Graetz, *Annalen der Physik und Chemie*, vol. 25, 1885, pp. 337–357.

12 "Measurements of Average Heat-Transfer and Friction Coefficient for Subsonic Flow of Air in Smooth Tubes at High Surface and Fluid Temperatures," by L. V. Humble, W. H. Lowdermilk, and G. D. Desmon, NACA TR 1020, 1951.

13 "Heat and Momentum Transfer in Turbulent Flow of Mercury," by S. E. Isakoff and T. B. Drew, Heat Transfer Discussions, London, England, September, 1951.

14 H. W. Iversen, personal communication.

15 "Point Unit Thermal Conductances for Viscous Flow of an Oil in a Horizontal Tube With Parabolic Velocity Distribution at Entrance," by T. W. Jackson, MS thesis, University of California, Berkeley, Calif., 1946.

16 "Variation of the Eddy Conductivity With Prandtl Modulus and Its Use in Prediction of Turbulent Heat Transfer Coefficients," by R. Jenkins, Heat Transfer and Fluid Mechanics Institute Preprints, Stanford University Press, June, 1951.

17 "Heat Transfer to Molten Lead-Bismuth Eutectic in Turbulent Pipe Flow," by H. A. Johnson, J. P. Hartnett, and W. J. Clabaugh, Trans. ASME, vol. 75, 1953, pp. 1191–1198.

18 "Heat Transfer to Mercury in Turbulent Pipe Flow," by H. A. Johnson, W. J. Clabaugh, and J. P. Hartnett, Trans. ASME, vol. 76, 1954, pp. 505–511.

19 "Forced Convection From Nonisothermal Surfaces," by J. Klein and M. Tribus, University of Michigan Engineering Research Institute Report, August, 1952, ASME Paper No. 53—SA-46, unpublished.

20 "Investigation of Heat Transfer at High Heat Flux Densities: Experimental Study With Water of Friction Drop and Forced Convection With and Without Surface Boiling in Tubes," by F. Kreith and M. Summerfield, Progress Report No. 4-68, Ordcit Project, Contract No. W-04-200-ORD-455, Jet Propulsion Laboratory, California Institute of Technology, Pasadena, Calif., April, 1948.

21 "Der Warmeübergang an einen turbulenten Flussigkeits oder Gasstrom," by H. Latzko, *Zeitschrift für angewandte Mathematik und Mechanik*, vol. 1, 1921.

22 "Mass Transfer Between Solid Wall and Fluid Streams," by C. S. Lin, R. W. Moulton, and G. L. Putnam, *Industrial and Engineering Chemistry*, vol. 45, 1953, pp. 636–646.

23 "Druckverlust und Warmeübergang im Anlauf de Turbulenten Rohrstromung," by W. Linke and H. Kunze, *Allgemeine Warmetechnik*, Jahrgang 4, no. 4, 1953.

24 "Heat Transmission," by W. H. McAdams, McGraw-Hill Book Company, Inc., New York, N. Y., second edition, 1942.

25 "Heat Transfer to Molten Metals," by R. C. Martinelli, Trans. ASME, vol. 69, 1947, pp. 947–959.

26 "Sul Moto Di Fluidi In Regime Tuibolento Nel Tratto Iniziale Dei Tubi Con Distribuzione Logaritmica Di Velocita," by L. Pascucci, *Atti di Guidonia*, vol. 4, 1953.

27 "Forced Convection Heat Transfer in Thermal Entrance Regions," by H. F. Poppendiek, Part I, Oak Ridge National Laboratory, Oak Ridge, Tenn., 1951.

28 "Forced Convection Heat Transfer in Thermal Entrance Regions," by H. F. Poppendiek and L. D. Palmer, Part II, Oak Ridge National Laboratory, Oak Ridge, Tenn., 1952.

29 "Experimental Determination of Heat Transfer to an Oil in a Horizontal Pipe With Reynolds Noduli in the Transition Region," by F. E. Romie, Jr., MS thesis, University of California, Berkeley, Calif., 1947.

30 "Phase Equilibria in Hydrocarbon Systems—X. Thermodynamic Behavior of Liquid Mixtures of n-Butane and Crystal Oil," by B. H. Sage and W. N. Lacey, *Industrial and Engineering Chemistry*, vol. 28, 1936, pp. 106–111.

31 "A Mathematical Analysis of the Turbulent Heat Transfer in a Pipe With a Surface Temperature Discontinuity at Entrance," by V. D. Sanders, MS thesis, University of California, Berkeley, Calif., November, 1946.

32 "Untersuchungen über laminare und turbulenten Strömung," by L. Schiller, Forschungsarbeiten auf dem Gebiet der Ingenieur-Wesen, *Zeitschrift für angewandte Mathematik und Mechanik*, Heft 248, 1922, p. 96.

33 "Die Entwicklung der Geschwindigkeitsverteilung bei der turbulenten Rohrstromung," by V. L. Schiller and H. Kirsten, *Zeitschrift für Technische Physik*, vol. 10, 1929, pp. 268–274.

34 "Temperature Gradients in Turbulent Gas Streams," by W. G. Schlinger, V. J. Berry, J. L. Mason, and B. H. Sage, *Industrial and Engineering Chemistry*, vol. 45, 1953, pp. 662–666.

35 "Calculations Relative to the Thermal Entry Length for Fluids of Low Prandtl Number," by R. A. Seban and T. Shimazaki, University of California, Division of Engineering Research Report, series no. 16, issue no. 4, Berkeley, Calif., January, 1950.

36 "Finite Difference Calculations of the Thermal Entrance Length for Low-Prandtl-Number Fluids With Constant Heat Rate Boundary Condition," by R. A. Seban, J. P. Hartnett, and S. Levy.

37 "Point Unit Thermal Conductance for the Viscous Flow of an Oil in a Horizontal Tube With Uniform Velocity Distribution at Entrance," by J. A. Stahn, MS thesis, University of California, Berkeley, Calif., 1946.

38 "Laminar Swirling Pipe Flow," by L. Talbot, *Journal of Applied Mechanics*, Trans. ASME, vol. 76, 1954, pp. 1–7.

39 "Experimental Determination of the Thermal Entrance Length for the Flow of Water and of Oil in Circular Pipes," by J. P. Hartnett, PhD thesis, Division of Mechanical Engineering, University of California, Berkeley, Calif., February, 1954.

Discussion

R. G. DEISSLER.[3] The paper gives information on heat transfer in the entrance region for a range of Prandtl numbers where few or no data existed. The results are therefore of fundamental interest.

The author's conclusion that the entrance length is unaffected by Prandtl number is not entirely in agreement with the analytical results of the writer. The analysis mentioned in the paper was for gases; the analysis for higher Prandtl numbers was not available at the time the paper was written. Incidentally, the writer's curve shown in Fig. 10 of the paper is plotted too low. The original curves from reference (6) of the paper were quite difficult to read in the region plotted in Fig. 10.

The writer has made a comparison between his analysis for Prandtl numbers above 1 and the experimental data for water and oil. This comparison is given in Fig. 13 of this discussion.
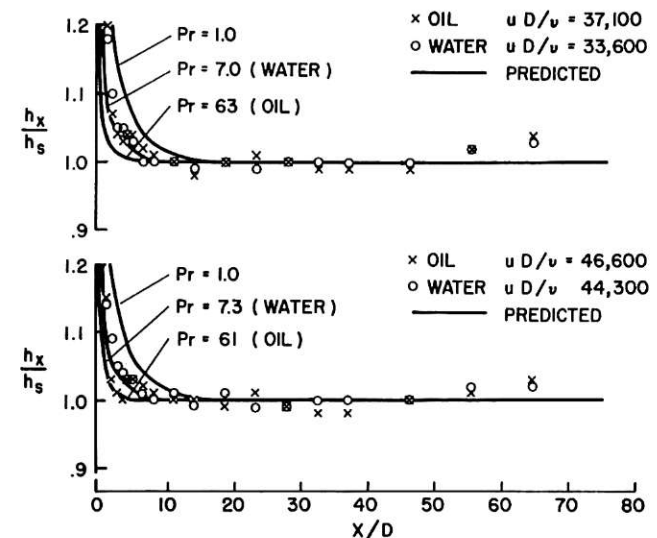

FIG. 13 COMPARISON OF WRITER'S ANALYSIS WITH EXPERIMENTAL DATA FOR WATER AND OIL

Data from Figs. 10 and 11 of the paper are compared with the predicted curves for corresponding Reynolds and Prandtl numbers. The curves for a Prandtl number of 1 and a Reynolds number of 30,000 are shown for comparison. The data for water are in very good agreement with the predicted curves for water in both figures. The circles represent the water data. The data for oil in the upper figure lie above the predicted curve for oil. In the lower curve, some of the points for oil are in agreement with the predicted curve, but some lie above it. It appears that no definite conclusion can be drawn in the case of oil. In general,

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[3] Research Engineer, National Advisory Committee for Aeronautics, Cleveland, Ohio. Assoc. Mem. ASME.

it would seem that the agreement between the analysis and experiment for oil is probably within the accuracy of the data. The experimental trends with Reynolds number are in agreement with those predicted; that is, the values of h_x/h_s at a given X/D decrease as Reynolds number increases, Fig. 12 of the paper.

The region very close to the entrance, that is, for X/D between 0 and 1 or 2, could be seen better by using an expanded scale on the abscissa. However, as indicated by the author, the heat losses to the electrical terminal at the entrance might be important in that region. Somewhat more accurate results very near the entrance might be obtained by using a wound wire or ribbon type of heater rather than direct resistance heating of the tube. In this way the heat loss through the electrical connection at the entrance would be reduced considerably.

S. LEVY.[4] The author has made a valuable contribution in a field where experimental data are practically nonexistent. The values of thermal-entrance lengths he has presented will find major application in the design of heat-transfer equipment and in the evaluation of available theoretical studies. To obtain his experimental results the author had to distinguish between heat-transfer coefficients different from each other by 5 and 1 per cent, and his effort is all the more appreciated in a field where the spread of experimental data always exceeds 5 per cent. It is felt, however, that a more complete understanding of the contents and results of this paper would be achieved by including the original data shown in Tables 2, 3, 4, and 5 of reference (39).[5]

Tables 3 and 5 give the local values of the ratio[6] h_x/h_s and may be used to derive an explicit function for h_x/h_s in terms of x/D, with more and more weight being placed upon the larger, and therefore more accurate, values of the ratio h_x/h_s. The explicit function in turn can be used to calculate the 5 and 1 per cent entry lengths, and probably will yield values more consistent than those obtained by the author by means of a graphical plot. With more emphasis given to the higher ratios of h_x to h_s, it is expected that whenever the 5 per cent entry length will decrease from run to run the corresponding 1 per cent entry-length value will exhibit the same trend, thus eliminating some inconsistencies noted in Tables 2 and 3 of the paper.

Tables 2 and 4 of reference (39) serve to demonstrate the extent of property variation in the present study. The property variation, though major in several instances, has not been mentioned by the author. Its existence is apparent in Fig. 4 of the paper where the test-section friction factor is seen to fall consistently below the friction factor in the calming section. If the test-section values are corrected by the factor $(\mu/\mu_w)^{0.14}$ as recommended in reference (24), good correspondence is obtained between the friction factor in the test section and the corresponding value in the last section previous to start of heating. The corrections ranging from 2 to 8 per cent also lead to satisfactory agreement with the recommended equation. Because of the greater dependency of oil viscosity upon temperature the role of property variation is larger for this fluid, and the friction coefficient or wall shear stress in the test section is expected to fall from 7 to 25 per cent below the calming-section values.

In comparing the experimental data with theory it is necessary, therefore, to realize that the velocity profile though fully developed in the calming region, undergoes slight changes in the test section because of system heating. Also, examination of Tables 2 and 3 of the paper may give the reader the erroneous idea

that the author was not always able to repeat his result at the same Reynolds number. In most cases where the Reynolds number was unchanged the difference in entry lengths may be explained in terms of different heat input, i.e., different property variation.

The comparison between theory and test as presented by the author indicates that the eddy diffusivity plays a major role in the laminar sublayer. Agreement with the analyses of Latzko (21), Deissler (6), and Berry (1) support this conclusion since the three theoretical studies do not account for the thermal capacity of a sublayer in which only molecular conduction prevails. In order to confirm the possible need for a re-evaluation of the existence or extent of the thermal sublayer, the writer derived the thermal-entrance length for the flow of a high-Prandtl-number fluid in a circular pipe.[7] The solution based upon a laminar sublayer of thickness y_1 is obtained readily from the book of Carslaw and Jaeger[8] if the linear-velocity profile in the sublayer is replaced by an equivalent constant-velocity distribution. The 5 per cent entry-length value becomes equal to [9]

$$\left(\frac{L_e}{D}\right)_{5\%} = 0.892\ y_1^{+3} \left(\frac{\mu C_p}{k}\right) \Big/ \left(\frac{uD}{\nu}\right) \sqrt{\frac{c_f}{2}}$$

where

$$y_1^+ = \frac{y_1}{D}\ \frac{uD}{\nu}\ \sqrt{\frac{c_f}{2}}$$

When the variation of properties is negligible $y_1^+ = 5$ and

$$\left(\frac{L_e}{D}\right)_{5\%} = 112 \left(\frac{\mu C_p}{k}\right) \Big/ \left(\frac{uD}{\nu}\right) \sqrt{\frac{c_f}{2}}$$

and the numerical results given in Table 5 are obtained.

TABLE 5   NUMERICAL RESULTS

uD/ν	$C_p\mu/k$	Prediction constant property	Test results	Prediction-effect of property variation (max)
5500	223	46	14	1.6
7830	260	35	7.2	0.97
10100	206	24	5.7	0.83
13300	170	16	4.6	0.86
14800	157	14	4.4	0.75
16200	90	6.6	3.7	0.75
16300	135	9.9	4.1	0.72
17200	138	9.6	3.7	0.72
17300	87	6.0	3.7	0.71
22900	98	5.2	2.2	0.66
24600	107	5.7	2.8	0.58
24600	107	5.7	2.6	0.52
29800	88	4.1	2.0	0.43
34600	63	2.4	2.8	0.46
46400	63	1.9	1.5	0.37
46600	61	1.8	1.7	0.33

The computed and test values are seen to decrease with Reynolds number[10] in direct contrast to the predictions of Latzko (21), Deissler (6), and Berry (1). Quantitatively speaking, the present theory always yields values larger than the experimental ones. This trend is to be expected since the value of y_1^+ is not equal to 5 but continually decreases below 5 in the entrance region. The value of the term y_1^+ at the end of the entrance region may be obtained from the results of Boelter, Martinelli,

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[4] Engineer, General Electric Company, Schenectady, N. Y. Assoc. Mem. ASME.

[5] All reference numbers refer to Bibliography at the end of the paper.

[6] All symbols, unless otherwise noted, are those defined in the author's nomenclature.

[7] "Heat-Conduction Methods in Forced-Convection Flow," by S. Levy, ASME Paper No. 54—A-142.

[8] Reference 5 in author's Bibliography.

[9] The 5 per cent entry length was chosen over the 1 per cent value because the experimental measurements at 5 per cent may be viewed with more confidence.

[10] The same trend is noted in results obtained by I. T. Alyadev, "Experimental Determination of Local and Mean Coefficients of Heat Transfer for Turbulent Flow in Pipes," NACA TM 1356, 1954.

and Jonassen,[11] or it may be computed from the measurements of the fully developed heat-transfer coefficient

$$\frac{hD}{k} = \frac{D}{y_1}$$

Values of y_1/D computed from the latter expression were substituted in the derived equation for $(L_e/D)_{5\%}$ and the corresponding entry lengths are tabulated with the constant property values. The two sets of predictions really represent an upper and lower limit of the term $(L_e/D)_{5\%}$ and examination of the results reveals that they play such a role. This bracketing effect coupled with the predicted decrease of entrance length with Reynolds number leads the writer to believe that the test results lend support rather than weaken the concept of laminar sublayer. Further, it is felt that a sublayer analysis accounting for the decrease in y_1 will give answers within the experimental range. With regard to the effects of Reynolds number it is interesting to note that, since at low Prandtl number the entry length is directly proportional to the Reynolds number, the entrance length must be independent of Reynolds number at some intermediate range of the Prandtl number.

The simplicity of the analytical solution at high Prandtl numbers suggests a method of measuring the thermal-sublayer thickness. By minimizing the effect of properties (fluid with relatively small property variation with temperature) or by controlling this variation so that y_1^+ remains about constant in the entrance region (test section maintained at constant temperature) the measured values of $(L_e/D)_{5\%}$ can be utilized to compute the laminar sublayer thickness y_1^+ by means of the derived expressions.[7] Such a technique may be preferable to the use of a temperature probe which disturbs the laminar nature of the sublayer and exhibits errors too close to the pipe wall. On the other hand, the entry-length measurements will require careful and complete experimentation of the type presented by the author.

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[11] "Remarks on the Analogy Between Heat Transfer and Momentum Transfer," by L. M. K. Boelter, R. C. Martinelli, and F. Jonassen, Trans. ASME, vol. 63, 1941, pp. 447–455.

AUTHOR'S CLOSURE

The general agreement of the experimental results presented by the author with the analytical results of Deissler is quite satisfactory. The Prandtl-number effect predicted by Deissler will be extremely difficult to verify experimentally in any detail although there appear to be some indications of this effect in the present experimental data as discussed in detail by Deissler.

In so far as obtaining more accurate information near the very entrance, this presents a formidable problem. Regardless of the type of heating used, there will be some axial heat conduction in the fluid itself which makes impossible, in practice, the obtaining of the initial conditions imposed in the analysis.

With respect to Dr. Levy's suggestion of deriving an explicit function for h_x/h_s in terms of x/D, this could easily be accomplished and might smooth out some of the inconsistencies in the results. However, it was felt that the statement that the thermal entrance length was approximately 10 to 15 pipe diameters over the range cited was sufficient for most applications; under those circumstances an explicit function was not considered essential.

The variation of the property values should certainly have been mentioned in the paper and the author is indebted to Levy for emphasizing this point. In particular, his suggestion that the correlation of the friction-factor results may be improved by the viscosity correction factor is a valuable criticism and results in improved agreement of the experimental values with the commonly accepted equations.

The approximate analysis proposed by Levy for high-Prandtl-number fluids predicts decreasing thermal-entrance lengths with increasing Reynolds number, and this is in agreement with the present experiment results. Contrary to the statement of Levy, this trend is also in agreement with the analysis of Deissler. However, Deissler's analysis shows decreasing thermal-entrance lengths with increasing Prandtl number, whereas Levy's approximate analysis yields the opposite trend, increasing entrance lengths with increasing Prandtl number. No conclusive evidence regarding this Prandtl-number effect is derivable from the experimental results.