detailed information needed for complete rationalization of the phenomenon. A local flow instability is a possible explanation, but no complete evidence is available for it. At values of $S/D > 0.60$, the heat-transfer coefficients again follow the original trend of the data. A beginning of transition is indicated, but the recovery factors, in general, do not indicate the development of a completely turbulent flow. The run at the lowest Reynolds number appears to indicate separation rather than transition, and the recovery factor is substantially reduced in the region downstream of the separation point.

The results for the upper side of the ellipse at a 5-deg angle of attack reveal somewhat different characteristics in the demonstration of laminar, transition, and turbulent boundary-layer flow. Because of the rapid acceleration at the stagnation point no details are available for this region; however, the first instrumentation point, at the end of the major axis, gives a heat-transfer coefficient as high as the maximum value obtained with zero angle of attack. The boundary layer remains laminar for a relatively short time, transition to turbulence occurring rather early, probably because of the greater instability of the decelerated flow. Excellent correspondence exists between the recovery-factor measurements and the heat-transfer coefficient measurements, with the recovery factor attaining a value of approximately the cube root of the Prandtl number as soon as the boundary layer appears to become turbulent. The differences in the transition point for the two sides of the ellipse tested may be due to some undetected difference in the surface condition of the model.

Acknowledgment

The results given herein were obtained in connection with Air Force Contract AF-33(038)-12941 and this support is gratefully acknowledged. L. J. Gee performed the experimental work and F. M. Sauer contributed substantially in the entire effort.

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Discussion

E. R. G. Eckert.[4] The paper constitutes a contribution which will be welcomed by those interested in convective laminar and turbulent heat transfer in boundary-layer-type flow. The results should be useful in checking available calculation procedures especially in so far as the influence of a temperature variation along the surface and of frictional heating on heat transfer is concerned. For an investigation of the influence of a pressure variation along the surface the experiments are less suited since the large axis ratio of the ellipse used makes conditions over the major part of the surface quite similar to those on a flat plate. It will be interesting to check how well the surface temperatures, calculated with available heat-transfer coefficients obtained from low-velocity relationships, together with adiabatic wall temperatures obtained on the unheated cylinder with low heat conductivity, agree with measured surface temperatures.

Remarkable are the low values of the recovery factor measured on the rear part of the cylinder surface and presented in Fig. 3 of the paper. Such low values on surfaces in separated flow regions were first observed by Eckert and Weise on circular cylinders and have been investigated recently and extensively by L. F. Ryan.[5] Obviously, they are connected with vortex-type flow.

A. N. Tifford.[6] Because of wall interference, compounded by compressibility effects, the experimental data presented in the paper are not correct for the general case of an elliptical cylinder of 1:4 axis ratio. Rather, the data are valuable, as indicated in the Introduction, primarily for providing a check on theoretical methods of prediction of the distribution of the local heat-transfer rate. However, the paper itself makes no more than a qualitative comparison of theory and experiment. For this reason it is found to be incomplete. The fact that similar experimental data have been presented in the past without a detailed comparison with theory does not justify all future papers being in kind. After all, it has been only within the past few years that relatively precise theoretical methods of heat-transfer calculation (at least for the laminar boundary layer) have become available.

Essentially, two separate experimental studies have been reported: (*a*) The variation of the recovery factor on the surface of an unheated elliptic cylinder, and (*b*) the temperature distribution on the surface of an elliptic cylinder when the heating rate per unit surface area is held constant. On the basis of these data, the Nusselt-number variations along the surface have been given. In this presentation the heat-transfer coefficient has been based on the local temperature difference between the insulated cylinder surface and the heated cylinder surface. Contrary to the inference of the paper, however, there is nothing uniquely correct about this method of accounting for the effect of frictional heating. What does particularly recommend it is the lack of a significant pressure-gradient effect on the frictional heating factor. Computational studies—as well as the present data—have shown that Pohlhausen's square-root rule satisfactorily specifies the insulated-surface temperature in the laminar-flow region regardless of surface pressure-gradient conditions.

The relative invariability of the heat-transfer-coefficient distribution along the surface with changes in angle of attack and compressibility is primarily due to the constant surface-heating rate imposed. Any relative change in the operating conditions at a point on the surface tends to change the local heat-transfer coefficient and the local surface temperature in compensating directions. Thus a much smaller than expected change in local heat-transfer coefficient occurs. For example, if the relative local velocity at a particular point increases, the local heat-transfer coefficient thereby tends also to increase. Since the heating rate is held constant, the local surface temperature must decrease. This decrease in local surface temperature, as compared with the surface just ahead, introduces thermal gradients in the boundary layer which tend to decrease the local heat-transfer coefficient, and so on. (Of course, an equilibrium condition normally is rapidly established.)

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[4] Professor, Department of Mechanical Engineering, University of Minnesota, Institute of Technology, Minneapolis, Minn. Mem. ASME.

[5] "Experiments on Aerodynamic Cooling," by L. F. Ryan, Mitteilung Institut Aerodynamik, Eidgen. Technische Hochschule, Zurich, Switzerland 1951, pp. 7–52.

[6] Aeronautical Engineering, The Ohio State University, Columbus, Ohio.

Two things are to be learned from the foregoing: (1) For the most precise check by means of the present data of general theoretical methods of calculation of heat transfer for nonisothermal surfaces, the surface-heating rate should be calculated on the basis of the measured surface temperature distribution and then compared with the actual heating rate; and (2) inherently less precise methods of heat-transfer calculation are satisfactory for surfaces subjected to a constant surface heating rate.

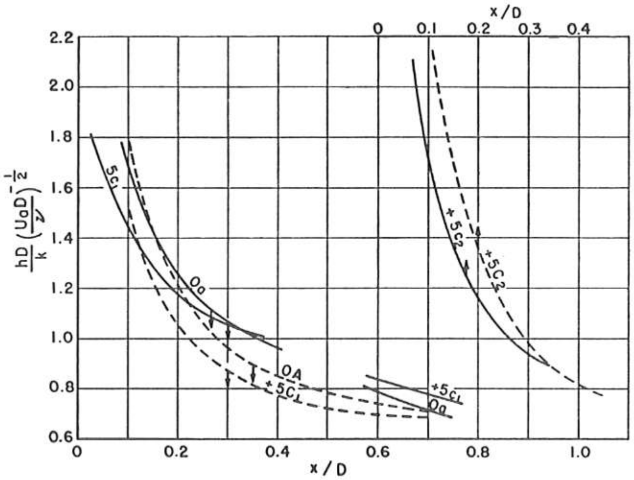
In summary, some very interesting experimental data have been presented but their analysis seems rather incomplete.

AUTHORS' CLOSURE

The utilization of the results for comparison with methods for predicting local heat-transfer coefficients has been noted in the paper and called for by one of the discussions. For the region in which the boundary layer is laminar, Eckert[7] has reviewed a number of methods and has presented a method simple enough to be of utility and applicable in the case of isothermal surfaces. Seban[8] has presented a similar method, modified to account for variable surface temperature, the modification being approximate and inducing some further uncertainty into the results. That method, when applied for the experimental velocity distribution over the elliptic cylinder and for the case of constant heat rate that obtained in the experiments, yielded the results shown in Fig. 6 of this closure for the laminar flow region in the three altitudes of the model presented in Figs. 2, 3, and 5 of the paper. As in those figures, the stagnation region is eliminated in favor of a larger ordinate scale.

The comparison between prediction and experiment is favorable, although the discrepancy is as much as 20 per cent in some regions. At the stagnation point, not shown in the figure, the heat-transfer coefficient predicted from the theoretical velocity distribution is always lower than that measured, the difference again not exceeding 20 per cent. Far from this region, at about $0.2 < S/D < 0.8$ for the results in Figs. 2 and 3, the free-stream velocity becomes almost uniform and Dr. Eckert has noted that the "flat-plate" equations might be applicable there. If used, they yield results about 10 per cent lower than the more elaborate prediction and this indicates their utility provided the pressure gradient is small. The comparison of the measured values and those predicted by accounting for and by ignoring the variation of pressure indicate that the theory does not account sufficiently for either the effect of pressure variation or for the surface-temperature variation.

The rapid decrease in the recovery factor in the rear region of the cylinder, evident in Fig. 3, does not appear consistently with equal Reynolds numbers, and was more evident in the runs shown in Fig. 3 than in those shown in Fig. 2. This has not been found in experiments on a 1:3 elliptic cylinder, and may depend on attaining some appropriate type of flow in the wake region immediately adjacent to the cylinder.



Zero angle of attack:

Oa experiment at $\frac{U_aD}{\nu} = 607{,}000$

OA prediction for $\frac{U_aD}{\nu} = 928{,}000$

5-Deg angle of attack, under side:

$5c_1$ experiment at $\frac{U_aD}{\nu} = 915{,}000$

$5C_1$ prediction for $\frac{U_aD}{\nu} = 917{,}000$

5-Deg angle of attack, upper side:

$5c_2$ experiment at $\frac{U_aD}{\nu} = 915{,}000$

$5C_2$ prediction for $\frac{U_aD}{\nu} = 917{,}000$

FIG. 6 COMPARISON OF EXPERIMENTAL AND PREDICTED RESULTS

It is recognized that different bases can be chosen for the temperature difference used in the evaluation of the heat-transfer coefficient, but the present method is consistent, as noted by Dr. Tifford, with the values of the recovery factor deduced by the adiabatic wall-temperature measurements. This is a sufficient argument for the elimination of other methods which yield heat-transfer coefficients which would refer to a different kind of surface-temperature distribution and yet not clearly specify a recovery factor.

There seem to be no grounds for considering the constant heat-rate surface condition to give a more invariable heat-transfer coefficient than would be the case with an isothermal wall. For the theoretical wedge flows, the variability of the heat-transfer coefficients with distance along the wall is the same for constant surface temperature and for constant heat rate. For the elliptic cylinder, since both cases coincide at the stagnation point, there is a slightly greater variation predicted for the isothermal case, but the relative difference between the constant heat-rate and constant wall-temperature cases is small.

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[7] Refer to (6) of the authors' Bibliography.

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