

a node  $r$ , representing a reradiator surface or a radiosity node  $I_{rn}' = 0$ .

The over-all solution of the problem is given as a relationship between the bounding values

$$[I'] = [\bar{Y}'] [E'] \dots \dots \dots [44]$$

or its inverse

$$[E'] = [\bar{Y}']^{-1} [I'] \dots \dots \dots [45]$$

Matrix  $[\bar{Y}']$  which appears in the foregoing as the key to the solution is obtained by the use of a transformation matrix  $[A]$  relating the branch potential differences with the node-pair potentials as follows

$$[E] = [A] [E'] \dots \dots \dots [46]$$

The structure of this matrix is easy to deduce

	1n	2n	3n	4n... (n-1)n	
12	1	-1	0	0	0
13	1	0	-1	0	0
⋮					
1(n-1)	1	0	0	0	-1
1n	1	0	0	0	0
23	0	1	-1	0	0
24	0	1	0	-1	0
⋮					
2(n-1)	0	1	0	0	-1
2n	0	1	0	0	0
⋮					
(n-1)n	0	0	0	0	1

⋮ [47]

since

$$E_{12} = E_2 - E_1 = (E_n - E_1) - (E_n - E_2) = (+1)E_{1n}' + (-1)E_{2n}'$$

$$E_{13} = E_3 - E_1 = (E_n - E_1) - (E_n - E_3) = (+1)E_{1n}' + (-1)E_{3n}'$$

etc.

The transpose of this matrix relates the node-pair current with the branch current generators

$$[I'] = [A]_t [I] \dots \dots \dots [48]$$

and annihilates the  $i$  vector

$$[A]_t [i] = 0 \dots \dots \dots [49]$$

as can be verified, say, by performing the indicated operations.

Premultiplying both sides of Equation [38] by  $[A]_t$ , using Equations [41] and [49]

$$[A]_t [I] = [A]_t [\bar{Y}] [E]$$

whence with Equation [46] and [48]

$$[I'] = [A]_t [\bar{Y}] [A] [E']$$

so that

$$[\bar{Y}'] = [A]_t [\bar{Y}] [A] \dots \dots \dots [50]$$

It should be noted that  $[\bar{Y}']$  is an  $(n-1)$  by  $(n-1)$  matrix. It is obtained by simple multiplications shown in Equation [50] and, as demonstrated in Equation [44], it serves directly as a solution to a whole class of problems involving the determination of net fluxes induced by a given potential field.

## Discussion

VICTOR PASCHKIS.<sup>9</sup> The paper is very interesting because it applies a powerful technique to an important problem. It may be pointed out that this technique has been used previously for study of certain radiation problems in electric furnaces.<sup>10,11</sup> In this case, provision was also made in the computing network to represent conduction between different parts of the radiating body.

### AUTHOR'S CLOSURE

The author thanks Dr. Paschkis for pointing his attention to the earlier work on network analog computers for problems in radiation heat transfer. However, the technique used by Dr. Paschkis differs fundamentally from the method proposed in the present paper in that he considered node potentials as proportional to surface temperature, while, according to present method, node potentials represent radiosities and black-body emissive powers. Consequently, Dr. Paschkis had to deal with a nonlinear network which restricted considerably the scope of application, while the radiation networks described in the present paper are essentially linear and their scope is restricted only by the assumption of diffuse radiations with either gray or monochromatic system elements. In fact, his method was restricted to enclosures made up of black bodies only, and, as an example, he treated solely the simple case of such an enclosure consisting of four sources, one sink, and a single refractory surface.

Moreover, it should be noted that the present paper is by no means concerned only with analog computers—a field in which Dr. Paschkis' contributions are well known.

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<sup>10</sup> "Elektrisches Modell zur Verfolgung von Waermestrahlungsvorgaengen insbesondere in elektrischen Oefen," by Victor Paschkis, *Elektrotech. Maschinenbau*, vol. 54, 1936, pp. 617-621.

<sup>11</sup> "Industrial Electric Furnaces and Appliances," by Victor Paschkis, Interscience Publishers, Inc., New York, N. Y., vol. 1, 1945, p. 27; vol. 2, 1948, p. 75.