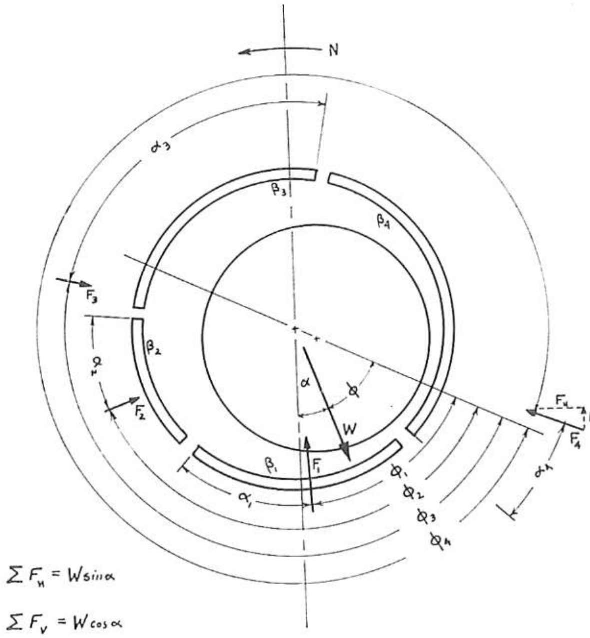

Fig. 7 Example of grooved bearing

grooved bearing which would then be a function only of the Sommerfeld number (for a given α/β). The case of grooved bearings composed of symmetrical arcs and loaded over the center of one of the arcs is given in [2]. By similar methods it is possible to obtain solutions for any arbitrary arrangement of grooves and loading.

C **Noncircular Bearings.** In the case of noncircular bearings, such as shown in Fig. 8, the additional complication is that not only do α/β and ϕ differ for each arc but also the eccentricity ratio ϵ. Thus for any fixed position of the journal, each arc will have different α/β, ϵ, and ϕ values. Aside from the thus greatly increased number of operations, the procedure of calculation is the same as for circular bearings. This consists of the following steps:

(*a*) Fix the journal at a given eccentricity and attitude angle with respect to the geometric center of the full bearing.

(*b*) Calculate the resulting ϵ and $(\alpha + \phi)$ values for each individual lobe.

(*c*) Obtain from the appropriate table the values of S, ϕ, q_{in}, and q_z corresponding to a given ϵ and $(\alpha + \phi)$. This may involve interpolation and crossplotting of the presented data.

(*d*) Add vectorially all values of S. This will yield a resultant $1/S$ and a resultant α (as well as ϕ) for the full bearing.

(*e*) Add algebraically the values of q_{in} which will yield the lubricant flow; and the values of q_z to yield the side leakage.

This procedure of first fixing the shaft position and then finding the corresponding load and load angle is, of course, the inverse of the practical problem in which usually the loading is known and the quantity to be established is the locus of the journal. However, once a general relationship between S, α/β, and the journal locus is established, it is then easy to revert to the practical procedure of determining eccentricity and flow for a given set of operating conditions.

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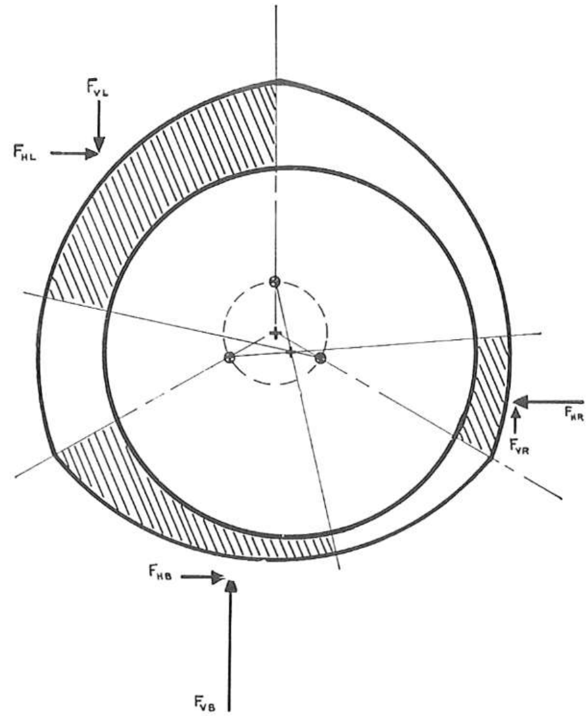

Fig. 8 Example of noncircular bearing

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DISCUSSION

W. A. Gross[3]

Mr. Pinkus has performed a valuable service by publishing computed characteristics of finite partial journal bearings for incompressible films with only positive pressures. His results approximate liquid film behavior with vaporization. It would also be valuable to tabulate, for these bearings, the properties which include the effects of subambient pressure. These properties are applicable to both gas and liquid film bearings at low bearing numbers.

The Gümbel-type boundary conditions (in which subambient pressures in an incompressible film are ignored) were apparently used for simplicity, anticipating the probable accuracy for describing liquid film characteristics. Does the author have solutions for which Swift-Stieber boundary conditions (in which the pressure gradient is presumed to vanish where the pressure falls to ambient) apply? Such results would be valuable for estimating the validity of the Gümbel boundary condition assumption.

Additional information is desirable for assessing the accuracy of the results for application to actual bearings. Although we accept the validity of the Reynolds differential equation, we do need to know the accuracy with which the Reynolds difference

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equation for the grid used approximates the Reynolds differential equation. Finally, we need to know the accuracy with which the difference equations have been solved. Practically, this means that both the grid size and, in the absence of a matrix solution, that the convergence indicator be appropriately chosen.

A stable iteration scheme will yield solutions which approach asymptotically the correct value. The appropriate convergence indicator for a chosen accuracy may then be established. A grid size appropriate for the specified accuracy may be chosen by comparing solutions for two or more similar grid sizes as shown by the discusser.[4] The author's evaluation of these two effects would add significantly to the value of the paper.

Since the difference equation used is that described by reference [5], the author apparently still prefers to approximate film thickness gradients rather than expressing them precisely. Since the film thickness is prescribed by the assumed eccentricity ratio and attitude angle, the gradients at grid points may be specified exactly. They may therefore be computed once-for-all before beginning the iteration. A routine such as the author uses may be preferable for handling irregular films, and a separate subroutine can be set up for such cases. One runs the risk, however, of encouraging computational instability by using this latter method. In fact, it is probably better to approximate an irregular film thickness by an analytic function. This function and its gradients can then be precisely specified for each grid point.

Author's Closure

Dr. Gross's discussion is most welcome as it provides an opportunity to clarify, among others, the problem of boundary conditions. The author has not used in this paper, nor in any of his other papers, the Gümbel boundary conditions. These conditions are shown in Fig. 9 as curve 1. The author has used in the present paper, as he has consistently done in the past, the boundary conditions

$$p = \frac{dp}{d\theta} = 0 \tag{6}$$

at the trailing end of the fluid film, as shown by curve 2 of the figure. This differs from the experimentally observed shape of the pressure wave only in so far as it eliminates the subatmospheric loop as shown by the dashed line of Fig. 9. This subatmospheric loop is of significance in the solution of gas bearing problems; it is of negligible importance in practically all cases dealing with liquid lubricants. The manner in which conditions (6) have been satisfied in the numerical solution of the Reynolds equation is given in Ref. [5] of the paper.

There is no direct way of evaluating the accuracy with which the finite difference results approximate the correct solution. An analytical solution of the Reynolds equation would first be needed, but such solutions have not, as yet, been obtained. One way to check the degree of accuracy would perhaps be to use the case of an infinitely long bearing for which an analytical solution does exist. However, this computed accuracy will certainly not be valid for the case of finite bearings. The accuracies involved will also vary with such items as eccentricity, length of arc, load angle, etc. It is thus difficult to talk about a single degree of accuracy. A knowledge of the over-all range of accuracies involved would certainly be of interest.

The paper does give the grid size used as well as the convergence indicator; the grid is given as varying from 50 to 200 points depending on length of arc and L/D ratio and the convergence used for the integrated pressure field is 0.004. Actually the total resultant force obtained from integrating the pressure field can be made to approach the same accuracy even though the grids may vary in density. This can be accomplished in either of two ways: Additional pressure points, on a linear scale or graphically, can be inserted between the known pressure values and ordinary step integration used; or trapezoidal summation can be used. A 7×7 grid integrated trapezoidally yields the same resultant force as a 14×14 field using step integration, with a saving of 75 per cent of computer time. Useful information about the relations between grid size, convergence, time, and computer cost are given in Ref. [3] of the paper.

Dr. Gross correctly points out that there was no need for using a finite difference expression for the film thickness. Considerately, he also provides an answer to his question and that is that the finite difference form used here is more universal by embracing also discontinuous and irregular film shapes. The author would not endorse the recommendation of approximating such irregular films by some analytical expression. While this seems to be a permissible procedure in thrust bearings, journal bearings depend strongly on the exact shape of the fluid film.

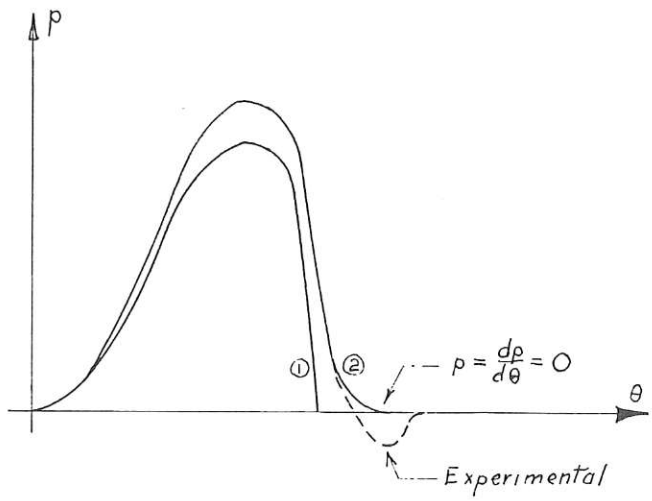

Fig. 9 Boundary conditions for the solution of the Reynolds equation

[4] Fig. 2.1, "Numerical Analysis of Gas Lubricating Films," First International Symposium on Gas Lubricated Bearings, Washington. D. C., October, 1959, U. S. Govt. Printing Office.