of the Reynolds number and the same starting-length function appeared. However, the experimental data are not exact enough to establish definitely the relationship and to determine whether the correlations are identical.

In Fig. 12 values of the various functions of starting length are plotted versus $x_{st}/x$. The agreement between the theoretical and empirical results and between mass and heat transfer in turbulent flow is very satisfactory. The shape of a curve which would represent Equation [21] is shown in Fig. 9 only, not to overcrowd Fig. 12. Table 3 indicates that values of $\Psi$ according to this equation would be 0 to 11 per cent higher than those of the lowest curve in Fig. 12. For comparison the theoretical starting-length function for laminar flow is also shown.

## Summary and Conclusions

1 This paper describes an investigation of the evaporation of water from wetted plane surfaces to an expanding flat air jet.

2 Local rates of evaporation were obtained by means of a mass-flow meter, consisting of horizontal porous plates which formed the covers of individual compartments. Each compartment was supplied separately with water; the total evaporating porous surface was nearly 36 in. long and 15 in. wide. Dry "starting sections" could be obtained at will.

3 The air stream was discharged parallel to the system of porous plates from a horizontal rectangular-slot nozzle of $^1/_4$-in. gap. Initial velocities were varied from 50 to 230 fps.

4 Two types of tests were carried out. In the first the air of the jet and of the environment were of the same moderate humidity; in the second type the laboratory air was humidified artificially and the nozzle-outlet air dried. In this way concentration gradients from the wet surface to the jet and from the environment to the jet were established.

5 Distributions of velocity, temperature, and humidity at various heights above the evaporating surface were investigated. A traverse rake containing thermocouples and impact tubes was used to explore the jet, as was a dew-point meter of special design. It does not seem that humidity traverses above a surface evaporating into an expanding air stream have been performed in any previous investigation.

6 Distribution of local surface temperature was measured over the entire test plate.

7 A satisfactory correlation was obtained, showing that a modified Nusselt number, based on local coefficients, is proportional to the 0.8 power of a Reynolds number based on local maximum jet velocity and distance from the nozzle outlet. The range of Reynolds number was $0.16 \times 10^6$ to $1.3 \times 10^6$.

8 A strong influence of the ratio of dry starting length $x_{st}$, to total length, $x$, was observed and incorporated into the correlation.

9 Various theoretical and empirical formulas which account for the influence of $(x_{st}/x)$ on mass transfer and heat transfer were discussed and compared with the experimental data.

10 Our final correlation is in good agreement with Maisel and Sherwood's empirical representation of their evaporation data, notwithstanding considerable differences in configuration and type of flow.

11 Some heat-transfer measurements were performed in the same field as those of evaporation. They led to a correlation which is not greatly different from that of the mass transfer; however, further experimentation will be necessary to obtain a quantitative comparison.

## Acknowledgment

The authors wish to express their appreciation to the Air Force Wright Air Development Center for permission to publish this paper.

## Bibliography

1 "Heat Transfer From an Air Jet to a Plane Plate With Entrainment of Water Vapor From the Environment," by M. Jakob, R. L. Rose, and M. Spielman, Trans. ASME, vol. 72, 1950, pp. 859.

2 "Local Coefficients of Mass Transfer by Evaporation of Water Into an Air Jet," by M. Spielman, unpublished PhD thesis, Illinois Institute of Technology, Chicago, Ill., 1951.

3 "Temperature Measurements in High-Velocity Air Streams," by H. C. Hottel and A. Kalitinsky, *Journal of Applied Mechanics*, Trans. ASME, vol. 67, 1945, p. A-25.

4 "Heat Transfer," by M. Jakob, John Wiley & Sons, Inc., New York, N. Y., vol. I, 1949.

5 "Defrosting and Ice Prevention," by M. Jakob, Ten unpublished reports to the United States Air Force, Illinois Institute of Technology, 1947, 1948, 1949, and 1950.

6 "Evaporation of Liquids Into Turbulent Gas Streams," by D. S. Maisel and T. K. Sherwood, *Chemical Engineering Progress*, vol. 46, 1950, p. 131.

7 "Mass Transfer Effects in Turbulent Streams," by D. S. Maisel, unpublished PhD thesis, Massachusetts Institute of Technology, 1949.

8 "Heat Transfer From a Cylindrical Surface to Air in Parallel Flow With and Without Unheated Starting Sections," by M. Jakob and W. M. Dow, Trans. ASME, vol. 68, 1946, p. 123.

9 "Introduction to the Transfer of Heat and Mass," by E. R. G. Eckert, McGraw-Hill Book Company, Inc., New York, N. Y., 1950.

10 "An Analytical Investigation of Convective Heat Transfer From a Flat Plate Having a Stepwise Discontinuous Surface Temperature," by M. W. Rubesin, unpublished ASME Paper No. 48—A-43, 1948.

11 "Influence of Curvature Upon the Heat Transfer From Cylinders to Gas Streams Parallel to the Axis," by W. Tessin, unpublished PhD thesis, Illinois Institute of Technology, 1951.

# Discussion

D. S. Maisel[12] and T. K. Sherwood.[13] The authors have measured the rate of evaporation of water from a series of successive plane strips into a jet of air flowing parallel to the surface. These data, together with similar information obtained by the writers, confirm the fact that mass and heat-transfer rates are influenced by the presence of a "starting length," i.e., an initial length of plane over which momentum transfer without heat or mass transfer occurs. In addition, the present data are correlated excellently by an expression developed from our earlier data. While this is pleasing, there appear, at first glance, several differences in experimental conditions which should be noted.

1 Conditions of flow are quite different. The authors used an air jet resulting in an inverted form of velocity profile over the transfer surface. This is compared to our earlier work wherein the plane surface was suspended in the center of the air stream in a tunnel drier.

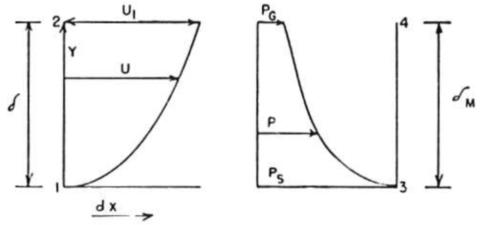
2 In some experiments by the authors an inverted humidity gradient was obtained by steam injection into ambient air.

3 We used average coefficients for 1, 2, and 3 planes, each 2 in. in downstream length, with the Reynolds number determined by the distance to the downstream edge of the wet surface. The authors calculate the Reynolds number based on the downstream distance to the center of the 3-in. section.

It may be helpful to dwell briefly on the conditions of flow at a plane boundary to help understand the mechanism of mass and momentum transfer. As shown in Fig. 13, herewith, a velocity and a concentration gradient exist at the interphase boundary. The conditions at this point can be expressed rigorously by means of a material balance. Note that momentum and heat balances may be similarly derived for these cases if desired. The crux of the situation lies, therefore, in determining the partial

[12] Esso Laboratories, Development Division, Standard Oil Development Company, Linden, N. J.

[13] Massachusetts Institute of Technology, Cambridge, Mass.




$$\frac{d}{dx}\int_0^{\delta_M}(P - P_G)\,U dY = \left(\frac{dP}{dY}\right)_W$$

$\delta_M$ = mass-transfer boundary layer
$\delta$ = momentum-transfer boundary layer must determine
(1) $P = F(Y)$ (2) $U = F(Y)$

FIG. 13 GENERAL BOUNDARY-LAYER EQUATION FOR MASS TRANSFER

pressure, velocity, and/or thermal gradients as a function of distance from the transfer surface. Several solutions for the mass-transfer case have been described. The results are generally quite similar and differ only to the extent that different functions were assumed in each for velocity and partial-pressure functions.

The authors, for example, cite the work of Eckert in deriving a relationship based on a parabolic velocity distribution. His relationship is shown in Fig. 14, together with the results from two other theoretical derivations. The first and the second, based on simple power series, obviously apply to conditions for laminar flow in the boundary which is characterized by the 0.5 power of the Reynolds number.

ECKERT (1950)

$$(N_{NU})_{HT} = 0.331\,(R_E)^{1/2}\,(P_R)^{1/3}\left[1-\left(\frac{X_{ST}}{X}\right)^{0.75}\right]^{-1/3}$$

MAISEL – SHERWOOD (1949)

$$(N_{NU})_{MT} = 0.323\,(R_E)^{1/2}\,(S_C)^{1/3}\left[1-\left(\frac{X_{ST}}{X}\right)^{0.75}\right]^{-1/3}$$

SEBAN (1951)

$$(N_{NU})_{HT} = 0.0289\,(R_E)^{0.8}\,(P_R)^{1/3}\left[(P_R)^{1/9}\left[1-\left(\frac{X_{ST}}{X}\right)^{0.9}\right]\right]^{-0.111}$$

FIG. 14

We have included the Seban relation, which was brought to our attention only recently, because it is so similar to the empirical expression which has been shown to apply to mass transfer from plane surfaces. This expression, incidentally, was derived for a turbulent boundary layer over an isothermal section preceded by an unheated section.

The point which has been troubling the writers is evident from these expressions. We believe $x_{st}$ and $x$ are independent, and it does not seem likely that a correlation based solely on the ratio $x_{st}/x$ can be complete. For a given air speed the laminar boundary layer breaks to a turbulent layer at some definite distance from the leading edge of the plate, and the position of the wet test surface relative to this break point is important; the rate of vaporization certainly will depend on whether the wet surface is under the laminar or the turbulent boundary layer, or under both. It seems evident, therefore, that both $x_{st}$ and $x$ must enter the correlation; the ratio $x_{st}/x$ is insufficient.

M. W. RUBESIN.[14] It is gratifying that the authors chose to emphasize the role played by boundary layers of momentum, heat transfer, and mass transfer which do not all begin at the same place. The point that the surface-temperature distribution in the case of heat transfer and surface partial-pressure distribution in the case of mass transfer are important factors in controlling the amount of heat or mass transfer cannot be overemphasized. Even after a fair amount of work dealing with these effects has been published (see Bibliography of paper) one may still find articles in the literature which ignore the effects of the surface temperature or partial-pressure discontinuity.

One aspect of this paper should be clarified. The end results of the paper are given in terms of dimensionless groups which implies their application to more general cases. The agreement of the data with Maisel and Sherwood's work seems to substantiate this general character. To determine the values of the local Reynolds number and local partial-pressure potential, however, it is necessary to know $v_j$ and $p_j$. As a designer is much more likely to know the values of the velocity and partial pressure at the exit of his nozzle, some means of relating $v_j$ and $v_o$ or $p_j$ and $p_o$ for various shaped nozzles and various free-stream partial pressures, if known, should be included.

MYRON TRIBUS.[15] The writer believes that the method of accounting for the unheated starting length used by Rubesin[16] and by Maisel and Sherwood, is more realistic than the authors' Equation [13] or [15]. The boundary conditions require that the equation have a singularity at the point where the temperature or vapor discontinuity occurs. Scesa[17] credits Seban with the development of a starting-length function of the form

$$\left[1 - \left(\frac{X_{st}}{X}\right)^{9/10}\right]^{-1/9}$$

This agrees fairly closely with the empirical equation of Maisel and Sherwood. A good test of these equations cannot be made unless the measurements are reduced to point measurements rather than averages over a short length of the plate. A method of comparing the various functions which are proposed to take into account starting length is to provide several steps in vapor pressure in order to accentuate the discontinuities in the region where $(X_{st}/X) = 1$. The important differences in these expressions, which are all approximations, is in the neighborhood of the singularity.

The authors do not draw any conclusions concerning heat transfer. The data of run II-9 show that if one uses the minimum jet vapor pressure at each station to compute the "equivalent free-stream dew point" (by definition) and the local maximum dry-bulb temperature (61.5 F and 84 F, respectively) the wet-bulb temperature is computed as 67.5 F. The plate temperature was measured at 68.2 F; hence the heat and mass-transfer rates must have been in almost the same ratio as over a wet-bulb thermometer.

The effect of starting length on heat transfer, if the foregoing result is typical, is the same as on mass transfer. Unfortunately, other runs could not be analyzed in this fashion as the other data are not given.

## AUTHORS' CLOSURE[18]

It is gratifying that Messrs. Maisel and Sherwood start their comments with elaborating item 10 of the Summary in the

[14] Ames Aero Works, Moffett Field, Calif

[15] Director, Icing Research, Engineering Research Institute, University of Michigan, Ann Arbor, Mich. Mem. ASME.

[16] See Authors' bibliography, reference (10).

[17] "Experimental Investigation of Convective Heat Transfer to Air From a Flat Plate With a Stepwise Discontinuous Surface Temperature," by Steve Scesa, MS thesis, University of California, 1951.

[18] In the absence of Mr. Spielman, this closure was prepared by Max Jakob, with confidence that he would agree with it.

paper, and add Seban's equation, which we had not known, to those discussed in the paper. Their next point is that the term $x_{st}/x$ occurring in all correlations, including theirs and ours, does not cover the whole mechanism of development of the boundary layers. However, it seems that not using an additional term, as, for instance, $x_{cr}/x$ where the subscript cr refers to the critical distance at which turbulence starts, does not seriously affect the correlation. This would mean that the separate influences of $x$ and $x_{st}$, which Maisel and Sherwood would like to have considered, actually are taken care of by the two independent functions of $vx/\nu$ and $x_{st}/x$ which occur in the various correlations. One might solve each of the correlating equations with respect to $x$, keeping $x_{st}$ constant, or with respect to $x_{st}$ with $x$ as parameter. Of course, as usual in similarity treatments, the relationship between the dimensionless groups used is a very simplified form which may be far from the form of an analytical solution of the differential equations.

Referring to the last item of Maisel and Sherwood's discussion, this author agrees with their statement that starting of the mass transfer in the laminar or turbulent section of the momentum boundary layer should yield different rates of evaporation. Hence, if a correlation should cover both cases, a third independent variable, for instance, $x_{cr}/x_{st}$ or $x_{cr}/x$, would have to be added.[19] However, because of the flow conditions past our nozzle,[20] it is quite unlikely that in any of our tests the boundary layer was in laminar flow at the start of evaporation; therefore the two independent variables used are sufficient.

Mr. Rubesin's remark that, for practical purposes, $v_o$ and $p_o$ should be used is also well justified. The velocity $v_o$ can be taken from Fig. 5 of the paper. As an average of the two lines in this figure

$$v_o = 1.97x^{0.435}\, v_j$$

with $x$ in foot-units.

The pressure $p_o$, on the other hand, can be calculated from the equation

$$N_p = 0.045\left(\frac{x}{a} - 14.4\right)^{0.80}$$

where $a$ = vertical width of our nozzle, and

$$N_p = \frac{p_o - p_j}{p_j - p_e}$$

This equation is based on Fig. 17 of reference (1); however, it is more exact than Equation [13] of that paper, for a reason given there.[21]

Mr. Tribus in his discussion seems to have overlooked the fact that Equations [13] and [15] of the paper are not those recommended in the present paper but are taken from reference (8) for comparison with our Equations [5] and [7], in which we used the starting-length function of Maisel and Sherwood. We did this because their function can be extrapolated to $x_{st}/x = 1$, while Jakob and Dow's empirical function, though perfectly representing the observed data, is not fit for extrapolation to the largest values of $x_{st}/x$. Hence our equations satisfy Tribus' requirement. We also agree with him concerning the importance of the crucial check in the neighborhood of $x_{st}/x = 1$.

In an example from our tests, Tribus demonstrates the agreement between heat and mass transfer. As mentioned already in the paper, we correlated the data of heat transfer occurring in the same field as mass transfer, and we found the form of our Equations [11] and [12] confirmed with not more scattering in the heat-transfer case than in the corresponding Fig. 10, and less scattering than in Fig. 11. However, the constant factors were 28 and 26 per cent larger than those in Equations [11] and [12], respectively.

Half of this difference might have been expected, according to an equation of Gamson, Thodos, and Hougen,[22] which can be converted to the form

$$\frac{N_{Nu}}{(N_{Nu})_{mod}} = 1.076\sqrt[3]{\frac{N_{Pr}}{N_{Sc}}} = 1.076\sqrt[3]{\frac{0.71}{0.59}} = 1.143$$

The other half may come from inaccuracies of the heat balance, because our apparatus had not been designed for measuring the heat exchange with the environment exactly, except for the upper surfaces of the test plates. Therefore we postponed publishing this part of the results.

Summarizing, it is a pleasure to feel oneself in agreement with the elucidating comments of four noted experts in the field under consideration.

[19] Fig. 15 in reference (8), however, shows that for different values of $x_{cr}/x$ the relationship in the fully developed turbulent range may be independent of $x_{cr}/x$.

[20] See second paragraph in section, Influence of Starting Length.

[21] See reference (1), p. 865, footnote 9.

[22] "Heat, Mass and Momentum Transfer in Flow of Gases Through Granular Solids," Trans. American Institute of Chemical Engineers, vol. 39, 1943, p. 1.