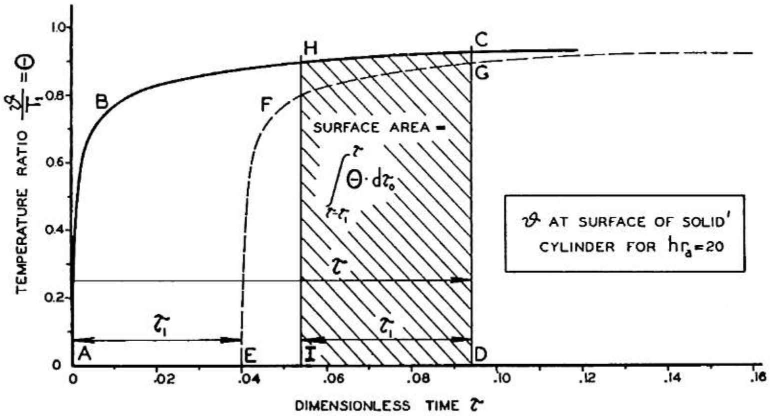

FIG. 9 SURFACE TEMPERATURE θ OF A SOLID CYLINDER AS A FUNCTION OF TIME FOR A SUDDEN CHANGE OF THE AMBIENT TEMPERATURE T_1

change at radius r represented by the ordinate at a point having as abscissa $(\tau - \tau_0)$ and situated on curve A-B-C representing the function Θ, Fig. 9. The integration is proportional to the surface A-B-C-D. However, if the temperature stops increasing beyond the point A, Fig. 8(a), we have to subtract the surface E-F-G-D corresponding to the time interval $(\tau - \tau_1)$. The value of the temperature θ at time τ is therefore proportional to the surface

$$ABCD - EFGD = ABCD - ABHI = IHCD$$

i.e., to the surface enclosed by the axis of time τ, the curve Θ, and the two verticals I-H and C-D separated by the lapse of time τ_1. The value of θ can be plotted as a function of time τ after multiplying the areas by the slope $\dot{f}(\tau_0)$, according to Equation [43].

In case of a nonlinear variation of temperature T, Fig. 8(b), the curve is split up into a series of straight lines and the preceding method is applied for each straight-line portion of the polygonal approximation. It must be remembered that each preceding increase of temperature ΔT_1 remains constant when the next sloping line is considered.

The foregoing theory for temperatures applies equally well to thermal stresses.

Example. Fig. 10 is an example applied to the determination of the variation with time of the tangential thermal stresses at the surface of a solid cylinder and for different laws of variation of the ambient temperature. Each stress curve is derived by graphical method from the basic curve 1. It is interesting to note the relatively small diminution of the maximum stress for slower heating.

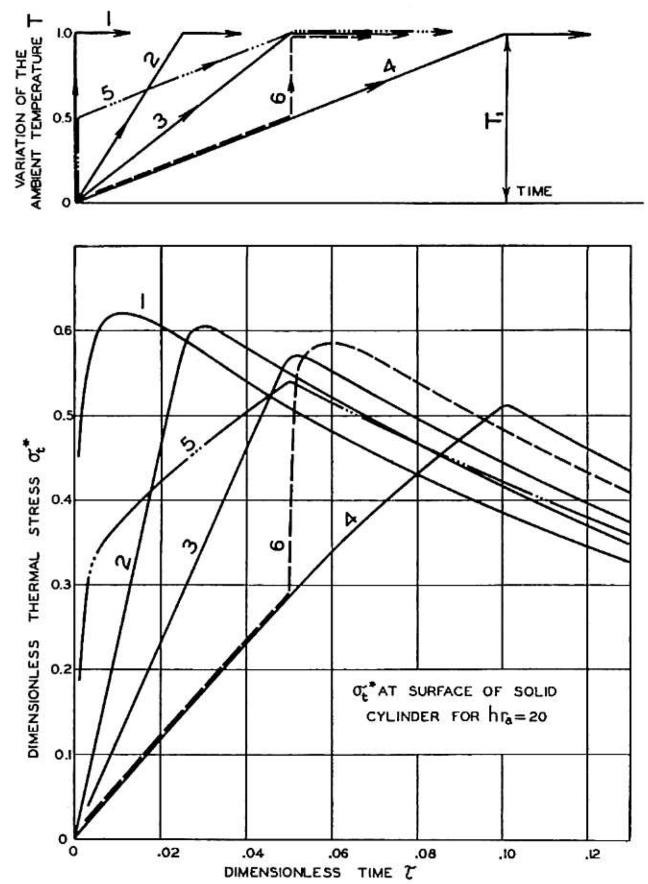

FIG. 10 TANGENTIAL THERMAL STRESSES AT SURFACE OF SOLID CYLINDER FOR DIFFERENT LAWS OF VARIATION OF AMBIENT TEMPERATURE T

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Discussion

T. McLEAN JASPER.[8] This paper is highly theoretical and the writer's purpose is to supply certain facts which have been obtained in the testing of pressure vessels to destruction so that the author may fit them into his mathematical interpretation of what happens to pressure vessels under the testing conditions, if such a fitting can be made.

The Lamé formula for the maximum stress is the one which we

[8] A. O. Smith Corporation, Milwaukee, Wis. Mem. ASME.

have applied in pipe, relatively thin solid-wall vessels, and thick-layer vessels. We find that it discovers for us the yield and bursting strength when the cylinders are long, are made of steel, are not autofrettaged or precompressed, and are tested at ordinary temperatures. When steel cylinders of pressure vessels are about two diameters long, we find that the heads or ends retard the building up of the maximum stresses in the middle length of the cylinder so that the yield and burst pressure is about 13 or 14 per cent higher than the Lamé formula indicates. However, when the cylinder is about four or more diameters long we have discovered, if the longitudinal weld is as strong or stronger than the steel in the cylinder, that the yield and bursting stress is closely indicated by the normal strength tests such as the normal plate specifications require for and when the Lamé formula is applied.

We have discovered that large thick solid-wall vessels do not follow the foregoing statement. We have burst five such vessels which we had hoped would reach a hydraulic pressure of 20,000 psi before bursting. The highest bursting pressure for the five large solid wall vessels was slightly over 17,000 psi, or approximately 85 per cent of the anticipated value. The average bursting pressures of these five solid wall vessels were about 80 per cent of the anticipated value.

We also have burst five large thick-layer vessels, and the bursting pressures have all been above the minimum of the steel specifications used. Some of these burst vessels have been slightly precompressed so that in these cases the yield and ultimate strengths were slightly increased above the anticipated stress at yield and ultimate bursting pressure when compared with the physical tensile-test properties of the steel and when the Lamé formula was used. It should be mentioned here that the total amount of energy contained in the water and steel at the burst pressures for these vessels was well over 2,000,000 ft-lb in each case. The solid-wall vessels shattered into many fragments, but the layer vessels were always in one piece, thereby presenting a much less dangerous situation should failure occur in any service in which they were used.

We have made several hundred tests on relatively thin solid-wall cylinders about 40 ft long and varying in diameter from 14 to 30 in., on many of which we measured the yield and ultimate pressures and compared them with the actual tensile test properties of the steel. The short cylinders in this series of tests led us to testing nothing shorter than 4 diameters long, if we would wish to check with the material physical tests, and also through the Lamé formula anticipate the minimum bursting pressure of a long cylinder.

About 30 full-size pressure vessels varying in diameter up to 10 ft, and in length up to 50 ft, have been tested to destruction in which the yield and ultimate pressures have been recorded. With the exception of the 5 large thick solid-wall vessels mentioned, and the first 2 vessels tested without proper head shapes and reinforcement of openings, we have found that the Lamé formula will anticipate the yield and burst qualities of such vessels very closely. For these reasons we believe that the Lamé formula not only anticipates the yield strength of a nonprecompressed layer vessel, but it anticipates its bursting strength as well.

Since the author refers to layer vessels in particular, some particular data will be presented on two layer vessels which were made from identical plates. Each plate was cut in half and wrapped upon a 20-in-OD length of pipe $^1/_2$ in. thick. Upon each piece of pipe were wrapped twelve $^1/_4$-in. layers, making a cylinder with a nominal inside diameter of 19 in., and a nominal outside diameter of 26 in. The vessels were marked A and B.

The Lamé formula was applied to the actual dimensions of the cylinder as follows (the actual dimensions were slightly different from those given.)

$$S = P\,\frac{D_1^2 + D_2^2}{D_1^2 - D_2^2} = 3.219\,P$$

Vessel A was tested as wrapped and vessel B was stress-relieved at 1150 F, and held at temperature for $3^1/_2$ hr, and was then cooled slowly.

The test values are given in Tables 1 and 2.[9]

TABLE 1 AVERAGE PHYSICAL PROPERTIES OF STEEL IN VESSELS DURING TEST

Coupons from vessel	Yield strength, psi	Ultimate strength, psi	Elongation in 2 in., per cent	Remarks
A.......	39770	58845	29.5	(Not stress-relieved)
B.......	37670	56260	32.3	(Stress-relieved same as vessel B)

TABLE 2 STRESSES AT YIELD AND FAILURE OF VESSELS

	Yield by water column		Ultimate bursting point	
Vessel	Pressure, psi	Stress, psi	Pressure, psi	Stress, psi
A......	13400	43100	18800	60400 (not stress-relieved)
B......	11500	37000	17375	55900 (stress-relieved)

The calculation of stresses in Table 2 is obtained by multiplying the pressure by 3.219, which is a strict application of the Lamé formula to the original dimensions of the two cylinders. In considering vessel A that was not stress-relieved, it is evident that a slight precompression is obtained in both the yield pressure and ultimate bursting point. This was due to the wrapping method. When vessel B was stress-relieved, however, this small precompression was relaxed which indicated that the Lamé formula applies to layer-vessel stresses due to pressure not only at the yield but at the ultimate also. This had been suspected a considerable time before in the testing of solid-wall relatively thin vessels.

The reader should refer to the publications mentioned for a more complete application of precompression of layer vessels and a general discussion of the same.

In discussing the transitory and steady thermal stresses in vessels, the author has outlined the mathematical considerations quite completely. However, rarely are pressure vessels built in which the restraints necessary to develop large steady thermal stresses are present. At the operating temperature which is constant over the operating period there are not large thermal stresses present unless different steels are involved in the different layers in which the coefficients of expansion may be different. If, for instance, the inner layer is, say, 18-8 corrosion-resistant steel whose coefficient of expansion is, say, 50 per cent greater than that of the remaining carbon-steel layers then if the temperature changes are great there will be a considerable difference. If, however, the inner layer is of 15 per cent chrome steel in which the expansion factor is similar to that of plain-carbon steel, then the temperature stresses for steady operation are negligible. Transitory thermal stresses of large values are often inflicted by careless operation. In the building of welded solid-wall thick vessels the codes require careful handling during stress-relieving. Some operators ignore the implication of care taken in the building of pressure vessels when careless operations are allowed. No one may design to eliminate the danger. Even ice in a pipe will burst it when careless temperature changes are allowed.

The paper should be studied carefully with the possibility of determining where it may help in the proper application of operating conditions, particularly where solid construction is involved. It will be observed how carefully the writer has avoided discussing the mathematics of the author. He is familiar with the references listed. For pressure vessels, and particularly thick walls, we have the problem of the plus-plus-minus zone of the three-dimen-

[9] Test values are taken from a paper, "Multilayer Construction of Thick Pressure Vessels," Trans. AIChE, vol. 37, 1941, p. 889.

sional system. The compression value may be high and is certainly equal to the pressure on the inner surface of the cylinder. This value is slowly reduced in a solid-wall vessel by a diminishing shearing-stress system until at the outer surface it is zero.

The writer adheres to the maximum -energy theory in this discussion. In the plus-plus-minus zone representing this theory, a relatively large compressive stress reduces the pressure at breakdown of a pressure vessel very materially. This has been thought of as contributing to the test values obtained in the five solid heavy-wall vessels referred to early in this discussion.

Something should be said of the thick-layer vessels which also may be considered as being in the plus-plus-minus zone of the stress system. Why do they not act as the thick solid-wall vessels have acted? The writer does not know the complete answer, but it may be suggested that horizontal shear cannot be transmitted from one layer to the other, and therefore, each separate layer acts in a similar manner to that of a catenary of a suspension bridge made of many wires. Suffice to say that in five-layer vessels tested to destruction, each one has acted as if it were a two-dimensional problem within the plus-plus quadrant of the stress system. The facts, therefore, speak for themselves.

The problem which applies to the casing of deep wells with steel tubes is one which involves the plus-minus zone of the two-dimensional system. These structures are relatively thin and therefore the pressure values for failure are relatively small. This problem has been brought to solution by experiment. The results follow very closely the maximum-energy idea. In this case the collapse stress at failure is always within the elastic range of the steel.

We found in the layer-vessel tests, which are reported herein, and which are on cylinders about four diameters long, that if we applied a Poisson's ratio[10] of about 0.25, the Lamé formula also checked within a small value clear to the ultimate bursting pressure if the maximum-energy idea is used. The engineer's method for obtaining the values of yield and ultimate strength are used in these tests. Most theories for failure do not go to the ultimate strength of steel, but break off at the elastic or yield strengths. Very rarely in pressure vessels do we get failure at this point.

Since the author has referred to the distortion theory of Maxwell-Hencky as applicable and since no experimental data are appended, it is to be hoped that some could be forthcoming to help the problem of the layer-vessel solution, or to contribute to the solution of pressure-vessel designs in general.

A. G. Thailing.[11] Has the author ever encountered the following situation? This primarily concerns itself, not with the usual conception of thermal stresses but with stresses created by the inability of a cylinder wall to equalize its temperature due to excessive internal temperature, creating high-temperature differential wall temperatures. These conditions create very high compression stresses on the internal fibers and high tensile stresses on external fibers.

Is it possible, after the use mentioned, for the vessel upon cooling to diminish in diameter internally and externally? Calculations of stresses due to temperature differentials indicate stresses beyond the elastic limits on the inside fibers, and with the material in a semiplastic condition. There could be a diminishing of inside diameters. Field observations indicate a slight diminishing of outside diameters. Do any of the author's calculations substantiate this condition and has he made any tests or field observations to indicate this condition as possible?

[10] "Poisson's Ratio and a Suggestion for Its Use in Stress Analysis," Proceedings of the ASTM, vol. 24, 1924, p. 1012.

[11] Senior Research Engineer, Carbide and Carbon Chemicals Company, South Charleston, West Va. Mem. ASME.

Author's Closure

The theory presented in the paper does not necessarily apply when extensive yielding is involved, and the true stress pattern in the wall of a vessel approaching bursting is not sufficiently well known to allow a rigorous discussion of the subject. However, the author will offer some comments to the interesting and appreciated discussion of Dr. McLean Jasper.

As pointed out in the discussion, Lamé's formula appears to be able to predict, with sufficient accuracy, both the bursting and the initial yielding pressures of vessels of small wall ratios. The relations developed in the paper are based on the energy of distortion-strength theory. However, for vessels of relatively small wall ratios, it will be shown that comparable values to those based on Lamé's formula can be obtained if the relations presented in the paper are used for predicting the bursting pressure.

Lamé's formula for the bursting pressure as proposed in the discussion is

$$p = S\frac{R^2 - 1}{R^2 + 1} \qquad [44]$$

in which R is the radius ratio of the vessel and S the ultimate strength of the material.

If we assume that Equation [31] of the paper is valid up to the bursting pressure, an average yield point situated between the yield strength and the ultimate strength must be introduced in order to take into account the effect of work-hardening. For simplicity the following formula is proposed

$$p = \frac{2}{\sqrt{3}} \cdot \frac{\sigma_0 + S}{2} \cdot \ln R \qquad [45]$$

since it is known that neither the yield strength σ_0 nor the ultimate stress S alone will predict bursting with sufficient accuracy.

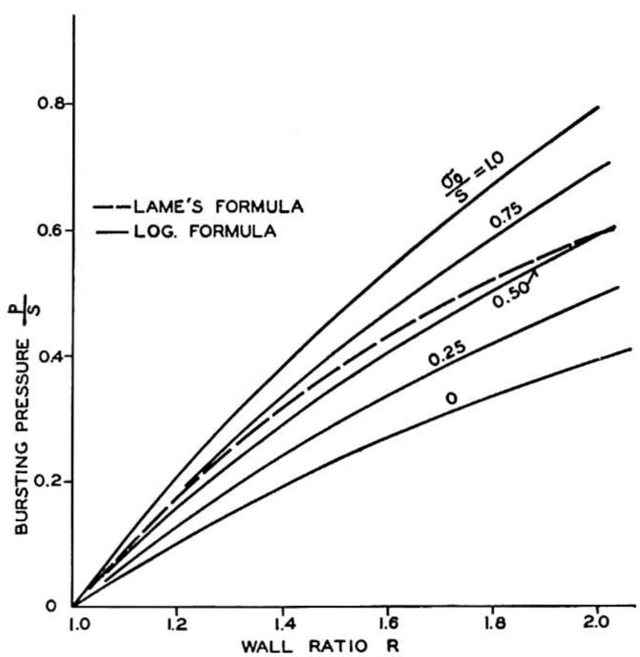

Fig. 11

Fig. 11 of this closure shows that for vessels of small wall ratios and made of steel having a ratio of the yield strength to the ultimate strength situated between 0.5 and 0.8, Relations [44] and [45] will give almost identical results.

From the data of tests carried out with solid-wall cylinders[12] it can be shown that whereas Equation [44] is not valid for wall ratios ranging from 2 to 3, Relation [45] can still be used for predicting the bursting pressure with sufficient accuracy. It is the belief of the author that the lower bursting pressure, observed by Dr. McLean Jasper, for thick solid-wall vessels might be attributed to the influence of the dimensional factor on the mode of yielding and to the yielding pattern which in a layer vessel, may be different from the one occurring in a solid-wall vessel.[13]

As shown in Table 3, herewith, Lamé's formula used for predicting initial yielding at the bore of a vessel of wall ratio larger than 1.17 gives corresponding internal pressures which are greater than those predicted by either the maximum-shear or the energy-of-distortion theories.

It is interesting to note that Dr. McLean Jasper's measurement of the pressure necessary to produce initial yielding at the bore of a layer vessel of wall ratio 1.37 is close to the value predicted by Lamé's formula. Results of tests on heavy solid-wall cylinders of wall ratios 2 to 3, however, reported in the references cited, agree with either the maximum-shear or the energy-of-distortion theories. This fact seems to indicate that a layer vessel starts yielding at a higher internal pressure than a solid-wall vessel. Before reaching a final conclusion on this point, further highly accurate tests on layer and solid-wall vessels are desirable since, owing to the complex mechanism of yielding, an accurate determination of the initial yield is difficult.

Dr. Thailing's question refers to a rather sharp temperature gradient across a heavy-wall cylinder. It is possible, under a great and rapid increase in temperature of the material adjacent to the bore, that yielding under compression occurs at the inner fibers, without appreciable plastic extension of the outer fibers. This yielding at the bore is made easier by the diminution of the yield strength with increasing temperature. During cooling of the vessel, a stress reversal will occur at the inner fibers which may then be at a sufficiently low temperature to resist complete reversal yielding. As a consequence, residual tensile stresses will appear at the bore together with a reduction in diameter. Therefore the external fibers will be submitted to a compressive stress introducing a corresponding reduction of the external diameter. The author has never observed such phenomenon.

TABLE 3 COMPARISON OF FORMULAS FOR PREDICTING INITIAL YIELDING

	Vessel wall ratio				
	1.1	1.17	1.4	2	3
Lamé............ p/σ_0	0.095	0.156	0.325	0.600	0.800
Energy of distortion. p/σ_0	0.100	0.156	0.284	0.433	0.512
Maximum shear.... p/σ_0	0.086	0.135	0.246	0.375	0.443

[12] "Influence of Residual Stress on Behavior of Thick-Wall Closed-End Cylinders," by T. H. Faupel and A. R. Furbeck, contributed by the Industrial Instruments and Regulators Division and presented at the Seventh National Instrument Conference, Cleveland, Ohio, September 9-10, 1952, of The American Society of Mechanical Engineers.

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