friction losses. These losses, which are not considered in Langhaar's analysis, affect the over-all pressure drop appreciably when the tubes are so short that the total pressure drop between the reservoirs is of the order of one velocity head. Since the test data at $(x/D\mathrm{Re}_D)$ less than 4×10^{-3} were obtained in tubes having square-edged entrances and aspect ratios L/D of 0.45, the experimentally measured pressure losses are larger than those predicted by Langhaar's analysis. The geometry of these short tubes actually approaches that of an orifice for which the flow rate can be related by Torricelli's equation in the form

$$\dot{Q} = C\frac{\pi D^2}{48}\sqrt{\left(2g_c\frac{\overline{\Delta p}}{\rho}\right)} \qquad [10]$$

The orifice coefficient C for the capillary tubes with an L/D ratio of 0.45 was found to be 0.76 ± 0.03 in the Reynolds-number range Re_D between 100 and 800. This result is in agreement with data obtained by Zucrow (12) in the same Reynolds-number range with benzol flowing through square-edged jets having an aspect ratio L/D of 0.33.

The experimental results obtained in the range of large values of $(x/D\mathrm{Re}_D)$ approach those predicted from the Poiseuille laminar-flow theory for a parabolic velocity distribution at values of $(x/D\mathrm{Re}_D)$ equal to 0.3.

Conclusions

From an experimental study of the flow characteristics of short capillary tubes the following conclusions can be drawn:

1 Measured values of the pressure drop between reservoirs upstream and downstream of capillary tubes with square-edged entrances are in agreement with Langhaar's theory for $(L/D\mathrm{Re}_D)$ larger than 4×10^{-3} and (L/D) larger than 2.

2 Measured values of the reservoir pressure drop for tubes having an aspect ratio L/D equal to 0.45 are considerably larger than those predicted by Langhaar's theory, but can be correlated by the usual orifice equation. The orifice coefficient for the range of Reynolds numbers Re_D between 100 and 800 was found to be 0.76 ± 0.03.

3 By proper selection of length-to-diameter ratios of short capillary tubes it is possible to simulate various flow characteristics in small-scale models for tests in atmospheric wind tunnels or in pneumatic control devices.

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Discussion

M. A. Rivas, Jr.[5] The experimental data presented in the paper are quite valuable due, in particular, to the care taken by the authors in their experiments. However, the writer takes exception to the lack of emphasis on what constitutes the governing design parameter for laminar flow in tubes (or capillaries). As has been shown by theoretical and experimental investigations (the authors' references (1), (3), (4), (5), and reference[6] of this discussion), the sole parameter which governs the flow is $L/D/\mathrm{Re}_D$ or $\mathrm{Re}_x/\mathrm{Re}_D{}^2$.

In particular, the writer is disturbed by the correlation presented in Fig. 3 of the paper where the exponent N in equation [5]

$$\dot{Q} = \text{const.}\ \overline{\Delta p}^N \qquad [5]$$

is depicted as a function of L/D alone. It will be shown that it is legitimate to write an expression as given by Equation [5], but that N, however, in this expression is a function of $L/D/\mathrm{Re}_D$ and not of L/D.

It is easily shown (e.g., from Equations [8] and [9]) that the total pressure drop between the two reservoirs (assuming that, as the fluid enters the tube, it forms a stream tube having the shape of a bellmouth—for a detailed discussion of flow on bellmouth entries (see reference[7] of this discussion) is given by

$$\overline{\Delta P} = \left(1 + 4\bar{f}_{\mathrm{APP}}\frac{x}{D}\right)\frac{1}{2}\rho V^2 \qquad [11]$$

or

$$\overline{\Delta P} = \left(1 + 4\bar{f}_{\mathrm{APP}}\frac{x}{D}\right)\frac{1}{2}\rho\left(\frac{Q}{A}\right)^2 \qquad [11a]$$

transposing

$$Q = \frac{A}{\sqrt{(\rho/2)}}\frac{\overline{\Delta P}^{1/2}}{\sqrt{(1 + 4\bar{f}_{\mathrm{APP}}x/D)}} \qquad [12]$$

For laminar flow in tubes (see Figs. 4 and 5 in reference[6] or Fig. 5 of the paper),

(a) If $(x/D)/\mathrm{Re}_D$ is very small $<10^{-5}$

then

$$4\bar{f}_{\mathrm{APP}}\frac{x}{D} <<< 1$$

and therefore, from Equation [12]

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[6] "Friction Factor in the Laminar Entry Region of a Smooth Tube," by A. H. Shapiro, Robert Siegel, and S. J. Kline, Proceedings of the Second U. S. National Congress of Applied Mechanics, June, 1954, pp. 733–741.

[7] "On the Theory of Discharge Coefficients for Rounded-Entrance Flowmeters and Venturies," by M. A. Rivas, Jr. and A. H. Shapiro, Trans. ASME, vol. 78, 1956, pp. 489–498.

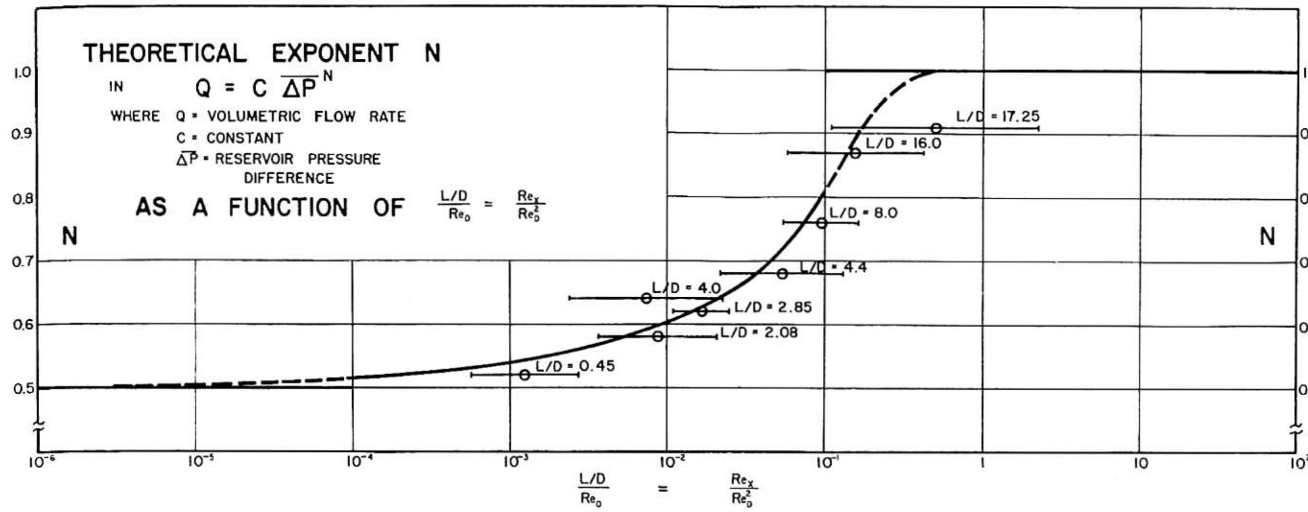

FIG. 6

$$Q \sim \overline{\Delta P}^{1/2}; \; N = 1/2$$

The same result is obtained if $4\bar{f}_{APP}(x/D)$ is constant.

(b) If $(x/D)/\mathrm{Re}_D$ is high >0.5 then

$$4\bar{f}_{APP}\,\frac{x}{D} \gg 1$$

and therefore, from Equation [12]

$$Q \sim \frac{\overline{\Delta P}^{1/2}}{\sqrt{\left(4\bar{f}_{APP}\,\frac{x}{D}\right)}}$$

but since, in this range

$$4\bar{f}_{APP}\,\frac{x}{D} = \frac{64\,\frac{x}{D}}{\mathrm{Re}_D} \sim \frac{1}{V} \sim \frac{1}{Q}$$

we then have

$$Q \sim \frac{\overline{\Delta P}^{1/2}}{\frac{1}{Q^{1/2}}} \quad \text{or} \quad Q \sim \overline{\Delta P}; \quad N = 1$$

(c) If $10^{-5} < \frac{x}{D}\Big/\mathrm{Re}_D < 0.5$

$$Q \sim \frac{\overline{\Delta P}^{1/2}}{\sqrt{\left(1 + 4\bar{f}_{APP}\,\frac{x}{D}\right)}}$$

but since

$$4\bar{f}_{APP}\,\frac{x}{D} = \varphi_1\left(\frac{\frac{x}{D}}{\mathrm{Re}_D}\right)$$

also

$$1 + 4\bar{f}_{APP}\,\frac{x}{D} = \varphi_2\left(\frac{\frac{x}{D}}{\mathrm{Re}_D}\right)$$

In particular we could define an exponent n such that

$$1 + 4\bar{f}_{APP}\,\frac{x}{D} \sim \left(\frac{\frac{x}{D}}{\mathrm{Re}_D}\right)^n \qquad [13]$$

or

$$1 + 4\bar{f}_{APP}\,\frac{x}{D} \sim \left(\frac{1}{Q}\right)^n \qquad [13a]$$

Substituting Expression [13a] into [12]

$$Q \sim \frac{\overline{\Delta P}^{1/2}}{\left(\frac{1}{Q}\right)^{n/2}} \qquad [14]$$

or

$$Q \sim \overline{\Delta P}^{\frac{1/2}{1-n/2}} = \overline{\Delta P}^N \qquad [15]$$

where

$$N = \frac{1/2}{1 - n/2} \qquad [16]$$

The exponent n can be calculated readily from the curve given in Fig. 5 of the authors' paper; however, the curve in Fig. 5 in reference[6] has been used instead, since it is in a form that yields a greater accuracy. Having calculated n, N is readily computed from Equation [16]. The results of these calculations are presented in Fig. 6.

The apparently successful correlation given by the authors in their Fig. 3 is explained as follows: If, from the authors' Table 2 and from their Fig. 3, we construct Table 3 and plot these data on Fig. 6 in the manner shown, where each solid horizontal line represents for each L/D the range of $(L/D)/\mathrm{Re}_D$ at its corresponding N-value, and each circle represents the mean value of the $(L/D)/\mathrm{Re}_D$ range, we can see how the authors were able to arrive at the smooth correlation shown in their Fig. 3. Therefore, it was quite

TABLE 3

L/D	Range of $L/D/\mathrm{Re}_D$		N
0.45	2.76×10^{-3}	5.60×10^{-4}	0.52
2.08	2.1×10^{-2}	3.62×10^{-3}	0.58
2.85	2.52×10^{-2}	1.1×10^{-2}	0.62
4.0	2.27×10^{-2}	2.38×10^{-3}	0.64
4.4	1.31×10^{-1}	2.2×10^{-2}	0.68
8.0	1.63×10^{-1}	5.47×10^{-2}	0.76
16.0	4.25×10^{-1}	5.8×10^{-2}	0.87
17.25	2.25	1.1×10^{-1}	0.91

fortuitous that the authors chose such an $(L/D)/\mathrm{Re}_D$ range for each of their L/D's, and had they extended the range of $(L/D)/\mathrm{Re}_D$ extensively for any of the L/D's chosen they would have observed a variation in the value of N for each value of the L/D's used.

One concludes from the foregoing that, although it is permissible to write an expression as given by Equation [5] of the paper, N in this expression must be regarded as a function only of $(L/D)/\mathrm{Re}_D$, the important sole parameter which governs the flow in tubes.

In regard to the deviations from tube-flow theory observed for low values of L/D, $(L/D = 0.45)$, the writer agrees with the authors that this is the result of a vena-contracta effect due to the sharp-edged entrance. This effect undoubtedly exists just as well in sharp-edge-entry tubes with the larger L/D's, but the effects become obscured by the wall-friction losses being of a much larger magnitude for tubes with larger L/D's. It is suggested that a systematic investigation, in which $(L/D)/\mathrm{Re}_D$ is kept constant and L/D (in this low-value range) is varied, would be extremely valuable. The present state of the art does not provide the designer with the criterion to distinguish at what values of L/D, for a particular $(L/D)/\mathrm{Re}_D$ range, deviations from tube-flow theory should be expected for tubes with sharp-edged entries.

A. H. Shapiro.[8] It is essential to the unambiguous interpretation of the experimental results given in this paper to appreciate that they refer to a capillary tube into which the flow is introduced from a large space via a square-edged orifice. Consequently the pressure drop depends both on the flow in the orifice and the flow in the capillary. Viewed in this light, the experimental results are less likely to be applied to circumstances in which they are not valid.

For example, it is true that the single dimensionless parameter, $x/D\mathrm{Re}_D$, governs the entire flow when the flow at the entrance of the capillary is uniform, and that it thus controls the dimensionless pressure drop in the capillary, $2\Delta p/\rho V^2$, for such a uniform entry. But, for a given orifice geometry, the governing number for the orifice is simply Re_D, and the latter controls the dimensionless pressure drop as the flow accelerates into the orifice. Furthermore, Re_D controls the character of the flow entering the capillary, and thereby the pressure drop in the latter. From all this we are forced to conclude that the dimensionless pressure drop for the entire system depends on two dimensionless groups, say

$$\frac{\Delta p}{\frac{1}{2}\rho V^2} = f\left(\frac{x}{D\mathrm{Re}_D}, \frac{x}{D}\right) \qquad [17]$$

rather than on $x/D\mathrm{Re}_D$ alone.

Accordingly, Fig. 5 of the paper must be used with some caution, for a more extensive series of tests, perhaps with less scatter, would surely reveal on this chart a series of curves, one for each value of x/D. The authors obtained what seems to be a single curve only because of the experimental circumstance that an increase in x/D was accompanied by a general increase in the experimental values of $x/D\mathrm{Re}_D$.

This point may be elucidated by considering two extremes. If x/D is very small, say less than 0.10, the effect of x/D itself vanishes, the system behaves like a sharp-edged orifice, and $2\Delta p/\rho V^2$ depends only on Re_D. Thus, if several orifices with very small (but different) values of x/D were tested, each would yield a different curve on the chart of $2\Delta p/\rho V^2$ versus $x/(D\mathrm{Re}_D)$, but all would yield the same curve on a chart of $2\Delta p/\rho V^2$ versus Re_D.

At the other extreme, consider large values of x/D, say always greater than 100. Then the nature of the inlet is relatively inconsequential, and all the experimental data may be expected to be in agreement with Langhaar's theory, even for very low values of $x/D\mathrm{Re}_D$.

The foregoing discussion suggests that it would be of interest to carry out further systematic experiments with the goal of establishing the individual curves of x/D in Fig. 5.

To the authors' account of theoretical and experimental work on laminar flow in tube entries, it might be added that in reference (6) several additional theories are presented. One in particular, based on the theory of thin, laminar boundary layers, is especially accurate for very small values of $x/(D\mathrm{Re}_D)$, where Langhaar's theory seems to be in error.

The best experimental data for short tubes are those of reference (6), also given in reference (8). They are in remarkable agreement with the theories, differing from the latter by less than 1 per cent in the range of $x/(D\mathrm{Re}_D)$ between 10^{-5} and 10^{-3}.

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J. F. D. Smith.[9] The authors of this paper have been loose in their dimensional treatment of the subject. The units specified are quite inconsistent and awkward. For example, in Equation [1] the authors divide a pressure drop in pounds (force) per square inch by a mass density in pounds (mass) per cubic foot. In the same equation V is given as a velocity in feet per second. The part in parentheses $(V^2/2)$ is raised to a variable power k, which means that the friction coefficient has dimensions in this case which vary with k.

The writer would suggest that the authors revamp the units and equations to insure dimensional consistency and rigor. For example, any factor raised to a variable power, as in this equation, must be dimensionless if no other dimensions are changed elsewhere.

This is not to be construed as a criticism of the results, but is a plea for a more satisfactory and rigorous mathematical presentation.

J. R. Sproat.[10] A paper was presented at the Appalachian Gas Measurement Short Course held at West Virginia University in August, 1955. Simple working formulas were developed by the writer for determining the dimensions of capillary tubes required to produce desired flow rates for gases and liquids. The concluding remarks of that paper may be of interest to those contemplating further investigations or use of capillaries:

"In the design of a capillary, the Reynolds number should be calculated to determine if the flow is laminar. If the Re_D is found too high, using the available diameter tube, it may be necessary to divide the flow, say in 10 parallel tubes.

"Roughness of the bore has no effect on the flow rate, because there is no flow at the wall of the tube.

"The capillary should be calibrated with dry air or a known gas such as nitrogen; boiled or distilled water. After calibration the capillary can be used on any other fluid if its viscosity is known. The flow will vary inversely as the viscosity; i.e., as the viscosity increases the flow decreases. Calibration is necessary due to the fact that the flow varies as the fourth power of the diameter and it is difficult to obtain tubing with exact bore through the entire length of section.

"If the capillary is to be coiled, calibration should be made after coiling.

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[10] Measurement Engineer, Carbide and Carbon Chemicals Company, So. Charleston, W. Va. Mem. ASME.

"The differential pressures should range from 20 to 300 inches of water.

"The length should be greater than 50 times the diameter.

"The area preceding the inlet should be at least 20 times the capillary area.

"A filter or screen should be installed upstream of the capillary.

"The temperature affects the diameter of the capillary and the viscosity of the flowing fluid; therefore, for accurate work a temperature bath or some suitable means should be provided to maintain uniform temperature of both capillary and fluid.

"Since the flow varies inversely as the viscosity, once the capillary has been calibrated with a fluid of known viscosity, such as air, nitrogen, or water, the viscosity of any other fluid may be determined. Several makes of viscosimeters use this method as a means of calculating viscosity.

"The capillary has many uses as a measuring device, especially for a small quantity of gas or liquid. They are accurate, inexpensive, and simple to construct."

Authors' Closure

The authors wish to thank the discussers for their interest in the paper. Some of the comments are pertinent and valuable and the authors will attempt to answer, as far as possible, the questions raised.

Lieutenant Rivas' comments represent a valuable contribution to the paper, but unless the assumptions made in the course of his analytical derivation of the flow exponent N are understood clearly, the application of his results could lead to erroneous conclusions. His assertion that the flow exponent N depends solely on the parameter $(L/D)/\mathrm{Re}_D$ is not entirely true. As pointed out explicitly in the paper as well as in Professor Shapiro's discussion, and implicitly also in the last paragraphs of Lieut. Rivas' discussion, the pressure drop in flow through short capillary tubes into which the flow is introduced from a large space via a square-edged orifice depends on L/D as well as Re_D, or $[(L/D)/\mathrm{Re}_D]$. Fig. 3 is an empirical correlation and therefore applies only within the range of the variables investigated. Its primary purpose is to show that by selecting appropriate values of L/D, flow exponents ranging from about 0.5 to 1.0 can be attained. The actual value of L/D required to obtain a specific value of N for a specified, but limited, range of Reynolds numbers Re_D depends also on the Reynolds-number range.

Lieut. Rivas' analysis can be used to predict N when $\Delta_1 p$ in Equation [7] of the paper is sufficiently small compared to $(\Delta_2 p + \Delta_3 p)$ so that deviations of the actual value of $\Delta_1 p$ from the assumed value of one velocity head are negligible. However, for small values of L/D, irrespective of the value of $(L/D)/\mathrm{Re}_D$, the theoretical curve shown in Fig. 6 does not apply because vena-contracta effects which are not considered in the analysis become important.

To elucidate this point and to illustrate the vena-contracta effects at least qualitatively, Fig. 7, based on Fig. 5 and Equation [16], has been prepared. In this figure curves of N, calculated on the assumption that $\Delta_1 p = \rho V^2/2g_c$, are shown as a function of L/D. Each curve is for a given value of Re_D. The crosses represent the experimental values of N from Fig. 3 and the accompanying vertical lines show the range of Reynolds numbers covered in the tests (see Table 2). The circles represent values of N calculated from experimental data reported by Linden and Othmer[11] for aspect ratios of 0.20, 0.30, and 0.58 in the Reynolds-number range indicated by the accompanying vertical lines.

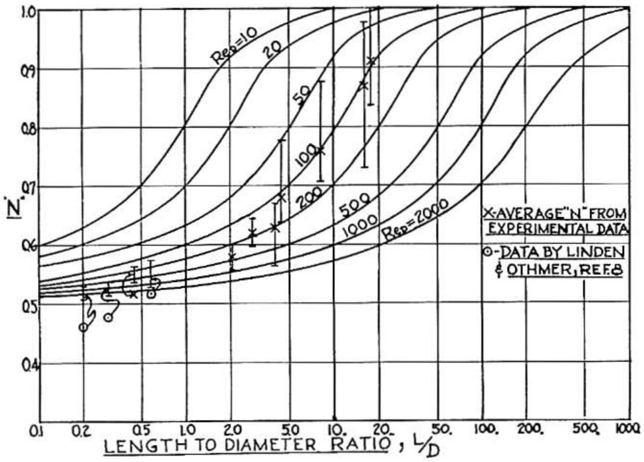

Fig. 7 Theoretical and Experimental Flow Exponent N Versus Length to Diameter Ratio for Various Reynolds Numbers

An examination of Fig. 7 shows that N depends both on L/D and Re_D or $(L/D)\mathrm{Re}_D$. The observed values of N represent average values for a specific length-to-diameter ratio over a limited Reynolds-number range. The experimental values agree fairly well with the predicted values for $L/D > 2$. Fig. 7 further shows, as pointed out by Lieut. Rivas, that if a much wider range of Reynolds numbers had been covered with any of the test sections, the relation between $\overline{\Delta p}$ and $\dot{Q}$ could not have been approximated by a single straight line. It is also apparent that for small values of L/D the experimental results deviate appreciably from the values predicted by Lieut. Rivas' analysis. For very small values of L/D the data of Linden and Othmer yield values of N below 0.5 which is the lower limit of the analysis. For the purpose of relating $\overline{\Delta p}$ to $\dot{Q}$ empirically, it would therefore appear acceptable, and maybe even preferable, to treat N, the flow exponent for the system investigated, as a function of L/D within a limited, but specified, range of Reynolds numbers rather than to treat N solely as a function of $(L/D)/\mathrm{Re}_D$ as suggested by Lieut. Rivas.

It should also be noted that in Fig. 5 of footnote 6 which was used by Lieut. Rivas for his calculations, no experimental data for $(L/D)/\mathrm{Re}_D$ above 1×10^{-3} are shown. In the range of $(L/D)/\mathrm{Re}_D$ between 1×10^{-3} and 5×10^{-1} in which the boundary-layer flow gradually merges into fully developed Poiseuille flow and the variation of N takes place, the results of the various theories shown differ by as much as 10 per cent from one another.

In an effort to determine which of the theories is valid in the range of the experimental results of this investigation, the experimental data are compared in Fig. 8 of this closure, with the analytical results summarized by Shapiro, et al.,[6] which was published after this paper was submitted. The solid line represents the average of the results of this investigation for tests with $(L/D) > 4$. As predicted by Shapiro, et al.,[6] the experimental results agree in the range of $(L/D)/\mathrm{Re}_D$ between 5×10^{-3} and 5×10^{-1} more closely with results obtained by differential-equation methods (1 and 5) than with results calculated by integral methods.

Professor Shapiro's comments emphasize succinctly the importance of the entrance conditions. In an effort to shed more light on the pressure drop, the authors reviewed available data, including those for very small values of L/D where conditions approach flow through a sharp-edged orifice.[12]

An incidental observation made in the course of this study is that orifice coefficient in the laminar-flow range may be affected

[11] "Air Flow Through Small Orifices in the Viscous Region," by H. R. Linden and D. F. Othmer, Trans. ASME, vol. 71, 1949, pp. 765–.277

[12] "Orifice Coefficients for Reynolds Numbers From 4 to 50,000," by H. W. Iversen, Trans. ASME, vol. 78, 1956, pp. 359–364.

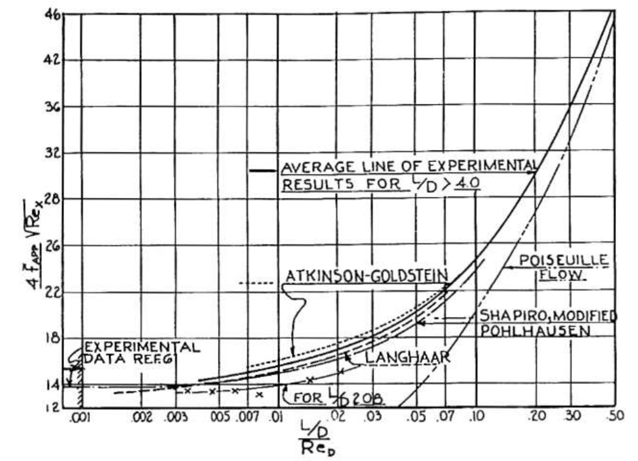

Fig. 8 Comparison of Theoretical and Experimental Results for Integrated Apparent Friction Factor

by L/D variations even when L/D is of the order of 0.1. The ISA and ASME codes specify the maximum thickness of orifice plates, t, as a fraction of the pipe diameter, D_p; i.e., $t < 0.02\ D_p$. For a small orifice in a large pipe it is therefore quite possible that the pressure drop through the orifice, although designed according to specifications, will deviate from the results predicted from a simple orifice equation (e.g., footnote 8). To safeguard against such deviations it is suggested that the maximum orifice thickness be specified in terms of the orifice diameter.

The information available to date is unfortunately insufficient to give quantitative answers to the questions raised by Professor Shapiro's valuable and pertinent comments which, however, are broader in scope than the limited objectives for which the tests reported in this paper were conducted. Qualitatively, we can conclude that only for conditions when $\Delta_1 p$ in Equation [7] is of the order of $(\Delta_2 p + \Delta_3 p)$ will deviations from the pressure drop predicted from Equation [9] and Fig. 5 be appreciable.

The authors agree with Lieut. Rivas and Professor Shapiro that additional experiments, especially at Reynolds numbers between 500 and 2100 and in the low L/D range, are desirable. The tests reported by the authors were conducted within the scope of a large research project in the course of which experimental data for flow of air through capillary tubes in the pressure-drop range of 0.03 and 2.0 in. water were required. These conditions yield $(L/D)/\mathrm{Re}_D$ values between 1×10^{-3}, the maximum value for which data were available and 0.5, the value at which Poiseuille flow is approached. The data were published simply because they fall into a range of variables in which no experimental results were heretofore available. When comparing the accuracy of the experimental results of this study with those of Shapiro, et al.,[6] it is well to keep in mind that the pressure drop, which more than any other measurement limits the accuracy of the apparent friction coefficient at low Reynolds numbers, is much smaller in the range of variables covered by the authors than in the range of variables covered by Shapiro, et al. The original equipment was returned to the sponsor at the termination of the contract, but an improved version is at present being built at Union College to conduct further tests along the lines suggested by the discussers.

Dean Smith's comments regarding the units and the dimensional treatment of the subject seem to have been prompted by an error in the preprint of this paper, where instead of the proportional sign an equal sign was used in Equation [1]. Equation [1] is dimensionally consistent as shown and the parameters $\overline{\Delta p}/(\rho\ V^2/2g_c)$ and $(L/D\mathrm{Re}_D)$ are dimensionless quantities. One could, of course, express ρ in slugs/ft^3 instead of $\mathrm{lb_M/ft^3}$ and thereby eliminate the explicit use of g_c. This choice of units may be more satisfactory to some, but it would not affect the mathematical rigor.

Mr. Sproats comments on the application of capillary tubes to flow measurements will be of value to those contemplating the use of capillaries for this purpose. It would seem, however, that the minimum length should be specified in terms of $(L/D)/\mathrm{Re}_D$ instead of L/D. The experimental results of this study confirm theoretical predictions[6] which show that for the center-line velocity to approach the Poiseuille value within 1 per cent, $(L/D)/\mathrm{Re}_D$ must be at least 0.3, but probably close to 0.6. With the lower figure as a limit, the minimum length-to-diameter ratio required to establish a parabolic velocity profile at Re_D of 2000 is 600, which is twelve times larger than the value suggested by Mr. Sproats.