7 Lysen, Aufwand, Leistung und Wirtschaftlichkeit neuzeitlicher Werkzeugmaschinen, Verlag W. Girardet, Essen, Germany, 1953, pp. 186–187.

8 "Forschungsergebnisse beim Aussenrundschleifen," by Ernst Saljé, Werkstattstechnik und Maschinenbau, 43 Jg., no. 3, 1953, pp. 103–107.

Discussion

S. Doi.[4] It appears that the author has found some experimental results which cannot be explained by Arnold's hypothesis setting forth the falling characteristic of the cutting force in its relationship with cutting speed. It seems that the author is somewhat skeptical about Arnold's hypothesis. But the author has presented little discussion on this problem. Mr. Hahn[5] and the writer[6] believe that the cause of chatter is not explained by this hypothesis. The fundamental cause of the chatter and the presence of vibrational energy are not yet widely accepted.

The author measured the horizontal and vertical vibration of workpiece simultaneously without measuring cutting force. To properly investigate the cause of chatter it is necessary to measure the cutting force and vibration simultaneously.

It is desirable that a mathematical analysis be carried out with some understanding of the fundamental cause of chatter. Evaluating $P_1 = P_{01} + \alpha_1\dot{y} - \beta_1 z + \delta_1 y$ (Equation [1] of the paper), the author arrived at the equation of chatter (Equation [2]). Using this equation, the author has examined the stability and frequency of vibration. However, the writer is of the opinion that the values of $\alpha_1\dot{y}$, $\beta_1 z$, and $\delta_1 y$ during the chatter are of insufficient influence to develop the chatter vibration. As the author states, the amplitude of self-excited chatter is much larger than that of forced chatter.

The writer believes that the chatter vibration is a type of self-excited vibration caused by a lag in fluctuation of horizontal cutting force existing behind the horizontal vibration. Because of this lag, some amount of energy per cycle is available for maintaining or increasing the horizontal vibration of the workpiece. This delay in cutting force, the writer believes, is the peculiar phenomenon when metals are machined. If the horizontal vibration in the workpiece is initially started at its own natural frequency, the area of cut will fluctuate with that frequency, causing the vertical cutting force to do the same, resulting in the vertical vibration of the workpiece. Therefore the workpiece vibrates in the form of an ellipse (as shown in Figs. 3 and 15 of the paper) and the frequencies of components are equal.

The frequency of chatter shown in Table 1 is lower than the natural frequency of the main spindle in all cases except one case (lathe No. 3). According to the writer's experiments, the frequency of chatter is a little higher than the natural frequency and is in good agreement with Mr. Hahn's results.[5] The writer has observed that when the clearance is considerable in the main spindle bearing, or when the rigidity of the spindle in the horizontal or upward direction is much less than that in the downward direction, the frequency of chatter is lower than the natural frequency of the main spindle. The frequency of chatter depends upon the cutting condition and has no close connection with the fundamental cause of chatter.

In good agreement with the writer's experimental result are the following relations: That between the amplitude of chatter and width of cut, shown in Fig. 8; that between the amplitude of chatter and cutting speed, shown in Fig. 10, and that between the amplitude of chatter and wearland of the cutting edge shown in Fig. 16. These relations can be well explained by the writer's description.

[4] Professor of Mechanical Engineering, University of Nagoya, Chigusa-ku, Nagoya-shi, Japan.

[5] Reference (4) of the Bibliography of the paper.

[6] Memoirs of the Faculty of Engineering, University of Nagoya, Japan, vol. 5, no. 2, 1953.

R. S. Hahn.[7] This paper represents a considerable amount of work and the author is to be commended for the very interesting results he has presented. The general complexity of the problem of metal-cutting chatter is well illustrated by the numerous test results and graphs.

The author observes in many cases, for instance in Fig. 3, that the self-excited frequency occurring during the cutting process is less than the free "uncoupled" natural frequency, whereas he points out that Hahn (reference 4 of the paper) found the self-excited frequency to be greater than the uncoupled natural frequency. This might be explained by the fact that Hahn dealt with relatively flexible boring bars, in which case coupling with the workpiece acted as an increase in stiffness, whereas the contacting of the tool and work in the author's tests may have acted as coupling an additional mass to the system through a very stiff spring (the tool-work contact). Another feature which should be made clear regarding the various sets of tests is whether or not feedback effects from previous vibrations were possible. The writer has found that the action of chattered surfaces on the vibratory motion of the tool is very pronounced. Under feedback conditions it is possible to have self-excited vibrations at frequencies which differ somewhat from the natural frequency.

The writer agrees with the author that torsional vibrations can usually be neglected in comparison with the bending modes.

In connection with the results shown in Fig. 10, can the author connect the sudden drop in amplitude versus cutting speed with changes in the built-up edge? Or can this be explained on the basis of "destructive interference" in the feedback of energy from the vibration produced the previous revolution?

The author raises several questions about oscillograms Figs. 7, 8, 9, and 10 in Hahn's paper (4). The bars used by Hahn had essentially the same free natural frequency horizontally as they did vertically. The high-frequency pulse of "transitional" chatter in the horizontal or normal direction essentially "forced" the same frequency to appear in the vertical or tangential direction. The bar corresponding to Fig. 7 was weighted to lower its natural frequency.

The author's Equations [1] are an attempt to formulate force relations for "primary" chatter conditions; i.e., with neither "feedback" conditions nor "transitional" conditions (where a part or all of the tool is in the act of penetrating a work-hardened surface) present. In these equations the last term δy represents the change in force due to changes in rake angle which in turn are caused by deflection of the tool. In most practical cases this effect must be extremely small. On the other hand, since the clearance angle is small to begin with and the worn flat on the clearance surface even smaller, variations in effective clearance angle due to vibration can be important. If the instantaneous clearance angle C be defined as the angle between the clearance surface and the instantaneous velocity vector of the work relative to the tool it can easily be seen that

$$C = C_0 - \frac{\dot{z}}{v + \dot{y}}$$

where C_0 is the static clearance angle and v is the work speed. For large amplitudes of vibration the second term can completely nullify the first term, i.e., as the tool vibrates into the work the effective clearance can become zero.

Instead of considering the second-order effect of variations in rake angle due to vibratory displacements it might be well to

[7] Consulting Engineer, The Heald Machine Company, Worcester Mass. Mem. ASME.

consider the effect of variations in depth of cut on the deflection of the tool; i.e., does an increase in depth of cut cause the tool to deflect so as to decrease or increase the depth of cut? Gooseneck tools with the "center of deflection" above the cutting edge deflect away from the work as depth of cut is increased whereas tools with the center of deflection below the cutting edge tend to deflect into the workpiece with increasing depth of cut. Such tools are said to "dig in." This condition for static stability should be insured first before considering conditions for dynamic stability.

Author's Closure

The investigations reported and discussed in the paper were carried out two years ago. Meanwhile the field of vibrational research with reference to machine-tool development has widened considerably, increasing the complexity of the problems. Hence the results obtained at that time cannot be expected to agree completely with the relations and conclusions recently derived. It is rather evident that especially the basic relationships pertaining to machine-tool vibrations have to be sought for gradually. Therefore the author mentions in the beginning that the paper on the self-exited vibrations with two degrees of freedom should be considered as a discussion of the problems investigated.

Mr. Doi is quite right in stating that Arnold's hypothesis cannot be applied to all cases. The author is of the same opinion as Mr. Doi that the cutting force and the vibrations should be measured simultaneously. Vibrational investigations by Mr. Sokolowski (Russia) and by Mr. Doi show a lag of almost the same order of magnitude in the fluctuation between cutting force and vibrational motion. Referring to these investigations, it should be mentioned that Mr. Doi's test conditions did not meet the same requirements. The cutting speed was about 40 m/min, i.e., the range of the type 3 chip was covered, whereas cutting speeds up to 290 m/min were applied by Mr. Sokolowski, thus investigating the range most important to current practice. (Sokolowski, "Präzision in der Metallbearbeitung," Verlag Technik, Berlin—Precision in Metal Cutting.) For some materials the force-speed characteristic is a horizontal line. In these cases the coefficients α_1 and α_2 of Equation [1] become zero, and the expressions δ_1 and δ_2 can become small. However, the term β, showing how the cutting force is changed by the cutting depth, must not be neglected. The influence of the different elements may be estimated only due to the coupling of the vibratory components.

In his discussion Mr. Doi himself points out the importance of the coupling, stating that the horizontal vibration initially starts at its own natural frequency. Now the vertical cutting force is excited. There is not always an elliptical motion (see Fig. 3 of the paper) and the frequencies of the two components are different.

The frequency of chatter, Mr. Doi believes, does not depend directly on the fundamental causes of chatter, but the cutting conditions are closely related to the tendency to chatter. This fact, among others, was confirmed by Sokolowski. If, however, the tendency of the tool to chatter as well as the frequency depends on the cutting conditions, there must be a relationship between frequency and cutting conditions. In the present paper the author tried to find and illustrate these relationships.

Likewise, Mr. Hahn points out this relationship in his discussion. The solution of this problem will present considerable difficulty. As tests with boring quills of a relatively low natural frequency showed, the chatter frequencies are equal to the natural frequencies even in the case of different cutting conditions. The fact that the natural frequency in one case is smaller (according to Mr. Hahn's tests) and in another is larger than, or equal to, the chatter frequency (author's tests) leads to the assumption that not all the important components have been considered. However, the fact that the chatter frequency depends on the cutting conditions is confirmed.

In Mr. Hahn's opinion, the fact must be considered that if the vibrating uncoupled element has a small spring constant, it will acquire greater stiffness by coupling, but if the vibrating uncoupled element possesses a great spring constant, it will—after coupling—act during the cut like an element with a smaller spring constant. This explanation led to the following considerations:

Under completely equal cutting conditions the uncoupled natural frequency of the vibrating element is kept constant; however, the ratio of the spring constant to the mass is changed, thus altering the term k_w/kg in Mr. Hahn's equation of frequency. In this way can be shown the cases to which Mr. Hahn's relation may be applied and how coupling and spring constant influence the frequency under the same conditions.

A feedback effect caused by chatter marks of the previous vibration will be the larger, the smaller is the ratio between the cutting depth and the depth of the chatter marks. In these cases self-excited as well as forced vibrations occur. If the forced vibration becomes larger than the self-excited vibration, the chatter frequency (in this connection no longer being a real chatter frequency in the general sense) will be determined by the frequency of the chatter marks. However, Fig. 21 of the paper shows that in spite of the periodic alternations of the chip's cross section the self-excited vibration may be of a higher magnitude. The turning tool starts to vibrate almost with its natural frequency in spite of the presence of forced vibrations.

The drop of the amplitude (Fig. 10) cannot be explained. Meanwhile, the tendency of the amplitude, plotted against the cutting speed, was proved by further investigations. Beyond this, the Russian literature also mentions these characteristics. This drop cannot be interpreted by the built-up edge, which occurs only up to a cutting speed of 50 m/min. A "destructive interference" is impossible. Each test was run for several minutes until the vibration was completely built up. The frequency of the chatter marks is equal to the chatter frequency.

The author appreciates Mr. Hahn's hint on the variation of the cutting force due to the effective clearance angle. This effect also was taken in consideration by Mr. Sokolowski. It may be assumed that the influence of the effective clearance angle is more important to chatter than that of the rake angle. Measurements of the cutting force show a sudden increase of the cutting force if the clearance angle becomes zero.

A further valuable suggestion is made by Mr. Hahn. From the investigations recently carried out, it can be seen that the magnitude and direction of the deflection of a tool with a great natural frequency, but with a relatively small static spring constant, must be considered. Some vibrational phenomena may be explained by this fact.