

- 3 The film thickness was calculated at the beginning, middle, and end of each section assuming that the surfaces were rigid.
- 4 The viscosity was calculated from Equation (14) assuming  $p_j = 0$ .
- 5 Using results obtained in steps 3 and 4, Equation (11) was solved for pressure in the middle of every section.
- 6 Using results from step 5, Equation (13) was solved for new temperature field.
- 7 New viscosity was now calculated using Equation (14).
- 8 With the pressures obtained in step 5, deformation was calculated using Equation (15).
- 9 New film thickness was calculated by Equation (16).

$$h_0 = h_{\min} + \frac{x^2}{2R} - H(\xi) \quad (16)$$

- 10 New pressures were then recalculated using viscosity from step 7 and film thickness from step 9.
- 11 This process was repeated until the pressures, temperatures, and deformations converged within a prescribed limit of error.

In this analysis the maximum prescribed error was specified to be less than 0.1 per cent. The

$$\frac{\sum_{j=1}^m |p_j^k - p_j^{k-1}|}{\sum_{j=1}^m |p_j^k|} < 0.001 \quad (17)$$

After some study of the effect of mesh size on error and time for convergence we set  $m = 50$  for these calculations and it took approximately forty-five iterations  $k \approx 45$  before convergence to the prescribed error occurred. Inspection of pressure distribution at the  $k$  and  $k - 1$  iteration showed maximum pressure variation at any point to be less than 2 per cent.

Using the foregoing equations and the low VI oil characteristics given in Appendix 2, Table 1 was calculated. Likewise, using the elastohydrodynamic analysis outlined in steps 1 to 11 comparison on performance with the low and high VI oils was made and is given in Table 2. The characteristics of the low and high VI oils are given in Appendix 2. In the numerical analysis a table of "look up" obtained from Fig. 9 was used for the viscosity calculations rather than Equation (14). This was done in order to obtain higher accuracy.

## APPENDIX 2

### Properties of the Blended Oils

Pressure psig ↓	High VI		Low VI	
	Temp. → 100° F	Viscosity in Centipoises 210° F	100° F	210° F
0	57.1	6.83	56	5.4
2000	83	9	84	7
4000	110	11	120	9
6000	150	14	174	11
8000	200	17	252	14
10,000	257	21	370	18
20,000	970	51	2520	57
40,000	11,100	250	$1.1 \times 10^5$	570
60,000	$1.3 \times 10^5$	1060	$4 \times 10^5$	5400
80,000	$1.5 \times 10^6$	4110	$1.6 \times 10^6$	43,000
100,000	$1.7 \times 10^7$	15,400	$6.7 \times 10^6$	$3.2 \times 10^5$
VI	102		15	
Density at 76°F	0.85 gm/cm <sup>3</sup>		0.90 gm/cm <sup>3</sup>	

**Acknowledgment.** These oils were blended and supplied for this program with the foregoing data by Mr. H. A. Hartung of the Atlantic Refining Company.

## DISCUSSION

### H. Poritsky<sup>3</sup>

This paper considers (for the first time so far as I know) rolling contact between cylinders, including effect of *change of viscosity with both pressure and temperature*, as well as the effect of *deformation contact surfaces*. It is, therefore, in my opinion, a more complete treatment than has been hitherto available. Calculations of this kind will eventually enable engineers to choose a lubricant most suitable for any design of gears, cylindrical rollers, or cams, purely from the physical characteristics of the lubricant and the load, speed, and other features of the mechanical design.

The effect of temperature is calculated on the assumption that all the heat developed is taken up by the oil film. An estimate should be made of the effect of heat conductivity, since it is certainly true that the contacting surfaces absorb heat, and in the process of heating up will keep the film temperature down.

If possible, a comparison should be made between the rate of heat flow into the lubricant and the power losses in the lubricant in a gear set, as obtained, for instance, by D. W. Dudley of the General Electric Company.

The finite difference equation which replaces Equation (10), namely, Equations (12, 13), is solved, along with the integral relation (15), by a successive approximation process, outlined in steps (1-11) on pages 221, 222. It would be of interest to describe the calculation in more detail, stating how many points on the contact strip are used, how many repetitions were necessary to secure convergence, how accurate the final value may be, etc.

I take particular exception to the use of Equation (15) for the calculation of the deformation of the surface due to a given pressure distribution. No mention is made of what the relation between  $\xi$  and  $x$  is, nor is any calculation of the displacement beyond the pressure area given. Equation (15) must be based on conformal mapping of the half plane on a semi-infinite strip, with the pressure interval going into the edge of the strip. A method which lends itself to greater accuracy is described in Poritsky, reference [8], in which the displacement due to the Hertz effect is obtained analytically (both inside and outside the Hertz area) and the added deformations due to the "tails" of the pressure distribution are computed by a numerical method which is much simpler than the one used by the authors. Finally, I am surprised that no linear term  $A + Bx$  has been added to the film thickness corresponding to a small relative rotation of the contacting members, since a rigid displacement may also be added to any solution of an elastic problem.

Incidentally, the same paper (Poritsky, reference [8]) gives a proof of the Hertzian nature of the load distribution which can be easily carried over to the case where both the pressure and temperature vary. This proof is based on the fact that the function

$$P = \int \frac{dp}{\mu}$$

reaches a nearly constant value for the high-pressure areas; this integral remains nearly constant even when the viscosity varies *both* with temperature and pressure, just so long as the viscosity is large. One thus does not have to depend upon the complicated numerical integrations of the equations mentioned to reach the conclusion that the pressure distribution is nearly Hertzian over the major part of the contact area.

The curves of Fig. 4 give photoelastic constant-shear curves for both dry and wet lubrication. These curves might be compared with the curves computed by H. Poritsky in "Stresses and Deformations of Cylindrical Bodies in Contact With Application to Con-

<sup>3</sup> Consulting Engineer, General Electric Company, Schenectady, N. Y. Mem. ASME.

tact of Gears and of Locomotive Wheels," *Journal of Applied Mechanics*, vol. 17, 1950, pp. 191-201. Figs. 10 and 12, in which are computed curves of constant shear for a Hertzian load accompanied by a shearing load which is proportional to it.

Finally, since ball-bearing tests are used to confirm the theory, it is surprising that the differences between the contact stresses, displacements, and fluid flow between balls and races, and rolling cylinders is never mentioned.

## R. Rhoads Stephenson<sup>4</sup>

### Part 1

The theoretical work in this paper is quite remarkable in that an energy equation has been solved in conjunction with the elastohydrodynamic problem. The energy equation gives an approximation to the temperature field in the lubricant film and thus allows the effect of viscosity variations with temperature to be included. The authors make some assumptions, however, which do not appear to be valid in the cases studied. For purposes of discussion, it will be useful to present a brief derivation of the energy equation in the form used in the paper.

Making the usual assumptions of lubrication theory, the energy equation in differential form may be shown to be [25]:<sup>5</sup>

$$\mu \left( \frac{\partial u}{\partial y} \right)^2 = \rho c_v u \frac{\partial T}{\partial x} - K \frac{\partial^2 T}{\partial y^2} \quad (18)$$

where  $K$  = thermal conductivity,  $c_v$  = specific heat at constant volume,  $\rho$  = mass density, and the other symbols are the same as defined in the paper. The left-hand side of this equation represents the heat generated due to viscous dissipation and the terms on the right represent the convection of heat along the film and the conduction of heat across the film, respectively.

For the moment I will assume, as the authors did, that the temperature is uniform across the film so that the last term in (18) is zero. (This assumption implies an adiabatic situation. However, assuming adiabatic conditions only requires that the temperature gradient is zero at the surfaces and it is possible to have large temperature variations across the film. For an example, see [26].)

The velocity distribution within the film is given by

$$u = \frac{h^2}{2\mu} \left( \frac{dp}{dx} \right) \left\{ \left( \frac{y}{h} \right)^2 - \left( \frac{y}{h} \right) \right\} + (U_2 - U_1) \left( \frac{y}{h} \right) + U_1 \quad (19)$$

where  $U_2$  is the velocity of the upper surface and  $U_1$  is the velocity of the lower surface in the  $+x$  direction. Note that the first two terms correspond to the velocity distribution that would occur in Poiseuille flow and the remaining terms represent that which would occur in Couette flow. Substituting equation (19) into the simplified form of equation (18) and integrating across the film yields the energy equation:

$$\frac{h^3}{12\mu} \left( \frac{dp}{dx} \right)^2 + \frac{\mu}{h} (U_2 - U_1)^2 = \rho c_v \left[ -\frac{h^3}{12\mu} \left( \frac{dp}{dx} \right) + \frac{(U_2 + U_1)}{2} h \right] \frac{dT}{dx} \quad (20)$$

Again the terms on the left represent the heat generated due to viscous dissipation. The first term may be associated with Poiseuille flow and the second with Couette flow. Under the condition of pure rolling studied in the paper,  $U_1 = U_2$ , and the Couette term vanishes.

<sup>4</sup> Graduate Student, Department of Mechanical Engineering, Carnegie Institute of Technology, Pittsburgh, Pa.

<sup>5</sup> Numbers 25 to 29 in brackets designate References at end of this discussion.

### Part 2

The foregoing equation has been based on the assumption that there are no temperature variations across the film. It is doubtful, however, whether this assumption is valid for heavily loaded contacts. An approximate analysis has been developed which allows the assumption of adiabatic surfaces (and hence uniform temperature across the film) to be checked. When applied to the physical situation studied in the paper, it is found that a considerable amount of heat will be conducted across the film and that the assumption is not valid.

The method is as follows:

(1) We assume that the surfaces are adiabatic, solve the resulting equations, and find the lubricant temperature at the surface as a function of  $x$ .

(2) We make use of the fact that the adiabatic condition implies that the surface temperature of the metal must be the same as that of the lubricant. Therefore a point on the surface, traveling through the contact region, will experience a known temperature rise.

(3) The amount of heat required to raise the surface temperature at the known rate is determined.

(4) This quantity of heat is compared with the total heat generated within the contact region. If it is large compared with the total heat generated, then the surfaces cannot possibly be adiabatic. If it is small, then the adiabatic assumption is justified.

Step 3 is a difficult problem and an approximate analysis is useful in determining this quantity of heat. Consider a thin slice of material cut from the roller (or gear tooth) as shown in Fig. 11.

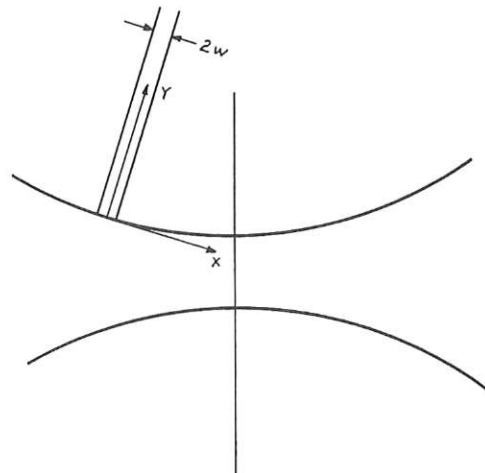


Fig. 11 Co-ordinate system on the roller

This slice extends to infinity in the  $\pm z$  directions and in the  $+y$  direction (this is a new co-ordinate system set up on the slice). The thickness in the  $x$  direction,  $2w$ , is taken very thin, and the planes  $x = \pm w$  are assumed to be insulated. As the slice moves through the contact region its surface is subjected to a known temperature rise. Knowing the surface velocity, the surface temperature can be specified as a function of time. Therefore the temperature in the slice is only a function of  $y$  and the time  $\tau$ .

The differential equation governing this situation is

$$\frac{\partial^2 \theta}{\partial y^2} = \frac{1}{\alpha} \frac{\partial \theta}{\partial \tau} \quad (21)$$

where  $\alpha$  is the thermal diffusivity and  $\theta$  is the difference between the actual temperature and the initial temperature. The initial temperature is assumed to be uniform. This is reasonable since this material has been out of the contact region for the longest



possible time and the temperature variations probably have had a chance to die out. The boundary conditions are:

$$\theta = 0 \quad \text{at} \quad \tau = 0 \quad \text{for} \quad y \geq 0 \quad (22a)$$

$$\theta = f(\tau) \quad \text{at} \quad y = 0 \quad \text{for} \quad \tau \geq 0 \quad (22b)$$

$$\theta \rightarrow 0 \quad \text{as} \quad y \rightarrow \infty \quad (22c)$$

It is consistent with the adiabatic assumption to consider the lubricant and the surface to be at the same temperature *before* entering the contact region. Hence  $f(0) = 0$ .

The differential equation (21) subject to the boundary conditions (22) can be solved once  $f(\tau)$  is specified. Detailed information on the temperature distribution is not available for the solution presented in the paper so a linear variation is assumed, e.g.,  $f(\tau) = a\tau$ . For this function the temperature distribution is given by

$$\theta(y, \tau) = a \left\{ \left( \tau + \frac{y^2}{2\alpha} \right) \operatorname{erfc} \left( \frac{y}{2\sqrt{\alpha\tau}} \right) - y \left( \frac{\tau}{\pi\alpha} \right)^{1/2} e^{-\frac{y^2}{4\alpha\tau}} \right\} \quad (23)$$

and the heat flux at the surface is

$$Q = -K \frac{\partial \theta}{\partial y} \Big|_{y=0} = 2Ka \left( \frac{\tau}{\pi\alpha} \right)^{1/2} \quad (24)$$

The total heat added during the contact time  $\tau_0$  is

$$Q_T = \frac{4Ka}{3(\pi\alpha)^{1/2}} \tau_0^{3/2} \quad (25)$$

The foregoing solution depends on the assumption that the slices are insulated from one another so that there is no heat conduction in the  $x$  direction. This is a conservative assumption in the sense that conduction in the  $x$  direction would counteract large surface-temperature gradients and would tend to make the surfaces *less* adiabatic (more isothermal). The amount of heat required to raise the surface temperature will thus be underestimated.

This method will now be applied to the case for the high VI oil shown in Table 2 of the paper. For steel disks  $K = 6.95 \times 10^{-3}$  Btu/ft sec deg F and  $\alpha = 1.26 \times 10^{-4}$  ft<sup>2</sup>/sec. The temperature increase of approximately 300 deg F will be assumed to take place over the width of the Hertzian contact region,  $2b$ . Based on the maximum pressure of 120,000 psi we find that  $2b = 7.35 \times 10^{-3}$  feet. The surface speed of 785 ft/min gives a contact time  $\tau_0$  of  $5.65 \times 10^{-4}$  seconds. Therefore we find that  $a = 5.3 \times 10^5$  deg F/sec and using equation (25) we find that

$$Q_T = 3.3 \text{ Btu/ft}^2$$

For comparison purposes, the total amount of heat generated in the lubricant will be calculated. Nearly all of the viscous dissipation takes place within the Hertzian region, so it is consistent to use

$$Q' = \frac{1}{2b} \int_{-b}^{+b} \frac{h^3}{12\mu} \left( \frac{dP}{dx} \right)^2 dx \quad (24a)$$

Since detailed information for the viscosity variation, film thickness, and pressure distribution is not presented in the paper,  $Q'$  will be estimated by using typical values to evaluate the integrand. In the elastohydrodynamic problem,  $h$  is nearly constant over the contact region and is about 25 per cent greater than the minimum film thickness. Hence  $h$  will be considered to be  $10^{-5}$  inches. The kinematic viscosity will be taken as 100 centistokes and  $\rho = 1.7$  slugs/ft<sup>3</sup>. The pressure gradient will be approximated by  $P_{\max}/b$ . Using these values, it is found that

$$Q' \approx 0.76 \text{ Btu/ft}^2 \text{ sec}$$

During the contact time the total amount of heat generated is

$$Q_{T'} = 4.5 \times 10^{-4} \text{ Btu/ft}^2 \quad (25a)$$

The calculation of  $Q_{T'}$  is admittedly very approximate (especially in taking the average of the square of the pressure gradient). Perhaps the authors would numerically evaluate equation (24a) so as to provide a more accurate result. It is hard to believe, however, that the estimate would be off by 4 orders of magnitude. Hence it may be concluded that there is not enough heat available to raise the surface temperature as fast as was anticipated. Therefore a large amount of the heat generated in the film will be conducted across the film to the walls rather than convected along the film. In this case the complete energy equation (18) should be used.

On the other hand, if  $Q_{T'}$  was much greater than  $Q_T$ , it would probably be valid to neglect the conduction term in equation (18), although it would be wise to check the effect of conduction in the  $x$  direction in the metal.

One may ask if the assumption that the slice extends to infinity in the  $y$  direction is valid. Using equation (23) it can be shown that at the end of the contact time the temperature is just beginning to be felt at a depth of about 0.01 in. Hence all of the temperature variations take place in a very thin surface layer.

### Part 3

It is interesting that a deformation equation for finite cylinders was used in the elastohydrodynamic calculations. A development of equation (15) can be found in reference [27]. Equation (15) is based upon the assumption that the pressure distribution is symmetric about the line  $\eta = 0$ . This is true for the Hertzian contact problem but it is *not* true for elastohydrodynamic lubrication. This equation implies that the deformations are also symmetric about the line  $\eta = 0$ , which does not agree with previous solutions [28, 29].

The method of reference [27] can be used to give a deformation equation for any pressure distribution. The combined deformation of both surfaces for cylinders of the same material and radius is

$$H(\xi) = \frac{4(1-\nu^2)R}{\pi E} \int_{-\pi/2}^{\pi/2} \left\{ \ln \tan \frac{1}{2} |\eta - \xi| + \ln \left[ \frac{\cos \frac{1}{2} |\eta - \xi|}{\cos \frac{1}{2} |\eta + \xi|} \right] + \cos \eta \cos \xi \right\} P(\eta) d\eta \quad (26)$$

where the integration is carried out over the entire pressure region.

It is easily shown that for large values of  $R$ , and hence small angles of contact, equation (26) reduces to

$$H(x) = \frac{2(1-\nu^2)}{\pi E} \int_{-\infty}^{\infty} \ln(x-s)^2 P(s) ds + \text{constant} \quad (27)$$

which is the relation normally used for the deformation of infinite half spaces [29].

### References

- 25 W. F. Cope, "The Hydrodynamical Theory of Film Lubrication," *Proceedings of the Royal Society of London*, series A, vol. 197, 1949, p. 201.
- 26 W. B. Hunter and O. C. Zienkiewicz, "Effect of Temperature Variations Across the Lubricant Films in the Theory of Hydrodynamic Lubrication," *Journal of Mechanical Engineering Science*, vol. 2, 1960, pp. 52-58.
- 27 T. T. Loo, "Effect of Curvature on the Hertz Theory for Two Circular Cylinders in Contact," *Journal of Applied Mechanics*, vol. 25, TRANS. ASME, vol. 80, 1958, pp. 122-124.
- 28 D. Dowson and G. R. Higginson, "A Numerical Solution to the Elasto-Hydrodynamic Problem," *Journal of Mechanical Engineering Science*, vol. 1, 1959, pp. 6-15.
- 29 D. Dowson and G. R. Higginson, "The Effect of Material Properties on the Lubrication of Elastic Rollers," *Journal of Mechanical Engineering Science*, vol. 2, 1960, pp. 188-194.

### Authors' Closure

We wish to thank Dr. Poritsky and Mr. Stephenson for their prepared discussions. It has been most gratifying to see the amount of interest that a number of other investigators have recently shown in our studies. We sincerely hope that others will attack related aspects of this problem for we feel that valuable contributions can be made in this virgin field of thin film lubrication and its effect on failure of machine elements.

Both discussers bring up the point of heat conduction. We have clearly stated that heat conduction has been neglected in the presented study. In fact our sixth recommendation suggests that future investigations should include heat conduction. Our eighth recommendation goes further and suggests that thermal stresses and thermal deformations caused by thermal gradients in the metal be considered in future analysis.

Our criterion for rolling-element fatigue suggests that thermal stresses caused by thermal gradients within the metal play a major role on fatigue for different lubricants. We also recommend the establishment of quantitative data on fatigue as a function of load, speed, and temperature.

It must be recognized, however, that the amount of conduction will vary in different applications. In gear applications where sliding is considerably higher than in rolling elements the temperature rise is considerably greater. We have made some estimates between the heat flow into the lubricant and the power loss in a gear set as reported by D. W. Dudley of General Electric and found that the comparison is remarkably good.

Mr. Stephenson introduced tremendous errors in his evaluation of equation (24a) by making the following two assumptions

$$\frac{dp}{dx} = \frac{p_{\max}}{b} \quad \text{and} \quad \mu = \text{constant} = 100 \text{ centistokes.}$$

The pressure gradients are considerably higher than  $p_{\max}/b$  on both the leading and trailing edges of the contact zone, as seen from Fig. 2. Numerical integration of equation (24a) with only six  $dx$  increments gives a value of  $Q' = 1972$  Btu/ft<sup>2</sup> sec rather than the  $Q' = 0.76$  Btu/ft<sup>2</sup> sec computed by Mr. Stephenson.

Recent measurements, performed by Dr. D. R. Whitney of General Motors Research Laboratory, of the average fluid film temperature within the contact zone agree well with our calculated values. If anything, the experimental results are on the high side. (The technique is based on the British work of Dr. Crook where the product of RC is calculated from the measurements of total resistance and capacitance. This procedure cancels out the effect of geometry and RC is proportional to temperature. For an order of magnitude change in RC there is approximately 100 deg F change in temperature.) These results indicate that the lubricant remains virtually adiabatic and that little heat conduction takes place during the short contact time. The question of conduction deserves considerably more attention along both theoretical and experimental lines. In our present program we are concentrating on this problem.

With reference to the question raised by Dr. Poritsky on numerical analysis we used approximately 50 mesh points and the spacing between them was not always equal. It takes somewhere between thirty and sixty iterations for the results to converge. The number of iterations increases with load and speed. We set the limit of accuracy to within 0.1 per cent in the total pressure variation between successive iterations. We expect that more points will be required when heat conduction and heavier loads are considered.

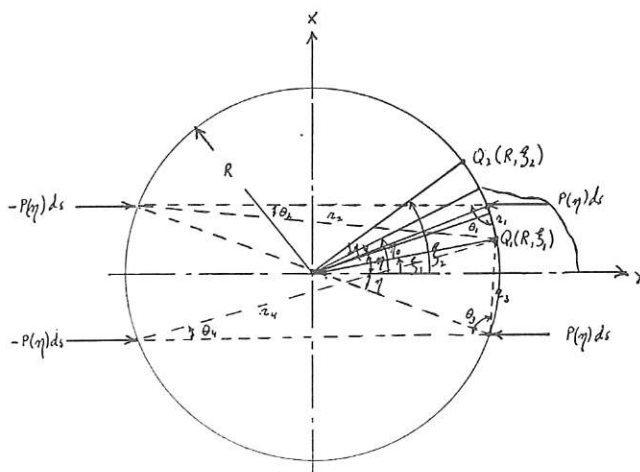


Fig. 12

With reference to the elastic equation the last term of equation (15) considers the deformation caused by antisymmetric pressure. This makes the comment by Mr. Stephenson invalid.

The comments raised by Dr. Poritsky are discussed next.

- (1) The relation between  $\xi$  and  $x$  as seen from Fig. 12 is  $x = R \sin \xi$ .
- (2) Calculation beyond the pressure area.

It should be noted that equation (15) results from

$$dH(\xi) = \left\{ \frac{k+1}{4\Lambda\pi} \left[ \ln \frac{r_2}{r_1} + \ln \frac{r_4}{r_3} \right] + \frac{1}{4\Lambda\pi} [(\cos 2\theta_1 - \cos 2\theta_2) + (\cos 2\theta_3 - \cos 2\theta_4)] - \left[ \frac{2(k-1)}{4\Lambda\pi} \cos(\eta) \frac{y}{R} \right] \right\} P_1(\eta) ds + \left\{ \frac{k+1}{4\Lambda\pi} \left[ \ln \frac{r_2 r_3}{r_1 r_4} + \frac{(\cos 2\theta_1 - \cos 2\theta_2)}{(k+1)} - \frac{(\cos 2\theta_3 - \cos 2\theta_4)}{(k+1)} \right] \right\} P_2(\eta) ds \quad (28)$$

The first term corresponds to the symmetric case and the last to antisymmetric. For  $Q_1(R, \xi_1)$  where  $\xi_1 < \eta$  it is evident from Fig. 12 that

$$\left. \begin{aligned} r_1 &= 2 \sin^{1/2}(\eta - \xi) \\ r_2 &= 2 \cos^{1/2}(\eta + \xi) \\ r_3 &= 2 \sin^{1/2}(\eta + \xi) \\ r_4 &= 2 \cos^{1/2}(\eta - \xi) \\ 2\theta_1 &= \pi + \eta + \xi \\ 2\theta_2 &= \eta - \xi \\ 2\theta_3 &= \pi - \eta - \xi \\ 2\theta_4 &= \eta + \xi \\ \frac{y}{R} &= \cos \xi \\ ds &= R d\eta \end{aligned} \right\} \quad (29)$$

Substituting the above relations into equation (28) we obtain equation (15). For the case of  $\xi_2 > \eta$  (outside the load  $P(\eta) ds$ ) we obtain from Fig. 12 the following

$$\left. \begin{aligned} r_1 &= 2 \sin^{1/2}(\xi - \eta) \\ r_2 &= 2 \cos^{1/2}(\xi + \eta) \\ r_3 &= 2 \sin^{1/2}(\xi + \eta) \\ r_4 &= 2 \cos^{1/2}(\xi - \eta) \\ \cos 2\theta_1 - \cos 2\theta_2 &= -2 \cos \eta \cos \xi \\ \cos 2\theta_3 - \cos 2\theta_4 &= -2 \cos \eta \cos \xi \end{aligned} \right\} \quad (30)$$

Substituting these relations into equation (28) we obtain the same form exactly as equation (15). Thus this equation is valid inside and outside the pressure region.

(3) The derivation of equation (15) is based on a direct application of the Cauchy Integrals to the first fundamental problem of elasticity for the case of a circle. Naturally, complex representation was employed in formulating this boundary-value problem. Other techniques may be used in deriving the above

results, such as conformal mapping, infinite series, etc. However, it is doubtful if a conformal mapping of the half space problem with a concentrated load at the free surface to a semi-infinite strip will be of any use to this problem.

(4) We question the basis for the statement, "A method which lends itself to greater accuracy is described in Poritsky's reference [8] . . .," since equation (15) can be solved analytically. On the other hand if numerical method is employed to ease the computation it should be no less accurate than the numerical method used to compute the deformation due to the "tails" of the pressure profile used in Ref. [8].

(5) The linear term can be added quite readily to the computations of film thickness but thus far we did not find it necessary. We might add that other investigators which have studied the elasto-hydrodynamic problem under isothermal conditions have not added it either.

We wish to thank once again the discussers for the interest they have shown in studying our paper.