the validity of the analysis is to be judged upon the experimental data, then extreme care must be exercised in maintaining accurate geometries.

Stability curves similar to Fig. 4 could result from this endeavor and such curves would be of value. Could the author comment on the occurrence of two critical points on the curves of constant mass m?

In closing the author alludes to experimental work being done in verification of a distributed parameter approach to stability. Is work also currently being done in line with the suggested experimental program of this paper and, if so, are data available?

Author's Closure

Dr. Hays is quite right with regard to the omission in equation

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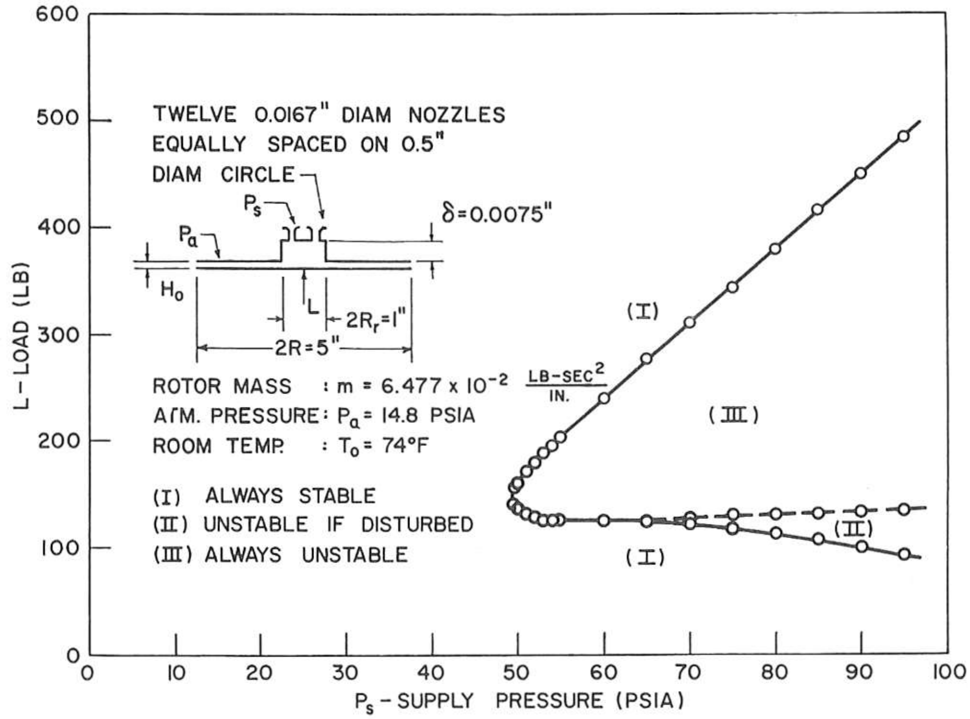

Fig. 8 Stability locus of externally pressurized, air-lubricated thrust bearing

(24). The correction also applies to the dimensionless equation (26), in which $-l/D \leq \zeta \leq +l/D;\ \theta_a \leq \theta \leq \theta_b$ must be added to complete the statement of the boundary condition for the recess (or inner groove) perimeter.

The author did not intend to minimize the difficulties involved in numerical and experimental techniques for obtaining accurate data of pressure distribution, mass flow-rate, and all that is required for the determination of the stability coefficients and bearing characteristics in general. It may be more convenient to carry out the integration in equation (33) along the outer pad perimeter instead of along the recess perimeter. One may also consider the feasibility of solving this type of Reynolds equation by means of the electrolytic tank analogy as an alternative to the digital computer. In suggesting an experimental method for the measurement of pressure and flow, it has not been anticipated that the schematic diagrams in Figs. 6 and 7 would be interpreted as anything else but as schematic diagrams. The author appreciates the helpful suggestions offered by Dr. Hays and wishes to assure him that he has also considered many additional refinements which one would have to incorporate in the proposed apparatus.

Quite frequently, having defined a given set of quantities and parameters, the author is content to state the end results of his analysis in most general terms and delegate to others the task of their interpretation. This writer believes that the suggested determinations of important characteristics of externally pressurized bearings represent a logical supplement to the stability analysis, but that the essential contribution and main subject of this paper is the method of simplifying the characteristic equation. This fact is not reflected in Dr. Hays's discussion.

In reply to Dr. Hays's first question, our experience indicates that there may be more than one "critical point" for externally pressurized, gas-lubricated journal and thrust bearings. A rotor supported in multi-pad journal bearings was stable at low values of the supply pressure. An increase of the supply pressure was accompanied by a self-sustaining oscillation, but at a relatively high value of the supply pressure the rotor became stable again. The evaluation of the coefficients λ and A represents no special difficulty in the case of a simple geometry, bidirectional, parallel-surface thrust bearing. Equations (15) and (16) may be used to obtain stability loci similar to those shown schematically in Fig. 4. The appended Fig. 8 shows an experimentally determined stability locus for a unidirectional, circular thrust bearing, with a centrally located recess. In this case the variable parameters were the load and the supply pressure. The lower-right branch of the locus, which corresponds to relatively high values of supply pressure and of gap width, indicates a nonlinear behavior. In this region, for a given supply pressure and in the absence of large disturbances, it is possible to attain a relatively high load value before the onset of auto-oscillation. The load must then be reduced by a considerable amount before the bearing becomes stable again. This "hysteresis" phenomenon does not occur on the entire upper and on the lower-left branch of the locus.

In reply to Dr. Hays's second question in reference to the distributed parameter approach to stability, this has been reported in "An Analytical and Experimental Study of the Stability of Externally-Pressurized, Gas-Lubricated Thrust Bearings," by L. Licht and H. G. Elrod, Jr., Franklin Institute Report I-A2049-12, prepared for the Office of Naval Research, Contract Nonr-2342-(00), Task NR 061-113, February, 1961. A paper summarizing the experimental phase of this work is in the process of preparation. Unfortunately, there are, as of this time, no research funds available to this investigator with which to carry out an experimental program and the associated numerical work along lines suggested in this paper.